Analysis Of Credit Sale Risk Of Emerging Market Product

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Abstract

Credit transactions between enterprises are getting increasingly common, and credit sale (Credit Sale, CS) risk management has become an important part of modern business activity and management. Currently, the accounts in arrears between enterprises have become a very serious problem, due to the imperfect internal management mechanism and lack of effective credit sale risk management system. In order to make products quickly enter market to win time and achieve scale effect, credit sale has inevitably become the choice of emerging market enterprise. Based on this, we carry out the study to discuss the credit sale risk of emerging market enterprise. Firstly, we do the costs analysis of product credit sale of the emerging market enterprise, and establish a theoretical relationship between the accounts receivable holdings and the credit costs. Secondly, the general analytical expression of optimal accounts receivable holdings and the lowest credit costs is deduced; further, the concept and calculation method of risk-adjusted return on credit sale is proposed. Finally, based on the characteristics of emerging markets, we propose the concept of “credit sale risk capital”, which should be reserved in order to avoid the unexpected losses caused by the product credit sale risk, thus making the return of credit sale match the risk of emerging market enterprise.

Keywords: emerging market enterprise; credit sale; credit sale risk; risk costs

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1. Introduction

Credit sale is a powerful means of competition when the enterprise faces surplus commodities, competition, and few profit, however it may also bring a greater risk of loan losses. In the developed market economy countries, the credit sale risk management is considered to be the enterprise’s lifeblood. Without a sound and effective credit sale risk management system, the enterprise companies will lack sufficient market competitiveness and it eventually be weeded out market [1]. Credit sale risk stems from the external environment and internal management, among which the internal management plays a decisive role. In the internal management mechanisms, marketing, financial management, information management and credit risk management are intersect and indispensable [2].

Emerging Market is a relative concept, refers to the markets in the development of national, regional or an economy, compared with mature or developed market. For the emerging market enterprise, in order to make its product quickly enter and affect the market, credit sale has become an inevitable choice.

Based on the study of credit sale risk management of general enterprise [3], in this paper, we analyze the credit sale risk of emerging market enterprise combined with practical experience in credit sale risk management.

2. Credit costs analysis based on optimal accounts receivable holdings account holdings

According to the theory of modern credit management, the credit sale costs of the product include sale costs and credit costs. Further, the credit costs can be divided into bad debt costs, opportunity costs, management costs and shortage costs [4]. The sum up of four class costs constitutes the credit sale costs. In credit sale risk management, the optimal accounts receivable holdings usually can be determined by credit costs analysis.

2.1. Basic concept of credit costs

(1) Bad debt costs
Bad debt costs refer to the sale value that cannot be recovered after sale, and obviously the bad debt costs will rise with the increasing of account holdings.

(2) Opportunity costs
Opportunity costs refer to the missed value, and the opportunity costs caused by credit sale and overdue account equal to of the maximum credit sale costs. Thus, the opportunity costs are important to measure the level of enterprise credit sale risk management.

(3) Management costs
Management costs of credit sale refer to the sum of the costs of credit sale customer's credit investigation, credit evaluation, credit protection, accounts receivable management, exchange rate and all other related costs.

(4) Shortage costs
Shortage costs of credit sale refer to the loss caused by product sales without maximization, amount equal to
the increase return with maximization product sales.

(5) Credit costs
Credit costs is the comprehensive costs including the opportunity costs, bad debt costs, management costs, and shortage costs. In the coordinates of accounts receivable holdings and costs, the credit costs curve is a parabola with \( U \) distribution, while the profits curve in the coordinate is bell-shaped distribution. The lowest point of the parabola is corresponding with the credit costs of optimal accounts receivable holdings. Note \( A \) and \( B \) as the minimum and maximum holdings in accounts receivable; \( X^* \) represents the accounts receivable holdings with the lowest credit costs; \( E \) and \( B \) as the minimum and maximum accounts receivable holdings with the same management costs of \( X^* \). If accounts receivable holdings is little (accounts receivable holdings<\( A \)), the shortage costs is higher than the other three costs; if excessive receivables is enough (receivables holdings>\( B \)), the opportunity costs, bad debt costs and management costs will be greater than the shortage costs. Through marginal analysis and \( NPV \) method, we can calculate the shortage costs, bad debt costs, opportunity costs and management costs.

