# Multicriteria shape design of a sheet contour in stamping

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## Abstract

One of the hottest challenges in automotive industry is related to weight reduction in sheet metal forming processes, in order to produce a high quality metal part with minimal material cost. Stamping is the most widely used sheet metal forming process; but its implementation comes with several fabrication flaws such as springback and failure. A global and simple approach to circumvent these unwanted process drawbacks consists in optimizing the initial blank shape with innovative methods. The aim of this paper is to introduce an efficient methodology to deal with complex, computationally expensive multicriteria optimization problems. Our approach is based on the combination of methods to capture the Pareto Front, approximate criteria (to save computational costs) and global optimizers. To illustrate the efficiency, we consider the stamping of an industrial workpiece as test-case. Our approach is applied to the springback and failure criteria. To optimize these two criteria, a global optimization algorithm was chosen. It is the Simulated Annealing algorithm hybridized with the Simultaneous Perturbation Stochastic Approximation in order to gain in time and in precision. The multicriteria problems amounts to the capture of the Pareto Front associated to the two criteria. Normal Boundary Intersection and Normalized Normal Constraint Method are considered for generating a set of Pareto-optimal solutions with the characteristic of uniform distribution of front points. The computational results are compared to those obtained with the well-known Non-dominated Sorting Genetic Algorithm II. The results show that our proposed approach is efficient to deal with the multicriteria shape optimization of highly non-linear mechanical systems.

Keywords: Sheet metal forming; Initial blank shape; Springback; Failure; Multi-objective optimization

#### 1. Introduction

The sheet metal forming is of vital importance to a large range of industries as production of car bodies, cans, appliances, etc. It generates complex, of high geometrical precision, parts. However, the associated production technologies involve mechanical phenomena combining elastic-plastic bending and stretch deformation of the workpiece. These deformations can lead to undesirable problems in the target shape and performance of the stamped. To perform a successful stamping process and avoid the unwanted springback and failure defects, process variables should be optimized.

One of the most important issue in stamping process concerns initial blank shape optimization that can reduce if not eliminate design problems of the obtained product [1-5].

In general practice, techniques that are used in this optimization process were based on experiments and trial and error method which induce very high costs. Nowadays, growth and

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advances in computer science technologies are proved and numerical simulation tools are an efficient alternative, mainly making recourse to the finite element method (FEM).

In this context, several studies had been done to optimize forming parameters such as punch speed, blank holder force, friction coefficient, etc. [5-7]. Others investigated the optimization of geometrical parameters such as the radii of the punch and the die, the binder surface, etc [8, 9]. More recently, some studies were performed for the design optimization of tools in order to reduce the design time but without considering the quality of the desired workpiece [10].

We aim here to develop a numerical tool for the shape optimization of an initial blank in order to reduce springback and risk of failure. More precisely, the application targeted by this article is to efficiently optimize the initial blank shape used in the stamping of an industrial workpiece stamped with a cross punch, as presented in section 4. The stamping process is performed using the commercial FEA code LS-DYNA. The criteria considered are springback and failure. These two phenomena are the most common problems in the stamping process, and they present many difficulties in optimization since they are two conflicting objectives. To solve

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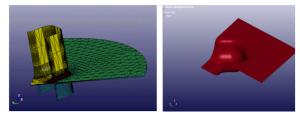


Figure 1. Simulation of stamping process (LS-DYNA).

Table 1. Process parameters used in simulation.
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Process Parameter	Value
Material	HSLA260
Young's modulus	196GPa
Poisson's ratio	0.307
Density	7750Kg/m <sup>3</sup>
Hardening coefficient	0.957
Punch speed	5m/s
Punch stroke	30mm
Blanck holder effort	79250N
Friction coefficient	0.125
Number of elements	5775

each single objective optimization problem, the approach chosen in section 5.1 was based on the hybridization of a heuristic algorithm, the Simulated Annealing (SA) [11], and a direct descent method, the Simultaneous Perturbation Stochastic Approximation (SPSA) [12]. This hybridization is designed to take advantage from both disciplines, stochastic and deterministic, in order to improve the robustness and the efficiency of the hybrid algorithm. For the solution of the multi-objective problem, we adopt methods based on the identification of Pareto front.

