5th Fatigue Design Conference, Fatigue Design 2013

Stress-strength interference method applied for the fatigue design of tower cranes

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Abstract

Crane owners want to maximize the efficiency of their equipment and teams on a construction site in order to minimize the amount of time needed to complete their work. This leads to very intensive crane use and stress on crane structures made primarily of welded steel elements (plates and beams). Therefore, it is necessary for Structural Engineers to consider fatigue resistance during the design process. The aim of the paper is to demonstrate the utility of a probabilistic approach for a better optimization of the fatigue design of tower cranes elements (e.g. jib elements).

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Selection and peer-review under responsibility of CETIM

Keywords: reliability, fatigue, crane

1. Introduction

Tower crane use on a construction jobsite is a limiting factor on the speed that work can be completed. Tower crane structures are made of steel plates or beams connected by welding and, due to intensive workload, fatigue resistance is an important aspect to consider for Structural Engineers. Classical design methods can lead to non-optimized structures with non-uniform safety margins, and consequently to a non-optimized distribution of safety factors related to fatigue. The proposed work takes place further to two projects called DEFFI (Reliability Approach in Fatigue Design for the Industry) and APPRoFi (Probabilistic Approach for Robust Fatigue Design). The goals of these projects were to experiment and to develop reliability approaches for the mechanical design against fatigue of industrial applications. Based on these previous works, the aim here is to quantify and optimize the safety margins with respect to fatigue of tower crane steel structures, by means of probabilistic approaches.
This study focuses on jib elements of a top slewing tower crane. The jib's function is to support the trolley that moves the load from one radius to another (see $R_1$ and $R_2$ in Figure 1). This jib structure is submitted to load variation depending on the radius and the live hoisted load.

All structures made by metal plates or beams connected by welding are subjected to fatigue phenomenon after many years of intensive cyclic use. This phenomenon is showing large variation for several reasons. First, there is the randomness of the loading on the structure, due to the variability of crane use by the owners. Second, the non-even fabrication process (manual welding) leads to material and geometry differences in the structure. Third, the great variety of geometries makes it impossible for life prediction to reach the same predictability in every single case. Fourth, complex structural modeling requires simplification hypothesis that may result in variation based on analysis techniques utilized.

European Standards used up to now for fatigue design of tower cranes [1] consider deterministic loading cycles regardless of time in service that lead to non-optimized safety margins. The aim of this work is to assess the fatigue damage probability of a crane structural member, namely the jib element, depending of the time in service. This kind of approach requires data collection concerning the resistance on one hand and real use of the crane on the other hand. These data are used to model random variables having an influence on the fatigue damage of the structure.

2. Stochastic modeling of fatigue life

2.1 Proposed fatigue analysis procedure

To determine fatigue life of a crane part, an analysis is performed by creating a model thanks to a Finite Element software in order to calculate the local stress state for relevant load cases. There are two load cases to be studied that would result in the most damage due to fatigue loading, i.e. the maximum load at the end of the jib and without load. Further on, these two load cases will be mentioned as load cases $A$ and $B$ (see Figure 2). To reduce the calculation time, welded structural assemblies are modeled using shell elements connected by MPC (Multi-point Constraint) equations. It allows modelization of the load transfer performed by the weld without actually modeling it. Then, a fatigue criterion inspired by the Dang Van criterion [2] is used (see equation (1)) to calculate an equivalent local stress $\sigma_{eq}$ under the assumption of a proportional stress state.

$$\sigma_{eq} = \sigma_{\max}^{(A,B)/2} + a_0 \cdot p_{\max}^{(A,B)}$$

$\sigma_{\max}^{(A,B)/2}$ is the amplitude of the maximum shear stress and $p_{\max}^{(A,B)}$ is the maximum hydrostatic pressure between $A$ and $B$ load cases and $a_0$ is a material coefficient. In [2], fatigue tests on welded specimens have permitted to determine a value close to $1/3$ for $a_0$. In the present study, this coefficient is considered as unknown and has to be calibrated.