2.2. Credit costs analysis

In practice, based on historical data and the expenditure costs in emerging markets, emerging market enterprise usually consider a variety of options of accounts receivable holdings, and then select a lower costs solution. In this paper, we will theoretically analyze the credit sale costs to explore general expression of optimal accounts receivable holdings and the lowest credit costs. Without loss of generality, we make the following basic assumptions:

\( H_1 \): The credit policy of credit sale is unchanged within the study period without considering the cash discount;
\( H_2 \): Bad debt costs and opportunity costs uniformly change;
\( H_3 \): Management costs change with step-wire;
\( H_4 \): Not against specific emerging market countries or regions, and monetary policy is basically stable.

Due to the changes in accounts receivable management costs are stable within a certain range, therefore, the lowest point of the credit costs curve should fall within the range of accounts receivable holdings (\( A, \min \{B, M\} \)), \( M \) represents the accounts receivable holdings when shortage costs is zero. \( M \) is located near point \( B \), and may be left or right at the point of \( B \). Since the credit costs is a parabolic curve, the lowest point of the parabola which corresponds with the minimum credit costs. So we can fit a quadratic parabola credit costs line:

\[
Y = a + bX + cX^2
\]  

(1)

\( X \) represents the accounts receivable holdings, and \( X \in (E, \min \{B, M\}) \). \( Y \) represents the credit costs, \( a, b, c \) is regression coefficient. We select \( N \) groups of samples \( (X^*, Y^*) \), and determine the regression coefficient \( a, b, c \) through the least squares method:
\[
\begin{align*}
    a &= \frac{\sum Y_i \sum X_i^4 - \sum X_i^2 Y_i \sum X_i^2}{N \sum X_i^4 - (\sum X_i^2)^2} \\
    b &= \frac{\sum X_i Y_i}{\sum X_i^2} \\
    c &= \frac{\sum X_i^2 Y_i - \sum X_i^2 \sum Y_i}{N \sum X_i^4 - (\sum X_i^2)^2}
\end{align*}
\] (2)

Since the formula (1) is unimodal function, the following optimization problem has a unique optimal solution.

\[
\min Y = a + bX + cX^2 \quad \text{s.t.} \quad X \in (E, \min\{B, M\})
\] (3)

The solution of the formula \((X^*, Y^*)\) shows in the following formula, \(X^* \in (E, \min\{B, M\})\).

\[
\begin{align*}
    X^* &= -\frac{b}{2c} + \frac{A + \min\{B, M\}}{2} \\
    Y^* &= a - \frac{b^2}{4c}
\end{align*}
\] (4)

Comparing \((X^*, Y^*)\) with \(E (X \in (A, E))\), for the credit costs curve is monotonically decreasing). Finally, we gain theoretical expression of accounts receivables holdings and the minimum credit costs:

\[
\begin{align*}
    \bar{X}^* &= \min\{E, X^*\} \\
    \bar{Y}^* &= DorY^*
\end{align*}
\] (5)

3. Risk-adjusted return on credit sale

3.1. Risk-adjusted Capital return on credit sale of emerging market enterprise

The expected loss of credit on credit (Credit Sale Expected Loss, CSEL) refers to the loss of the product shall bear during the credit sale activity. Due to the characteristics of emerging markets, there exists a great uncertainty whether products will be accepted by consumers when entering the market. Faced with this uncertainty from the market, emerging market enterprise must reserve value and bad debt capital (roughly at least equal to expected losses) to avoid affecting normal enterprise operations. On the other hand, if emerging market enterprise chooses credit sale, the market uncertainty will inevitably bring some losses on its credit sale business. In that case, emerging market enterprise should adjust performance evaluation objectives of credit sale business. According to the current credit status of emerging market, the credit risk that emerging market enterprise faced is enormous, and generally credit sale losses will exceed the pre-reserved bad debts capital, and the excess part is called unexpected loss of credit sale. In other words, emerging market companies not
only face greater credit sale losses expected, but also face unexpected credit sale losses [5].