To have a compromise between the convergence towards the front and the manner in which the solutions are distributed, we choose two appropriate methods in section 5.2 which are the Normal Boundary Intersection (NBI) [13] and The Normalized Normal Constraint Method (NNCM) [14]. These methods have the capability to capture the Pareto front and have the advantage of generating a set of Pareto-optimal solutions uniformly distributed. The last property is of important and practical use in the multicriteria optimization of non-linear mechanical systems. By reformulating the multiobjective problem to single-objective sub-problems and only with few points, these two methods can form a uniform distribution of Pareto-optimal solutions, which can help the designers and decision makers to select efficient solutions among the well represented Pareto front in the design space.

It is important to notice the necessity of solving the singleobjective sub-problems with global optimization approaches whereby we can obtain a global Pareto front, whereas the resulting optima using a gradient-based local optimization algorithm are only local Pareto-optimal solutions.

To check the efficiency of these multi-objective approaches, numerical examples were used to compare the obtained re-

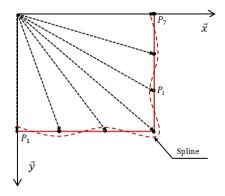


Figure 2. Blank contour parameterization.

sults with those obtained with a well-established technique in multi-objective optimization called Non-dominated Sorting Genetic Algorithm II (NSGAII) [15]. The results of initial blank shape optimization of the investigated test case, in order to reduce the springback and the risk of failure, were done, and are presented in the end of this section. Finally, a conclusion and perspective views are provided in Section 7.

## 2. Finite element modelling

Numerical simulation of metal forming processes is currently one of the most used technological innovations, which aim to reduce the high tooling costs, and facilitates the analysis and solution of problems related to the process.

In this study, the FEA code, LS-DYNA, was used to model and compute the stamping of an industrial workpiece. LS-DYNA is an explicit and implicit Finite Element program dedicated to the analysis of highly non-linear physical phenomena.

The aim was to study the influence of the initial blank shape on the stamping process of a blank with a cross punch (Figure 1). The blank was made of high-strength low-alloy steel (HSLA260) and was modelled using Belytschko-Tsay shell elements, with full integration points.

Due to symmetry, only the quarter of the blank, die, punch and blank holder were modelled and symmetric boundary conditions were applied along the boundary planes. Mechanical properties of materials and process characteristics are shown in Table 1.

## 3. Problem description

A sensitivity analysis done using FEA demonstrated that the overall dimensional quality is highly influenced by the initial dimensions of the blank. The initial blank design is a critical step in stamping design procedure; therefore it should be correctly designed.

This study aims to find the optimal initial blank shape that satisfies the design specifications during the forming process. To meet these specifications, it is mandatory to eliminate or at least minimize springback and risk of failure problems.

For this study, the geometry of the blank contour is de-

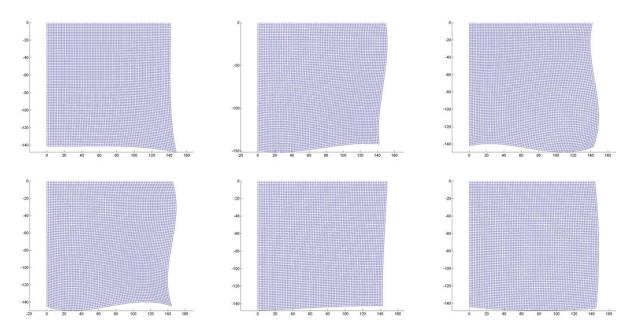


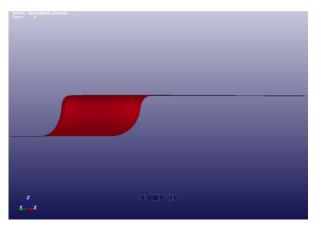
Figure 3. Examples of initial blank shapes using Spline curves.

scribed by parametric spline curves (Figure 2). Seven control points (P1...P7) are used to define the spline curves in order to have a wide variety of geometries. Some investigated shapes are described in Figure 3.

The maximum allowed variation of each control point is equal to 15mm in both axes in order to avoid distorted meshes.

