Abstract

A gradient DC magnet is designed using 3D magnetic field analysis code. This magnet, referred to as a $B_0$ magnet, is one of the key elements of a time-focusing device for ultra-cold neutrons (UCNs) based on a radio frequency gradient flipper—a so-called rebuncher. The magnet generates a guide and a potential field that interacts with the magnetic moment of UCNs and its main body comprises a C-shaped yoke made from iron. A field gradient is generated by a pole arrangement that includes an anisotropic inter-pole, which causes the fringe field to be uniform in the longitudinal direction even when the pole gap distance changes. The designed magnet has the following properties: (1) a maximum $B$-field of 10 kGauss, decreasing to 2 kGauss at a longitudinal distance of 25 cm, with a gradient of less than 400 Gauss/cm, (2) variations in the fringe field along the $y$ direction is less than 4% over a range of $-3 \text{cm} \leq y \leq 3 \text{ cm}$ at any $z$ position in the spin-flipping region. Tracking simulations show that the $B_0$ magnet is capable of accepting UCNs in the velocity range $2.3 \sim 3.3 \text{ m/s}$.

Keywords: ultra-cold neutron, neutron EDM, magnet, anisotropic inter-pole

1. Introduction

The neutron electric dipole moment (nEDM) has been measured by many researchers [1–3] because the existence of the nEDM opens the door to new physics beyond the Standard Model. Recent measurements using ultra-cold neutrons (UCNs) established an upper limit on the magnitude of the nEDM of $d(n) < 2.9 \times 10^{-26}$ e·cm. To improve the precision of such measurements, it is important to increase the UCN density in the storage bottle and decrease the bottle size.

Measurement of the nEDM has been proposed at the Japan Proton Accelerator Research Complex (J-PARC), where high intensity pulsed proton beam accelerators are available [4]. The R&D phase is already underway and is expected to be finished by JFY 2014. A commissioning and physics run will then be carried out. For this experiment, Shimizu et al. [4, 5] have proposed a novel method of increasing the density of the stored UCNs by the use of a spallation neutron source. This includes a device known as a rebuncher that is often used in charged-particle accelerators to squeeze the bunch length by rotating the bunch in longitudinal phase space [6–8].
This method can be applied to the nEDM experiment in the following manner (Fig. 1). UCN bunches produced by
the superthermal method are transported using a neutron guide to a UCN bottle. Each bunch is rotated in longitudinal
phase space ($\Delta t - \Delta v$) by the rebuncher which is located between the source and the bottle (Fig. 2). The bunch is focused
on the UCN bottle and enters it while a shutter located at the entrance to the bottle is open. The shutter is then closed.

By using the rebuncher, the UCN density in the bottle can be increased by a factor of 5 compared to the case of
simple transport with a proton beam power of 20 kW at 2 Hz [9].

Several methods for time focusing neutron bunches have been investigated to date, including travelling magnetic
potentials [10, 11], moving grating optics [12], and radio frequency gradient flippers [13]. The method used in
the present study is a modified version of the radio frequency gradient flipper method. This involves flipping neutron
spins using an oscillating magnetic field in the presence of a static magnetic field with a gradient in the longitudinal
direction.

In a magnetic field gradient ($B$), a neutron with a magnetic moment ($\mu$) is subjected to the following conservative
force,

$$ F(r) = -\nabla (\mu \cdot B). $$

(1)

Since the magnet has a finite size, the magnetic field is zero far from the inlet and outlet. In such a situation, if the
eutron spin is not flipped, the velocity of the neutron at the outlet of the magnet is the same as that at the inlet.

