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1S_0 pairing correlation in symmetric nuclear matter with Debye screening effects

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Abstract

The 1S_0 pairing of symmetric nuclear matter is discussed in the framework of relativistic nuclear theory with Dyson–Schwinger equations (DSEs). The in-medium nucleon and meson propagators are treated in a more self-consistent way through meson polarizations. The screening effects on mesons due to in-medium nucleon excitation are found to reduce the 1S_0 pairing gap and shift remarkably the gap peak to low density region.

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Compared with nonrelativistic framework, the relativistic nuclear theory can successfully describe the saturation at normal nuclear density. The basic meson exchange is normally considered as the nuclear saturation mechanism. The original σ – ω theory of quantum hadrodynamics model (QHD) developed by Walecka et al. and its various extensions have been widely used to discuss the properties of finite nuclei and nuclear matter [1–4].

Superfluidity of strongly interacting Fermi system is very important for understanding the properties of finite nuclei, such as the dramatic reduction of the moments of inertia in rotating nuclei or the energy gap in the spectra of many even–even nuclei [5,6]. The existence of superfluidity may also affect the dynamical

and thermal evolution mechanism of neutron stars because it is closely related to the emission of neutrino and cooling of neutron-rich matter. It is also argued that the superfluidity of nuclear matter can lead to the glitches of astronomy phenomena and attracts much attention in contemporary physics [7].

Although there are many works in the literature addressing the superfluidity of nuclear matter, the main results are obtained from the nonrelativistic nuclear theory and no definite conclusion can be made yet. We noted that since Kucharek and Ring [8] first derived the relativistic Hartree–Fock–Bogoliubov equation by using Green function method and the Gor'kov factorization analogously to nonrelativistic BCS theory, it was found that the superfluidity gap value is about three times larger than the “standard” value obtained with the nonrelativistic Gogny force [9]. To improve the description of superfluidity with relativistic

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nuclear theory, various approaches have been investigated, such as using external potential as input or various cut-offs of the integration momentum [10–12]. To our knowledge, there is even no definite result about the superfluidity gap (the gap value and peak position) for symmetric nuclear matter within nonrelativistic or relativistic nuclear theory up to now. However, it is widely accepted that the 1S_0 gap values at normal nuclear density should be very small.

Essentially one cannot expect that the softness of equation of state (EOS) describing the bulk property of nuclear matter is directly related with the superfluidity property of nuclear matter. For example, although the nonlinear σ - ω model with the possible embarrassing negative coupling constants b and c (which, in principle, lead to instability of nuclear system at high density scenario) in σ self-interaction terms $b\sigma^3 + c\sigma^4$ can give a very soft EOS with mean field theory (MFT) [7], the gap behavior is similar to that in the original version by using frozen meson propagators [8,12].

Theoretically, since it is difficult to make low energy calculations directly with quantum chromodynamics (QCD), one has to work with effective theories. As an effective theory, QHD-I model (and its extensions) has been widely used to discuss the effective meson masses under extreme environment in the past [13–16]. In principle, when one discusses the in-medium properties of nucleons and mesons, one has to take into account the back-interactions of nucleons with in-medium mesons. Therefore the *resummed* nucleon and meson propagators would form a closed set of coupled equations and should be solved simultaneously. With this self-consistent way, a softer EOS with an acceptable compression modulus \mathcal{K} in dealing with realistic nuclear matter can be obtained [13,17]. In the spirit of mean field theory, the exchanged mesons in determining nucleon propagator are not free but medium dependent. Their masses should be determined together with the nucleon mass through Dyson–Schwinger equations self-consistently, as indicated by Fig. 1.

It would be very interesting to analyze the in-medium effect of mesons on the superfluidity of nuclear matter in the framework of relativistic nuclear theory. In superfluidity state and with QHD-like Lagrangian, Dyson–Schwinger equations for the nucleon and meson propagators as indicated in Fig. 1 and the energy gap equation as indicated in Fig. 2 form a new

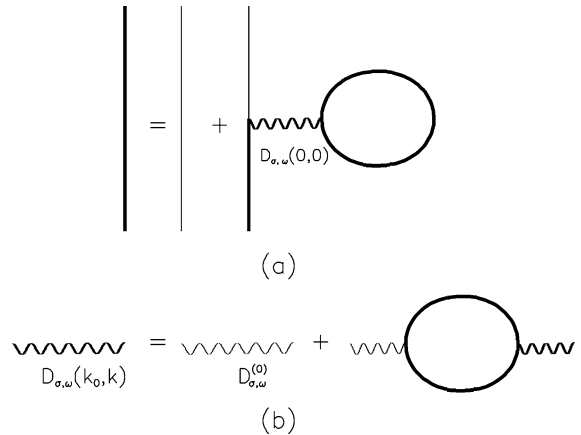


Fig. 1. Diagrammatic representations for the propagators of in-medium nucleons (a) and mesons (b).