Due to storage requirements and model complexity, the computational cost in sheet metal forming processes is very expensive. One usually uses surrogates (approximate functions, also known as metamodels), but the latter are also computationally expensive at higher dimensions (of the design space). Therefore, the building of metamodels is done using the sparse grid interpolation. Optimization process based on sparse grid interpolation is an optimal alternative in which criteria can be approximated with a suitable interpolation formula that needs significantly less points than the full grid. It is important to notice the potential applicability of sparse grids method especially for high dimensional problems. This potential was illustrated in several studies [16-20]. The basis of all sparse grids is the famous Smolyak's method [21], which provides a construction of interpolation functions with a minimum number of points in multi-dimensional space and extends adequately the univariate interpolation formulas to the multivariate case. The related interpolation algorithms are performed in a MATLAB Toolbox based on piecewise multilinear and polynomial basis functions. Additional tasks involving the interpolant are used in order to reduce the computational effort for function evaluations. We used the Sparse Grid Toolbox developed by Andreas Klimke [22, 23].

# 4. Calculation of criteria



In sheet metal forming, first the deformation is elastic and reversible; then, due to large strain, the linear behaviour is no longer valid, so the deformation is plastic.

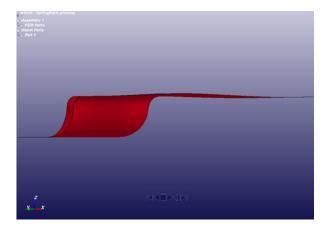


Figure 4. Simulation of the springback (LS-DYNA).

During this operation, the sheet metal is normally deformed to conform to the shape of the tools, except that upon unloading, the sheet looks for finding its original geometry due to the elastic component of deformation work previously stored as potential energy in the sheet. This phenomenon is called "Springback".

Simulation of springback involves two steps: loading (stamping) and unloading. Thus after the simulation of the stamping of the investigated workpiece, LS-DYNA generates an output file that contains all information about stresses and strains upon unloading. Based on this information, LS-DYNA can simulate the springback by an implicit integration scheme. Figure 4 shows a small deflection in the corner of the part that represents the springback phenomenon.

To estimate the springback, first the displacement in the z direction of each node is calculated by LS-DYNA after the springback simulation and then we extract the maximum value of this component for all nodes. Thus, the first objective function, namely the springback criterion, can be formulated as in Eq. (1).

$$f_1(\Phi) = \max_{1 \le i \le m} (UZ)_i \tag{1}$$

where  $\Phi$  is the vector of design parameters, *m* the total number of nodes, *i* the node number and  $(UZ)_i$  is the displacement in the z direction of the node *i*.

During sheet metal forming, localized deformations lead to local defect that appears in the stamped as sheet failure. To better characterize this sheet failure, it is first necessary to fully understand the formability of the sheet. In this sense, the concept of forming limit curve (FLC, see Figure 5) was introduced [24, 25]. It is determined by experimental tests in order to separate spaces representing homogeneous and localized strains For more reliability, we have considered a safety margin of 10% (advised to us by our steel industrial partners), which allow us to consider the curve below the FLC where cracking can start (if values are located beyond this curve). So, this curve describes the transition from the safe material behaviour to material failure. One of the aims of this study is to determine if the material can sustain the strains underneath the forming limit curve without failure.

First, from the strain tensor of each element, we calculated the principal strains in the average area. By placing these values of principal strains on the same diagram, we see that indeed, the elements whose principal strains are placed above the forming limit curve failed, we have seen this failure with the simulation by LS-DYNA in Figure 6.

To quantify the safety level of the stamped, we formulated the second objective function, namely the failure criterion, as Eq. (2).

$$f_2(\Phi) = \max_{1 \le i \le m} \left( \left( \varepsilon_1 \right)_i - 0.9 * \left( \varepsilon_f \right)_i \right)$$
(2)

The formula above represents the distance between the strain of the most critical element  $(\varepsilon_1)_i$  and the corresponding strain value in the limit curve  $(\varepsilon_f)_i$  taking into account the safety margin that we have considered.