The median number of cycles to fatigue damage $N_{cal}$ is calculated by Basquin's model [3]:
\[ N_{\text{cal}} = N_{\text{ref}} \left( \frac{\sigma_{\text{fat}}}{\sigma_{\text{ref}}} \right)^{-c} \]  

(2)

where \( N_{\text{fat}} \) and \( \sigma_{\text{ref}} \) are respectively the number of cycles and the stress of reference and \( c \) is a constant value.

### 2.2 Influential factors in fatigue

In addition to the stress range and the mean stress already taken into account in equation (1), some other influential factors can be considered in fatigue analysis. Some authors consider the base material, the stress gradient [4,5], the residual stress [6], the weld shape [7] or the plate thickness [8,5].

A Finite Element Analysis allows the assessment of the stress range and the mean stress of a studied welded part having known characteristics (base material and plate thickness). However, the residual stress and the weld shape are generally unknown and participate to the scatter observed in fatigue resistance as well as the base material which is considered here as negligible for the domain of interest (median life range from \( 10^4 \) to \( 10^6 \) cycles). To take into account the effect of the plate thickness, a classical approach inspired by IIW recommendations [9] is used to define a correction factor on \( \sigma_{\text{fat}} \). Hence, \( N_{\text{cal}} \) becomes:

\[ N_{\text{cal}} = N_{\text{ref}} \left( \frac{\sigma_{\text{fat}}}{\sigma_{\text{ref}}} \right)^{-c} = N_{\text{ref}} \left( \frac{\sigma_{\text{fat}}}{\sigma_{\text{ref}}} \right)^{-c} f_i(e) \]  

(3)

Where \( a_i = -cc \) is a model parameter, \( e \) and \( e_{\text{ref}} \) are respectively the actual and reference thicknesses and \( f_i(e) = \left( e_{\text{ref}} / e \right)^{a} \) is the correction factor on the fatigue life taking into account the effect of plate thickness. For example, in the IIW recommendations [9], \( e_{\text{ref}} \) is equal to 25 mm and \( \alpha \) is given between 0.1 and 0.3 depending on the type of welded joint. Concerning the stress gradient, an original approach is proposed, based on a simple definition of the linearized shear stress gradient \( \nabla \tau_{\text{max}} \) through plate thickness. The use of plate elements in FE models leads to express \( \nabla \tau_{\text{max}} \) by the shear stress on the top and the middle surfaces of the shell elements:

\[ \nabla \tau_{\text{max}} = \frac{1}{\tau_{\text{max}}^{(A-B)/2}} \frac{\partial \tau_{\text{max}}^{(A-B)/2}}{\partial e} \rightarrow \nabla \tau_{\text{max}} = \frac{\tau_{\text{max}}^{(A-B)/2}(\text{top}) - \tau_{\text{max}}^{(A-B)/2}(\text{middle})}{\frac{e}{2} \tau_{\text{max}}^{(A-B)/2}(\text{top})} \]  

(4)

where \( \nabla \tau_{\text{max}} \) ranges between 0 and 1 and is equal to 0 in pure tension. Thus, a new correction factor on shear stress gradient \( f_2(\nabla \tau_{\text{max}}) = \frac{10^{wV_{\tau_{\text{max}}}}}{V_{\tau_{\text{max}}}} \) is defined (\( \alpha_2 \) being a model parameter). By taking into account \( f_1 \) and \( f_2 \), the predicted number of cycles given by the FE models becomes:

\[ N_{\text{cal}} = N_{\text{ref}} \left( \frac{\sigma_{\text{fat}}}{\sigma_{\text{ref}}} \right)^{-c} \left( \frac{e_{\text{ref}}}{e} \right)^{a_1} 10^{a_2V_{\tau_{\text{max}}}} \]  

(5)

Note that thickness increase reduces lifetime, whilst gradient increase (i.e. increased bending or reduced thickness) improves life prediction, which conforms to experimental knowledge. To sum up, equation (5) accounts for three major influential factors on structural fatigue life: maximum hydrostatic pressure (i.e. stress ratio), thickness and gradient (i.e. bending ratio). Other existing factors (e.g. the material, residual stress, weld shape) are considered in the present work through their scattering effect.