However, if the spin is flipped in the magnetic field, the net velocity of the neutron is changed because the force is no
longer conservative; the neutron receives the following amount of kinetic energy at the magnet outlet,

$$ K = -\int_{-\infty}^{+\infty} F(z)dz, $$

(2)

$$ = -\mu \int_{-\infty}^{z_f} \partial B_T(z) / \partial z dz + \mu \int_{z_f}^{+\infty} \partial B_T(z) / \partial z dz, $$

(3)

where $z_f$ is the position along the beam axis where the spin is flipped and $B_T(z)$ is the magnetic field transverse to the
beam axis. If it is assumed that $B(\pm \infty) = 0$, Eq. (3) gives

$$ K = -2\mu B_T(z_f). $$

(4)

It should be noted that the kinetic energy depends only on the position where the spin is flipped if the magnetic field
distribution at that position is a single-valued function. So, by controlling the position of spin flip, the net change in
velocity can be controlled.

By a suitable choice of the magnet field configuration, fast neutrons can be made to decelerate and slow neutrons
to accelerate. This is achieved as follows. The arrival time of UCNs in the region of the magnetic field gradient
depends on their velocity. An rf field is applied to the UCN bunch when it passes through this region. This causes

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**Figure 1:** Schematic diagram of UCN transportation system for nEDM measurement and behavior of UCN bunches (shaded ellipses) without (upper figure) and with (lower figure) the rebuncher system.

**Figure 2:** Schematic illustration of the behavior of a UCN bunch in longitudinal phase space under the influence of the rebuncher.
neutron spins to flip based on the adiabatic fast passage NMR method [14]. The flipping frequency is determined by the static magnetic field produced by the gradient magnet. By tuning the flipping frequency to the arrival time at the gradient magnet, the flipping positions can be determined as a function of UCN velocity. After the UCN bunch passes through the gradient magnet and drifts for a certain distance in free space, it becomes focused in the $t$ dimension (time focusing).

The rebuncher differs from a conventional gradient flipper [13] with regard to four main points: (1) it is designed to be applied to UCN bunches, (2) the rf frequency is varied during passage of the UCN bunch through the gradient magnetic field region, (3) a much larger field gradient is required than for a conventional flipper, and (4) longitudinal-phase-space rotation is performed. A demonstration experiment to verify the basic rebuncher principles is currently being prepared, and will be carried out using BL05 [15] of the Materials and Life Science Experimental Facility (MLF) at J-PARC. The layout of the rebuncher is shown in Fig. 3. The UCNs are produced by a Doppler shifter which decelerates very cold neutrons originating from a beam port on BL05; they are then transported through a Ni-coated glass guide tube over a distance of about 6 m. After the UCNs pass through the rebuncher system, they are detected using a $^3$He counter.

A 3-dimensional (3D) longitudinal-gradient magnet was designed for the demonstration experiment. This is also the prototype magnet for the proposed nEDM experiment at J-PARC. This magnet is unique in that it is equipped with an anisotropic inter-pole, which was invented by Iwashita et al. [16].

The anisotropic inter-pole allows the creation of a magnetic field gradient in a constant gap magnet. Although this offers several advantages [16, 17], the actual magnet has not yet been fabricated. One advantage is that different regions of the fringe field are expected to have similar shapes when a constant pole width is used. Usually in electromagnets, to produce a field gradient, the gap distance of the iron pole is varied along the desired gradient direction. To obtain acceptable fringe field conditions, the pole width should be increased by $1.5 \sim 2.0$ times of the gap distance in addition to an effective field width. This results in large pole widths when the gap distance is large. However, if an inter-pole is used, the gap distance can be kept constant and the magnet can be designed with a constant pole width. This means that a magnet designed using 2D calculations will have almost the same field profile as one designed using 3D calculations, greatly reducing the required design time. In addition, magnets with compact sizes can be designed.

In this paper, the design of a 3D magnet, the calculated field profiles, and the results of a UCN tracking simulation for time focusing are described. The designed magnet is referred to as a $B_0$ magnet following the convention of NMR.