## 5. Optimization process

#### 5.1 The SA hybridized with the SPSA method

Currently, approaches from the hybrid meta-heuristics present promising prospects in optimization, in order to have a high rate of quality and also precision. We introduce in this section a hybrid method that has shown good results. The method is based on the simulated annealing (SA) method [8]

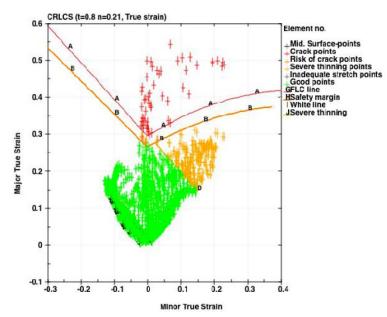


Figure 5. Forming limit diagram for HSLA260 steel-sheet.

hybridized with simultaneous perturbation stochastic approximation (SPSA) method [26].

The SA algorithm was developed by Kirkpatrick [8] and Černý [27]. It simulates the evolution of a heated system towards the equilibrium state (optimal configuration), and its most used variant, the algorithm of Metropolis [28], aims to start from an initial configuration and submit the system to a disturbance for each range of the control parameter *T*. If this disturbance generates a solution which improves the objective function *f*, we accept it; if it has the opposite effect, we draw a random number between 0 and 1, if this number is less than or equal to  $e^{\frac{-\Delta f}{T}}$ , we accept the configuration. Thus, at high *T*, the majority of moves in the space of configurations are accepted. By reducing progressively *T*, the algorithm allows few solutions optimizing the objective function; therefore, for very low *T*,  $e^{\frac{-\Delta f}{T}}$  is close to 0 and the algorithm rejects the moves that increase the cost function.

SA has many advantages that distinguish it from other optimization algorithms. First, it is a global optimization method, easy to program and applicable in several areas, on the other hand, it has some shortcomings such as the empirical regulation of parameters, the excessive calculation time and, at low T, the acceptance's rate of the algorithm becomes too small, so that the method becomes ineffective. Hence the idea of coupling the algorithm with a descent method in order to reduce the number of objective function evaluations.

One of the methods which fits the above approach and which answers the mentioned requirements is the method named *simultaneous perturbation stochastic approximation* (SPSA) [29].

It is a method based on gradient approximation from the perturbation of the objective function that requires only two evaluations of the objective function regardless of optimization problem dimension, which accounts for its power and relative ease of implementation.

Let us consider the problem of minimizing a loss function (x), where x is a m-dimensional vector. The SPSA has the general recursive stochastic approximation form as presented in Eq. (3).

$$\hat{x}_{k+1} = \hat{x}_k - \hat{a}_k \hat{g}_k(\hat{x}_k) \tag{3}$$

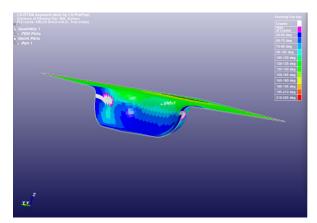


Figure 6. Simulation of the failure (LS-DYNA).

where  $\hat{g}_k(\hat{x}_k)$  is the estimate of the gradient  $g(x) = \frac{\partial f}{\partial x}$  at the iterate  $\hat{x}_k$ .

This stochastic gradient approximation is calculated by a finite difference approximation and a simultaneous perturbation, so that for all  $\hat{x}_k$  randomly perturbed together we obtain two evaluations of  $f(\hat{x}_k \pm \xi)$ .

Then, each component of  $\hat{g}_k(\hat{x}_k)$  is a ratio of the difference between the two corresponding evaluations divided by a difference interval following Eq. (4).

$$\hat{g}_{k_i}(\hat{x}_k) = \frac{f(\hat{x}_k + c_k \Delta_k) - f(\hat{x}_k - c_k \Delta_k)}{2c_k \Delta_{k_i}} \tag{4}$$

Here  $c_k$  is a small positive number that gets smaller as k gets larger and the vector  $\Delta_k = (\Delta_{k_1} \dots \Delta_{k_m})^t$  a m dimensional random perturbation vector; a simple and generally successful choice for each component of  $\Delta_k$  is to use a Bernoulli  $\pm 1$  distribution with probability of  $\frac{1}{2}$  for each  $\pm 1$  outcome.

There is a convergence condition which stipulates that  $a_k$  and  $c_k$  both go to 0 at rates neither too fast nor too slow, and that the cost function is sufficiently smooth near the optimum. Convergence is then faster when we approach an optimum compared to the random moves of the simulated annealing.