Suppose emerging market enterprise face a new credit sale product in emerging market, note \( RE \) (Risk Exposure) as the capital of emerging market enterprise affected by credit sale customers’ default risk, i.e. the credit risk credit exposure. \( EDF \) (Expected Default Frequency) is the expected default frequency of credit sale customers. \( LGD \) (loss given default) is the payment, not recovered under the EDF. Therefore, the expected losses of emerging market credit sale can be expressed as:

\[
CSEL = RE \times LGD \times EDF
\]  

(6)

\( CSEL \) is the core of calculating credit risk-adjusted returns on credit sale, which means the expected credit loss credit limit of credit sale [6]. Due to that we focus on credit sale risk discussion faced by emerging market enterprise in this paper, we can assume the credit sale exposure \( RE \) as the total credit sale amount of products (such assumption is based on emerging market credit sale risks have greater uncertainty); \( LGD \) as the payment when credit sale expires; further, note \( \alpha \) as payment recovery rate, so \( LGD \) rate is \( 1-\alpha \) (this indicator can be estimated from the history data of similar products in the emerging market, and adopt the average level is also feasible); the estimation of \( EDF \) can be achieved by the credit risk assessment method in emerging markets [7-8].

Based on the above analysis, risk-adjusted return on credit sale (\( RARCS \)) is calculated (the structure of \( RARCS \) is shown in Fig.1) as:

\[
RARCS = Credit\ Sale\ Income\ (CSI) - Credit\ Costs (CS) - CSEL
\]  

(7)

Since credit sale will bring emerging market enterprise a series of accounts receivable, therefore, the credit costs of credit sale should added into the credit sales costs, other than sales costs (\( SC \)).

![Fig.1. Risk-adjusted Capital Return on credit sale of emerging market enterprise](image)

3.2. Capital at Credit Sale Risk of emerging market enterprise

Because of the characteristics of emerging markets, the default rate and credit status of credit sale customer might changes unexpected [10] (for emerging markets, this is particularly important), therefore, the value of emerging market credit sale will usually fluctuates with unexpected losses, namely the unexpected loss on credit sale (\( ULCS \)), and this risk can be measured by the standard deviation of current of credit sale value: 

\[
\text{Fig.1. Risk-adjusted Capital Return on credit sale of emerging market enterprise}
\]
Wherein, \( V \) is the credit sale value of product during the studied period. Set the density of random variable \( L \) as \( f(L) \), and random factors causing the credit sale customers in emerging markets default, is independent with random variable \( L \), \( V_0 \) is the initial value of the credit sale product, so we gain

\[
\int f(L)dL = 1, \quad E(L) = \int Lf(L)dL = LGD, \quad E(L^2) = \int L^2 f(L)dL = \sigma_L^2 + LGD^2
\]

\[
E(V) = (1 - EDF)V_0 + EDF \int f(L)[V_0 - V_0 L]dL = (1 - EDF)V_0 + EDF[V_0 - V_0 \times LGD] = V_0 - EDF \times V_0 \times LGD
\]

\[
E(V^2) = (1 - EDF)V_0^2 + EDF \int f(L)[V_0 - V_0 L]^2dL = (1 - EDF)V_0^2 + EDF \times [V_0^2 - 2V_0 V_0 L + V_0^2 L^2]dL = (1 - EDF)V_0^2 + EDF \times [V_0^2 - 2V_0 V_0 \times LGD + V_0^2 (\sigma_L^2 + LGD^2)] = V_0^2 - 2 \times EDF \times V_0 \times LGD + EDF \times V_0^2 (\sigma_L^2 + LGD^2)
\]

Substituting formula (10) and (11) into formula (8), we gain:

\[
\text{var}[V] = E(V^2) - E^2(V) = V_0^2 \times [EDF \times \sigma_L^2 + LGD^2 \times (EDF - EDF^2)] = V_0^2 \times [EDF \times \sigma_L^2 + LGD^2 \times \sigma_{EDF}^2]
\]

Thus, the calculation formula of unexpected losses can be available as follows:

\[
\text{ULCS} = V_0 \times \sqrt{EDF \times \sigma_L^2 + LGD^2 \times \sigma_{EDF}^2}
\]

\( \sigma_L^2, \sigma_{EDF}^2 \) respectively is the variance of random variable \( L \) and \( EDF \) of credit sale customer.