### 2. $B_0$ magnet requirements

A schematic view of the gradient flipper system in the rebuncher is shown in Fig. 4. The system consists of the $B_0$ magnet and an rf coil. The $B_0$ magnet acts in a similar way to the linear-sweep static magnetic field in an NMR apparatus. The rf coil is driven by an rf amplifier and generates an oscillating magnetic field whose frequency is swept synchronizing with the UCNs time-of-flight. The rf coil consists of a copper plate wrapped around the neutron guide through which the UCNs drift. The neutron guide is a 1-cm-thick glass plate coated by nickel and has an inner aperture of 3 cm in the $x$-direction and 6 cm in the $y$-direction. Since both the guide and the rf coil must be inserted
into the gap space of the $B_0$ magnet, the gap distance should be more than 10 cm. Also, since the rf coil needs to be connected to a resonator system and a power line, the yoke should be C-shaped.

Figure 4: Schematic view of the gradient flipper system. The upper left figure shows a top-down view of the system. The rf coil winding the neutron guide tube is located within the $B_0$ magnet gap. UCN bunches travel from left to right along the $z$-axis and the $y$-axis represents the vertical direction. The upper right figure shows a cross section of the UCN guide tube and the rf coil. The lower figure shows the $B_x$ components generated by the $B_0$ magnet as a function of $z$.

The absolute strength of the $B_0$ field determines the net change in the UCN velocity. The $B_0$ magnet is required to generate a field that varies from 10 kGauss to 2 kGauss within a distance of 25 cm in $z$-direction with an average gradient of 320 Gauss/cm. To ensure that adiabatic conditions are in effect, the rate of change of the $B_0$ field from the viewpoint of travelling UCNs must be significantly smaller than the spin flipping speed, so that a lower field gradient is better. The required field gradient can be quantitatively calculated as follows, based on quantum mechanics [18, 19]. The neutron spin flipping probability is given by

$$p = 1 - \sin^2 \left( \frac{\pi}{2} \sqrt{1 + k^2} \right).$$

(5)

where $k$ is the adiabaticity parameter of the rotating frame and is expressed by

$$k(v, z) = \frac{\omega_1}{\omega_f}.$$

(6)

Here, $\omega_1 \sim \gamma_n B_1$ is the Larmor precession rate in the rotating frame and $\omega_f$ is the field rotation rate with respect to the $z$-axis. Then, $\omega_f$ is given by

$$\omega_f = \frac{1}{B_1} \frac{\partial B_0}{\partial z} v_n.$$

(7)

Here, $v_n$ is the neutron velocity and $B_1$ and $B_0$ are the amplitudes of the oscillating and static magnetic fields, respectively. Substitution of (7) into (6) yields

$$k = \frac{\gamma_n B_1^2}{v_n B_0 / \partial z}.$$

(8)

where $\gamma_n$ is the neutron gyromagnetic ratio. The experimental conditions for the demonstration are as follows. The amplitude of the oscillating field is determined by the maximum power of the rf amplifier. In the experiment, a
Figure 5: Design schematics for the $B_0$ magnet using third angle projection. Since the magnet is symmetric with respect to the $y - z$ plane, only one half of it is drawn. The upper figure shows a side view; the UCN beam direction is along the $z$-axis. The lower-left figure shows a front view and it can be clearly seen that the yoke is C-shaped. The lower-right figure shows the bottom view. The anisotropic inter-pole is inserted inside the gap between the main poles. The effective gradient region where spin flipping occurs is $-12.5 \, \text{cm} \leq z \leq 12.5 \, \text{cm}$.

1 kW amplifier will be used which can generate $B_1 \sim 7$ Gauss. As will be shown later, the velocity range of the rebunched UCNs is 2.3 to 3.3 m/s at the source position. Substitution of the conditions $B_1 = 7$ Gauss, $v_n = 3.3$ m/s, $\partial B_0/\partial z = 400$ Gauss/cm, and $\gamma_n = 1.83 \, \text{s}^{-1} \cdot \text{T}^{-1}$ into (8) yields $k = 6.8$, and substituting this into (5) yields $p = 0.980$. This spin-flipping probability is sufficient to demonstrate longitudinal-phase-space rotation.

The experimental requirements can be summarized as follows.

- $B$-field changes from 10 kGauss to 2 kGauss within 25 cm in $z$ direction,
- Gradient along beam axis: $dB_0/dz \leq 400$ Gauss/cm,
- Gap distance: $>10$ cm,
- Yoke shape: C-shaped,
- Effective field region: 6 cm×3 cm.