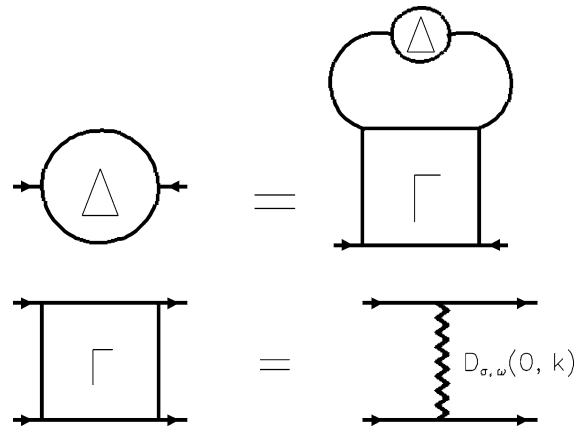


Fig. 2. Diagrammatic representations for the gap equation and the interaction kernel in instantaneous approximation with *screened* meson propagator.

closed set of coupled equations. The in-medium meson propagators D instead of the normally used bare ones $D^{(0)}$ will affect the kernel in the BCS gap equation. We will see below that the gap behavior with in-medium meson propagators is quite different from that with bare ones. We found that the polarization effects leading to screening have been widely discussed in the nonrelativistic framework of nuclear theory [18–23], this problem has not yet been addressed before in terms of relativistic field theory. In this Letter, we want to discuss the effects of in-medium effective potential for nucleon–nucleon interaction on the super-

fluidity due to screening by using the original renormalizable formalism of σ - ω model.

Let us start with the four-dimensional gap equation by using the standard Nambu–Gor’kov formalism in the ladder approximation of the meson exchanges as indicated by Fig. 2 [23–25],

$$\Delta^*(K) = i \int \frac{d^4 P}{(2\pi)^4} \langle P|\Gamma|K \rangle F^+(P), \quad (1)$$

where K is the four momentum $K = (k_0, \mathbf{k})$, $\langle P|\Gamma|K \rangle$ is the interaction kernel and

$$F^+(K) = \frac{-\Delta^*(K)}{[k_0 - \varepsilon(K) + i\eta][k_0 - \varepsilon(-K) - i\eta] - |\Delta(K)|^2}$$

is the Nambu–Gor’kov anomalous propagator with $\varepsilon(K) = E_k - E_{k_f}$ being the quasi-particle energy above Fermi-surface. For 1S_0 pairing, the gap equation can be reduced to [8,10]

$$\Delta(p) = -\frac{1}{8\pi^2} \int_0^\infty \bar{v}_{pp}(p, k) \times \frac{\Delta(k)}{\sqrt{(E_k - E_{k_f})^2 + \Delta^2(k)}} k^2 dk, \quad (2)$$

with $E_k = E_k^* + \lambda$ and $E_k^* = \sqrt{M_N^{*2} + \mathbf{k}^2}$. The quantity λ related with the baryon current is obtained from the tadpole self-energy of nucleon propagator with in-medium vector meson in Fig. 1

$$\lambda = \frac{g_\omega^2}{\bar{m}_\omega^2} \frac{\gamma}{2\pi^2} \int_0^\infty v_k^2 k^2 dk, \quad (3)$$

where $\gamma = 4$ is the spin–isospin degeneracy factor for symmetric nuclear matter, and v_k^2 is the BCS occupation number

$$v_k^2 = \frac{1}{2} \left(1 - \frac{E_k - E_{k_f}}{\sqrt{(E_k - E_{k_f})^2 + \Delta^2(k)}} \right). \quad (4)$$