In other words, due to the unexpected loss the potential losses other than the expected loss, the credit sale customer of emerging market enterprise should reserve enough capital for the uncertain potential loss, in order to maintain normal operations when that case of loss actually occurs. The reserved capital of emerging market enterprise is called “Capital at Credit Sale Risk” (Capital at Credit Sale Risk, \( CCSR \)), it can help to ease the financial pressures when the default of credit sale customer it appears frequently. “Credit sale risk capital” is usually measured by multiples of unexpected losses (as is shown in Fig.2), namely \( CCSR = \delta \times ULCS \), where \( \delta \) multiplier can be estimated based on and product’s market acceptance extent and the level of risk in emerging markets, or determined by the following formula (14).

Set \( L \) is a random loss variable, \( \varepsilon \) is the confidence level. At any time \( T \), to cover \( CSEL \) and \( CCSR \) of emerging market product with the probability \( \varepsilon \), should meet:

\[
\text{prob}\{L - CSEL \leq CCSR\} = \varepsilon \quad (12)
\]

or:

\[
\text{prob}\{\frac{L - CSEL}{ULCS} \leq \delta\} = \varepsilon \quad (13)
\]
From formula (12) and (14), we can gain the reserve enough capital for the uncertain potential losses (Capital at Credit Sale Risk, CCSR).

3.3. CS-RROC Model

According to the analysis mentioned above, we can gain the Credit Sale-Risk Return on Capital (CS-RROC) model:

\[
\text{CS-RROC} = \frac{\text{RARCS}}{\text{CCSR}} = \frac{\text{CSI} - \text{Credit costs} - \text{CSEL}}{\delta \times \text{ULCS}} \quad (14)
\]

The value of CS-RROC can measure the return of capital at credit sale risk from emerging market. In consideration of the unexpected losses, the emerging market enterprise can match the benefits and risks during the production’s credit sale activities, which will make the credit risk management more perfect.

3.4. Numerical analysis

Suppose an enterprise face the following situation: the amount of emerging market customers in product credit sale is 100,000 (RMB), the sales costs is 70,000(RMB),the credit sale management costs is 10,000(RMB), EDF is 15%, the random loss variable \( L \) subject to \( \beta \) distribution with parameter 1 and 8.

According to the formula of \( \beta \) distribution, we gain \( \text{LGD} = 11.11\% \), \( \sigma_{\text{EDF}} = \sqrt{\text{EDF} - \text{EDF}^2} = 0.3571 \), \( \sigma_L = 0.0994 \). From formula (7), we gain the \( \text{CSEL} = 10 \times 0.1111 \times 0.15 = 0.1666 \) (RMB); From formula (8), we gain \( \text{RARCS} = 10 - (7 + 1) - 10 \times 0.1111 \times 0.15 = 1.83335 \) (RMB); From formula (13), we gain \( \text{ULCS} \) as follow:

\[
\text{ULCS} = 10 \times \sqrt{0.15 \times 0.0994^2 + 0.1111^2 \times 0.3571^2} = 0.5528 \text{ (RMB)}
\]

Set the confidence level \( \epsilon = 95\% \), from the formula(15),we gain the capital multiplier \( \delta \):

\[
\delta = \frac{(0.312 - \text{CSEL})}{\text{ULCS}} = 0.2629
\]

Further, in order to control the \( \text{ULCS} \), we need to calculate \( \text{CCSR} = 0.5528 \times 0.2629 = 0.1453 \) (RMB). In other words, under the assumption mentioned above, if the Capital at Credit Sale Risk (CCSR) is 0.1453 (RMB), then it indicates that the emerging market enterprise will cope with the Unexpected Loss on
Credit Sale with probability of 95%.

Then, we can figure out the Credit Sale-Risk Return on Capital (CS-RROC):

\[
\text{CS-RROC} = \frac{\text{RARCS}}{\text{CCSR}} = \frac{1.8334}{0.1453} = 13
\]

In other words, the credit sale return is 13 times of credit sale risk capital after risk-adjusted of the product, which indicates that the credit sale effect in emerging market of the product is significantly.