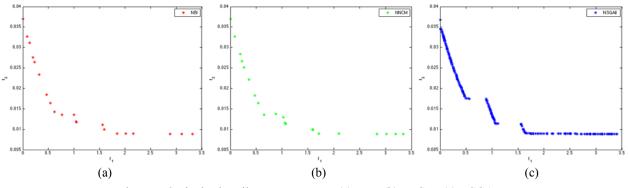


Figure 7. Springback-Failure Pareto Front: (a) NBI, (b)NNCM, (c) NSGAII.

To increase the accuracy of the simulated annealing, we need to implement the SPSA after each move minimizing the objective function and the Metropolis criterion always gives us the possibility to escape from local optima.

## 5.2 Multi-objective optimization

According to the principle established by Pareto, the solution of a multi-objective function is not unique; it is a set of solutions called "Pareto-optimal". Based on the Pareto's concept, a solution is Pareto-optimal if it is impossible to improve a component without degrading at least another one. The classical approach to solve engineering multiobjective problems is to generate an optimal Pareto front, very practical and easy to use by engineers, which requires first the capture of the Pareto front and second a good visualization of the front's points. This is the aim of these two methods: Normal Boundary Intersection (NBI) method [13] and Normalized Normal Constrained Method (NNCM) [14].

The proposed approaches were compared with the NSGAII algorithm, widely used and considered as representative of the state of the art and a reference algorithm in multi-objective optimization in various studies. The obtained results prove that the proposed approaches have also good performances compared with those obtained with NSGAII.

#### 6. Results of optimization of springback and failure

It is important to notice that our criteria (springback and failure) are not explicit functions. Additionally, the simulation of these two criteria is computationally very expensive. The FE model request around 45 min to predict these two criteria. However, the obtained metamodels using sparse grid interpolation need less than 1s to predict springback and failure on the same computation machine. To find the optimal initial blank shape, it was decided to perform the optimization process using the sparse grid metamodel. The construction of the sparse grid interpolant was based on the Chebyshev Gauss-Lobatto grid type and using the polynomial basis functions [22, 23]. This technique achieves a good accuracy with a competitive number of grid points.

The SA hybridized with SPSA was applied to minimize the corresponding criteria. According to the obtained results, the two multi-objective optimization approaches, NBI and NNCM, were used to find the set of Pareto optimal solutions in the criterion space i.e. springback criteria versus failure criteria. The NBI and the NNCM approaches were coupled with the SA hybridized with SPSA to obtain global solutions in each step of the two approaches. Figure 7 shows the Pareto frontier obtained with these two approaches. The same comparison was made with the NSGAII and the Pareto-optimal solutions are shown in the same Figure.

These Pareto curves confirm that the trade-off between the springback and the failure criterion exists and can help the engineers to better understand it. We may notice also that we can eliminate springback and reduce the impact of failure by optimizing the initial blank shape. The comparison of the obtained fronts shows that we can capture Pareto solutions by NBI and NNCM with fewer points than NSGAII, which requires a large number of populations, and several generations to obtain the Pareto front.

#### 7. Conclusion and perspectives

In the present paper, an estimation of the springback and the failure of a workpiece are achieved. Considering the complexity of finite element models of sheet metal forming, of non-linear large scale nature, the simulation and the estimation of these two criteria are very expensive.

To optimize these criteria, a hybrid approach is investigated to optimize each single-objective problem. This approach is based on hybridization of SA algorithm, which belong to meta-heuristic methods, and SPSA method, which is one of descent methods. The advantage of this approach was to find the global minimum in few steps.

The optimization problem in our study is multi-objective and criteria are antagonistic, whence the benefit of the NBI and the NNCM methods. These two methods have the power of generating a set of Pareto-optimal solutions uniformly spaced. This advantage was assessed and validated against many mathematical well-known benchmarks and successfully compared to the results of NSGAII method. Based on an accurate approximation of the two criteria using sparse grid interpolation method, the obtained Pareto Front of springback and failure criteria considering the initial blank shape were computed. We compared our results to those obtained with NSGAII. The main advantage of our approach was the possibility to obtain a Pareto front with few function evaluations compared with NSGAII method. In further works, we envisage to improve the current algorithms and extend the study to innovative approximation methods based on enrichment of metamodels in order to improve the Pareto front in each enrichment step and second apply these approaches in shape optimization of more complicated workpiece considering other criteria as wrinkling criterion.

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