The interaction kernel $\langle P|\Gamma|K \rangle$ in our treatment is approximated by the in-medium meson propagators instead of the bare ones. We use the static (instanta-

neous) approximation by neglecting the retarding effects [8]. Since the meson propagators $D_{\sigma,\omega}(0, \mathbf{k})$ with the vanishing temporal component of four-momentum are now medium dependent, the Debye screening effects will play an important role in the in-medium particle–particle interaction potential

$$\begin{aligned} \bar{v}(\mathbf{p}, \mathbf{k}) &= \langle \mathbf{p}s', \tilde{\mathbf{p}}s' | V | \mathbf{k}s, \tilde{\mathbf{k}}s \rangle - \langle \mathbf{p}s', \tilde{\mathbf{p}}s' | V | \tilde{\mathbf{k}}s, \mathbf{k}s \rangle \\ &= \mp \frac{M_N^{*2}}{2E^*(k)E^*(p)} \\ &\quad \times \frac{\text{Tr}[\Lambda_+(\mathbf{k})\Gamma\Lambda_+(\mathbf{p})\gamma^0\mathcal{T}^+\Gamma^+\mathcal{T}\gamma^0]}{(\mathbf{k} - \mathbf{p})^2 + m_D^{*2}}, \quad (5) \end{aligned}$$

where $\Lambda_+(\mathbf{k}) = (\not{k} + M_N^*)/(2M_N^*)$ is the projection operator of the positive energy solution and $\mathcal{T} = i\gamma^1\gamma^3$ is the time reversal operator. The tilde type-script “ \sim ” means the Gor’kov time reversal state and Γ is the corresponding interaction vertex of σ/ω with nucleons. The assymetrized matrix elements $\bar{v}_{pp}(p, k)$ in the gap equation Eq. (2) for 1S_0 pairing is obtained through the integration of $\bar{v}(\mathbf{p}, \mathbf{k})$ over the angle θ between the three-momenta \mathbf{k} and \mathbf{p}

$$\bar{v}_{pp}(p, k) = \int \bar{v}(\mathbf{p}, \mathbf{k}) d\cos\theta. \quad (6)$$

The effective nucleon mass M_N^* is determined by the relevant mass gap equation through tadpole self-energy of nucleon propagator with scalar meson

$$M_N^* = M_N - \frac{g_\sigma^2}{\bar{m}_\sigma^2} \frac{\gamma}{2\pi^2} \int_0^\infty \frac{M_N^*}{E_p^*} v_p^2 p^2 dp + \Delta M_{N,\text{vac}}^*, \quad (7)$$

with $\Delta M_{N,\text{vac}}^*$ being the vacuum fluctuation contribution

$$\begin{aligned} \Delta M_{N,\text{vac}}^* &= \frac{g_\sigma^2}{\bar{m}_\sigma^2} \frac{1}{\pi^2} \left[M_N^{*3} \ln\left(\frac{M_N^*}{M_N}\right) - M_N^2 (M_N^* - M_N) \right. \\ &\quad \left. - \frac{5}{2} M_N (M_N^* - M_N)^2 - \frac{11}{6} (M_N^* - M_N)^3 \right]. \quad (8) \end{aligned}$$

The polarization tensors $\Pi_{\sigma,\omega}(k_0, \mathbf{k})$ determining the in-medium σ and ω propagators are calculated by using Dyson–Schwinger equation as shown in Fig. 1. For brevity, here we list only the sigma meson self-

Table 1

The parameters determined at normal nuclear density with $k_f^0 = 1.42 \text{ fm}^{-1}$ in MFT, RHA and our self-consistent approach (labeled as SA) with $M_N = 939 \text{ MeV}$, $m_\omega = 783 \text{ MeV}$ and $m_\sigma = 550 \text{ MeV}$. We show also the compression modulus \mathcal{K} (in MeV), the *medium dependent* coupling constants (determining EOS) $C_s^2 = g_\sigma^2 M_N^2 / \bar{m}_\sigma^2$ and $C_v^2 = g_\omega^2 M_N^2 / \bar{m}_\omega^2$, the maximum of gap value Δ_f^m (MeV), the peak position k_f^m (fm^{-1}) and the “gap value” $\Delta(0)$ (MeV) at $k_f = 0$