4. Establishment of optimal portfolio of product credit sale

If the credit sale scale of a certain product of emerging market enterprise is large, in order to distract the credit sale risk, the enterprise may credit sale the product to multiple customers at the same time, avoiding putting the "eggs in one basket". Therefore, emerging market enterprise should use product credit sale portfolio. We determine the credit sale portfolio scheme through the applications of Markowitz optimization model (determine the proportion of each customer’s credit sale).

Suppose the number of credit sale customers is \( n \); \( r_i \) represents the return from credit sale to customer \( i \); \( x_i \) represents the weight of credit sale portfolio; \( \text{cov}(r_i, r_j) \) indicates the covariance of return on credit risk of customer \( i \) and customer \( j \); \( \mu \) represents the expected return of product credit sale portfolio; \( \overline{R}_i \) represents the risk-adjusted return on credit sale (RARCS) of customer \( i \). Through Markowitz portfolio optimization theory, we can establish the portfolio optimization model with the lowest credit sale risk, under the condition of expected credit sale return:

\[
\begin{align*}
\min & \quad \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \text{cov}(r_i, r_j) \\
\text{s.t.} & \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \\
& \quad \sum_{i=1}^{n} x_i \overline{R}_i = \mu
\end{align*}
\]

Or within the affordable product credit sale portfolio risk level, we establish the maximization of expected return optimization model:

\[
\begin{align*}
\max & \quad \sum_{i=1}^{n} x_i \overline{R}_i \\
\text{s.t.} & \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad \sigma_p^2 = \xi
\end{align*}
\]

\( \sigma_p \) is the standard deviation of product credit sale portfolio (in order to measure the product credit sale portfolio risk), \( \xi \) is the affordable product credit sale portfolio risk level.
Further, we note $X = (x_1, x_2, \cdots, x_n)^T$, $E = \text{cov}(r_i, r_j)$ is covariance matrix, $B = \begin{bmatrix} \mu \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} R_1 & R_2 & \cdots & R_n \\ 1 & 1 & \cdots & 1 \end{bmatrix}$, then formula (15) can be noted as matrix pattern as follow:

$$
\begin{align*}
\min \sigma_p^2 &= X^T EX \\
\text{s.t.} AX &= B
\end{align*}
$$

By introduction of Lagrange function to solve the optimization problem of (17, we can achieve the optimal weight of product credit sale portfolio:

$$
X^* = E^{-1} A^T (AE^{-1} A^T)^{-1} B
$$

Substituting formula (19) into formula (18), we gain the minimum product credit sale portfolio risk as follow:

$$
\sigma_p^* = \sqrt{X^{*T} EX^*} = \sqrt{B^T (AE^{-1} A^T)^{-1} B}
$$

From the above analysis, the emerging market enterprise can achieve the lowest product credit sale portfolio risk under the condition of expected credit sale return; or achieve the maximization of expected return within the affordable product credit sale portfolio risk level, and determine the proportion of each customer’s credit sale. Through the Markowitz portfolio optimization theory, we call the portfolio scheme meeting the conditions mentioned above as “effective” product credit sale portfolio, and call all the “effective” product credit sale portfolio as the “effective” set of product credit sale portfolio. The “effective” set’s trajectory in the plane of $(\overline{R}_i, \sigma_p)$ is called the effective frontier of product credit sale portfolio (as is shown in Fig.3.)

From what has been discussed above, the emerging market enterprise can choose the product credit sale portfolio scheme from the effective frontier of product credit sale portfolio, according to the affordable product credit sale portfolio risk level or the expected credit sale return, and determine the proportion of each customer's credit sale.

![Fig.3. the effective frontier of product credit sale portfolio](image-url)
5. Conclusions

Based on the study of credit sale risk management of general enterprise, in this paper, we discuss the similar problem of emerging market enterprise. The accounts receivable holdings and the credit costs is the key content of the enterprise’s product credit sale risk management. Therefore, the enterprise should set the credit sale risk capital system, in order to control the bankrupt risk. In this paper, we not only discussed the emerging market enterprise control the credit sale costs and the determination method of credit sale risk capital. In addition, we also explored the problem of the emerging market enterprise credit sale risk evaluation, credit sale customers’ recognition and selection, the credit sale policy making, which will provide great reference significance.

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