3. $B_0$-magnet design

Design schematics for the $B_0$ magnet are shown in Fig 5. The UCN bunch is transported along the $z$-axis and the main guiding field is generated in the $x$-direction. The yoke is made of iron and the gradient is generated by the pole.
shape shown in the lower-right figure in Fig. 5. To simplify the production process, the design uses many straight elements. The minimum gap distance is 100 mm and the pole width and length are 200 mm and 310 mm, respectively.

The anisotropic inter-pole is inserted within the gap. Detailed design schematics of the anisotropic inter-pole are shown in Fig. 6. The inter-pole consists of magnetic (iron) and non-magnetic (aluminum) plates that are stacked in the field-gradient direction. The pole is divided into a weak- and strong-field region. The packing factor is set to a small value of 0.33 in the weak-field region and, to avoid magnetic saturation of the iron plates, to a larger value of 0.67 in the strong-field region. Each of the laminated plates is given a trapezoidal shape to ensure that the overall shape of the inter-pole matches that of the main pole.

4. B-field calculations

The magnet was designed using the 3D analysis code TOSCA/Opera3D [20]. Figure 7 shows the calculated 3D model where the color contour map represents the magnitude of the B-field on the yoke surface. The coil is not shown in this figure so that the B-field on the pole surface can be seen. In the calculations, the total coil current, which is the coil current multiplied by the number of turns, is 47767.5 A-Turns for each coil. The calculated $B_x$-field (left) and $B_z$-field (right) on the median plane ($x=0$) are plotted as a function of $z$ in Fig. 8. In the upper part of each plot, the pole and return yoke shapes are drawn on the same scale as the $z$ axis. The meaning of the symbols is indicated in the plots. The rf-field (spin-flipping) region is $\pm 12.5 \text{ cm} < z < \pm 12.5 \text{ cm}$. It is found that the $B$-field decreases from 1 T to 0.2 T within 25 cm along the $z$-axis and the gradient is smaller than 350 Gauss/cm in the spin-flipping region.

The calculated $B_z$ and field gradient as a function of $z$ on the other planes are shown in Fig. 9 and 10. It can be seen that there are no large differences in the $B_z$ distribution compared to that for $x = 0$ cm. The gradient is less than 350 Gauss/cm at almost all positions and less than 400 Gauss/cm over the entire region. This distribution thus satisfies the requirements for demonstrating the rebuncher principle.

Figure 11 illustrates the uniformity and similarity of the fringe fields in the designed magnet. Figures 11(a), (b), (c), and (d) show normalized values of $B_x$ as a function of $y$ for $x = 0$, 0.5, 1.0, and 1.5 cm, respectively; the normalized $B_x$ is expressed as $B_x(x, y, z)/B_x(x, 0, z)$. In each plot, the closed circles, closed squares, closed triangles, inverted triangles, open circles, and open squares represent the normalized $B_x$ at $z = -12.5, -7.5, -2.5, 2.5, 7.5,$ and $12.5 \text{ cm}$, respectively. The differences between these curves in the range $-12.5 \text{ cm} < z < 12.5 \text{ cm}$ and across the inner width of the neutron guide ($-3 < y < 3 \text{ cm}$) are smaller than 1.5% for all $x$ positions, indicating that good similarity is achieved.
5. Summary and Discussion

A longitudinal gradient ($B_0$) magnet has been designed using 3D magnetic field analysis code. This magnet is a key component of a UCN rebuncher based on a radio frequency gradient flipper. The rebuncher differs from conventional flippers [13] in that (1) it is specifically designed for UCNs, (2) the frequency of the oscillation field is swept, (3) a large field gradient is required for the $B_0$ magnet and (4) longitudinal-phase-space rotation is performed. The $B_0$ magnet must be capable of generating a large field gradient varying from 10 kGauss to 2 kGauss within a distance of 25 cm, while satisfying the adiabatic requirement of less than 400 Gauss/cm. Unlike normal accelerator magnets such as quadrupole magnets or combined function magnets [21], the designed magnet includes a unique anisotropic inter-pole, which maintains a similar fringe field at all $z$-positions where the gap distance is different. This is highly advantageous in situations where the gap distance is greatly varied to produce a large field gradient.
The designed magnet has the following field characteristics:

1. A maximum $B$-field of 10 kGauss is achieved and it decreases to 2 kGauss within a longitudinal distance of 25 cm, while maintaining a gradient of less than 400 Gauss/cm.
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2. The fringe field is highly uniform along the y direction, with variations of less than 4% over the range \(-3 \text{ cm} \leq y \leq 3 \text{ cm}\) for any \(z\) position in the spin-flipping region.

These results indicate that the anisotropic inter-pole operates as expected and the field requirements for a UCN rebuncher can be achieved.

To confirm the performance of time focusing, a UCN tracking simulation was carried out using the calculated \(B\)-field map. In this simulation, all UCNs are assumed to travel parallel to the \(z\)-axis for simplicity. The coordinate system in the simulation was set to coincide with that of the magnet. Figure 12 shows time of flight (TOF) differences as a function of \(z\) from a UCN which travels along the \(z\)-axis with a constant velocity of 3.67 m/s. The solid and dashed curves correspond to the cases with and without the rebuncher, respectively. The trackings with the rebuncher are performed for UCNs with velocities of 2.3, 2.5, 2.7, 2.9, 3.1, or 3.3 m/s travelling in the \(z\)-direction along paths with \(x = 0 \text{ cm}\) and \(y = -3, -2, -1, 0, 1, 2, \text{ or } 3 \text{ cm}\). It can be seen that while the TOF difference at \(z = 400 \text{ cm}\) is 0.85 sec without the rebuncher, efficient time focusing occurs with the rebuncher.

Figure 13 shows the TOF differences at the outlet of the rebuncher (\(z = 405 \text{ cm}\)) as a function of the \(y\)-position of the UCN path. It can be seen that the spread in TOF differences is about 0.01 sec. This corresponds to the longitudinal-bunch spread after time focusing and includes the fringe field region. This spread is sufficiently small to establish the validity of the rebuncher approach. The results indicate that the \(B_0\) magnet can accept UCNs with velocities in the range 2.3 m/s to 3.3 m/s (\(\Delta v = 1.0 \text{ m/s}\)). It should be noted that, in practice, UCNs undergo reflection at the inner walls of the guide tube and the non-uniformity of the fringe field is averaged, so the actual spread in TOF differences is expected to be smaller than the value obtained here.

In the demonstration experiment, UCNs with velocities of 2.3–2.9 m/s (\(\Delta v = 0.6 \text{ m/s}\)) at the entrance to the \(B_0\) magnet will be rebunched. The decrease of the upper velocity limit is due to limitations imposed by the tuning range, the frequency sweeping speed of the rf coil and insertion of a vertical guide downstream of the Doppler shifter (length: 30 cm) where the UCNs velocity are decreased by gravity. Under these conditions, if the pulse duration of the UCN bunch is 0.5 sec without the rebuncher after it has travelled a distance of 6.0 m from the outlet of the Doppler shifter to the detector, this will be reduced to 0.01 sec with the rebuncher. The fringe field effect noted at the preceding paragraph is not included in this duration. Inclusion of this effect yields a duration of 0.014 sec or less. Therefore, an increase of 35 times or more in the instantaneous UCN intensity is expected for this initial velocity range.

In the proposed nEDM experiment to be carried out at J-PARC, UCNs with initial velocities of 4 m/s to 7 m/s will be rebunched [9]. In a practical \(B_0\) magnet, it is necessary that the pole width and gap size should be larger than those for the designed magnet because the cross section of the UCN guide tube is 30 cm×12 cm. However, the field
gradient and the field strength will be the same as in the designed magnet, so that a similar C-shaped iron yoke will be used in combination with the anisotropic inter-pole.

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