	g_σ^2	g_ω^2	C_s^2	C_v^2	\mathcal{K}	M_N^*/M_N	Δ_f^m	k_f^m	$\Delta(0)$
MFT	91.64	136.2	267.11	195.87	545.43	0.556	9.4	1.0	1.94
RHA	62.89	79.78	183.31	114.73	468.24	0.718	8.3	1.0	0.36
SA	48.90	53.40	123.17	66.078	338.00	0.794	1.9	0.5	0

energy explicitly,

$$\begin{aligned}
\Pi_\sigma(k) &= \frac{3g_\sigma^2}{2\pi^2} \left[3M_N^{*2} - 4M_N^* M_N + M_N^2 \right. \\
&\quad - (M_N^{*2} - M_N^2) \int_0^1 \ln \frac{M_N^{*2} - x(1-x)k^2}{M_N^2} dx \\
&\quad - \int_0^1 (M_N^2 - x(1-x)k^2) \\
&\quad \left. \times \ln \frac{M_N^{*2} - x(1-x)k^2}{M_N^2 - x(1-x)k^2} dx \right] \\
&\quad + \frac{g_\sigma^2}{\pi^2} \int_0^\infty \frac{v_p^2 p^2 dp}{E_p^*} \left[2 + \frac{k^2 - 4M_N^{*2}}{4p|\mathbf{k}|} (a + b) \right], \quad (9)
\end{aligned}$$

with

$$a = \ln \frac{k^2 - 2p|\mathbf{k}| - 2k_0 E_p^*}{k^2 + 2p|\mathbf{k}| - 2k_0 E_p^*},$$

$$b = a(E_p^* \rightarrow -E_p^*).$$

The effective masses \bar{m}_σ and \bar{m}_ω in Eqs. (3) and (7) are determined by the corresponding polarization tensors with vanishing four-momentum transfer, and the Debye screening masses m_σ^* and m_ω^* in asymmetricized matrix elements $\bar{v}_{pp}(p, k)$ are determined by the pole positions of corresponding spacelike propagators $D_{\sigma,\omega}(0, \mathbf{k})$ due to taking the static approximation [16]. For example, the transverse mode screening mass m_ω^* is determined self-consistently by

$$m_\omega^{*2} = m_\omega^2 + \Pi_\omega^T(0, i m_\omega^*), \quad (10)$$

where Π_ω^T is the transverse part of polarization tensor $\Pi_\omega^{\mu\nu}(k_0, \mathbf{k})$. In principle, the longitudinal mode

screening mass is different from the transverse mode one for in-medium vector meson due to the broken Lorentz invariance. However, neglecting this little difference does not affect the qualitative result in realistic numerical calculation.

Considering the in-medium meson effects on the property of nuclear matter, one should refix the parameters in the model. Noting that the effect of superfluidity on the bulk property is negligible, we fix the parameters by normal nuclear matter with saturation condition of binding energy $e_n = -15.75 \text{ MeV}$ at the normal nuclear density with $k_f^0 = 1.42 \text{ fm}^{-1}$. The relevant parameters are listed in Table 1.

The remaining task will be the numerical solution of the coupled equations indicated by Figs. 1 and 2. It should be noted that the relativistic kinematic factors guarantee the convergence of the gap equations such as Eq. (2) for the relativistic nuclear theory and lead to a definite result for the gap. In principle, the momentum integration upper bound in relevant equations such as Eq. (2) is infinity. However, a concrete upper bound must be used to give a numerical result by solving the gap integral equation. Strictly speaking, the gap value should not be sensitive to the adopted momentum upper bound and the sensitivity of momentum cut-off on the gap has been analyzed in such as in Refs. [8,10], which can be also reflected by the gap function indicated by Fig. 3(b). A concrete and large enough momentum upper bound $\Lambda_p = 20 \text{ fm}^{-1}$ has been used in this Letter for the description of screening effects. To focus on the characteristic due to polarization effects, the σ - ω mixing effects has been neglected, which will not affect the result qualitatively although it deserves further study.

The numerical results of the superfluidity gap equations are shown in Fig. 3. In the upper panel (a), we indicate the gap curves $\Delta(k_f)$ versus Fermi momentum

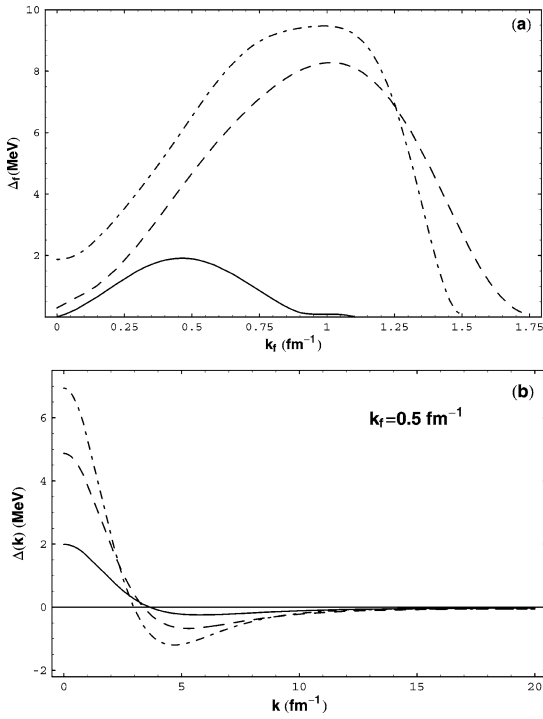


Fig. 3. (a) The pairing gap Δ_f at the Fermi surface as a function of density characterized by the Fermi momentum k_f . (b) The gap function $\Delta(k)$ as a function of momentum k for fixed Fermi momentum $k_f = 0.5 \text{ fm}^{-1}$. The dot-dashed lines correspond to the result obtained by MFT and dashed lines correspond to RHA [10], while the solid lines correspond to our self-consistent resummation approach.

k_f , and the gap functions $\Delta(k)$ at given Fermi surface momentum $k_f = 0.5 \text{ fm}^{-1}$ are shown in Fig. 3(b).

Compared with the previous superfluidity results of relativistic nuclear theory in the literature, the gap value we obtained is very small and the peak position is shifted to the low density region remarkably. As mentioned in the introduction, this interesting result is not due to the softer EOS but much attributed to the screened effective particle–particle interaction potential as indicated by Fig. 4. The key point is that the σ and ω propagators in the gap equations are not bare but in-medium ones determined self-consistently by Dyson–Schwinger equations. The effective nucleon–nucleon interaction potential with Debye screening of in-medium mesons leads to the change in the interaction force range. Different from the scenario of bare meson propagators used in the gap equation, the particle–particle potential is more sensitive to density,

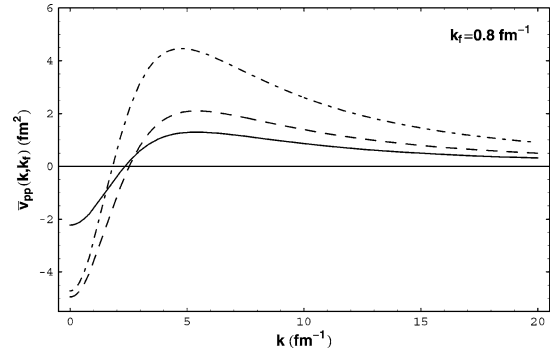


Fig. 4. Asymmetrized matrix elements $\bar{v}_{pp}(k, k_f)$ in momentum space at the Fermi momentum $k_f = 0.8 \text{ fm}^{-1}$. The line-styles are similar to those in Fig. 3.

which can be understood from the corresponding attractive and repulsive force range changes for different densities characterized by Fermi momentum k_f (not displayed obviously in Fig. 4).

It is clearly demonstrated in Fig. 3 that our self-consistent approach reduces the difference between the nonrelativistic and relativistic theories about the maximum gap value and the peak position of 1S_0 pairing correlation. The significant improvement by the self-consistent resummation approach for the particle–particle interaction leading to pairing is reflected on two aspects: one is at the saturation density with $k_f^0 = 1.42 \text{ fm}^{-1}$, the other is at $k_f = 0$. The improvement at $k_f = 0$ is crucial by noting that the MFT and RHA approaches with frozen meson propagators give unrealistic *nonzero* gap values $\sim 1.94 \text{ MeV}/0.36 \text{ MeV}$ [10].

Summarizing, with a set of more self-consistent equations for the resummed in-medium nucleon and meson propagators and superfluidity gap by Dyson–Schwinger Green function approach, we have investigated the 1S_0 pairing correlation in symmetric nuclear matter and compared our results with those obtained by MFT and RHA approaches. The Debye screening effects of in-medium meson propagators can reduce significantly the superfluidity gap value, while the gap peak position is shifted remarkably to low density region.

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