### 2.3 Factors calibration

In order to find the best set of parameters \( p = \{ \sigma_{\text{ref}}, c, a_0, a_1, a_2 \} \) that correct the theoretical fatigue life, a database was constituted from fatigue test results on T and X welded joints available in the literature (e.g. [8,10]) with varying parameters like material, thickness, stress ratio, etc. Parameters are selected during the calibration process to minimize the squared logarithmic difference between experimental and predicted life.
Considering that \( \varepsilon_{\text{ref}} \) and \( N_{\text{ref}} \) are fixed respectively to 6 mm and 250000 cycles, the factors \( p_j \) to calibrate in equation (6) are \( \varepsilon_{\text{ref}}, \varepsilon_0, \alpha_0, \alpha_1 \) and \( \alpha_2 \). The best set of parameters for the literature specimens is given in Table 1 and the corresponding scatter plot is depicted in Figure 3. Note that the obtained value of \( c \) is close to the one given by the FEM (3.322). Moreover, the small value of \( a_0 \) shows that the coefficient on the hydrostatic pressure seems to have very small influence for the studied specimens.

\[
\min_{p_j} f(p_j) = \sum_{i=1}^{n} \left( \log N_{\text{exp}}^{(i)} - \log N_{\text{cal}}^{(i)}(p_j) \right)^2
\]

\[\text{(6)}\]

Table 1 - Best set of parameters found

<table>
<thead>
<tr>
<th>( \varepsilon_{\text{ref}} )</th>
<th>( c )</th>
<th>( \varepsilon_0 )</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>3.39</td>
<td>0.005</td>
<td>0.464</td>
<td>2.49</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3 Scatter plot obtained after the factors calibration.

Figure 4 Distribution of the ratio between experimental and theoretical results.

2.4 Life prediction for a given load

In order to conduct the reliability analysis on the jib element, it is necessary to model the randomness of the resistance of actual structures. A database of fatigue tests on real structures is not available yet. Some results are available on more complex structures [11,12,13] but these results are not sufficient to model the resistance of crane structures. At this stage of knowledge, the authors take the assumption that the resistance of specimens of the literature is representative of the population of complex welded structures, including welded details utilized in tower crane. Therefore, the problem of finding a model of resistance for crane structures is reduced to modeling the randomness of the scatter plot depicted in Figure 3. For each point of the scatter plot, the ratio between the experimental life and the calculated life can be expressed as follows:

\[ N_{\text{exp}} = \omega N_{\text{cal}} \]

\[\text{(7)}\]

Therefore, the problem consists in characterizing \( \omega \). Assuming that \( \varepsilon \) follows a lognormal law (see Figure 4), it shows a mean value of 1.18 and a coefficient of variation of 62.3%. The latter value is higher than those usually found in works that focus on single joints studied separately. This higher scatter is the logical consequence of the gathering of multiple welded joints test results, thus accounting for a variety of geometries, materials, welders, etc.

3. Stochastic modeling of crane use

3.1 Definitions

To understand how tower cranes work, it is necessary to introduce the concept of load chart. The example of load chart depicted in Figure 5 shows the maximum load it is possible to lift according to the position of the trolley along the jib, i.e. the radius \( R \). For a given value of radius \( R^{(i)} \) at time \( t \), the load \( L^{(i)} \) that can be lifted by the crane is necessarily below the load chart, i.e. below \( L^{(i)}_{\text{max}} \) (see Figure 5).
Another important thing to consider is the motion sequence of a loading cycle. Figure 5 shows that a load cycle consists of four phases: (1) trolley in movement without load, (2) lifting of the load, (3) trolley in movement with the load lifted, (4) drop off of the load.

The concepts of load chart and load cycle will be useful for generating the load and radius time histories.

### 3.2 Load and Radius time histories

Recording has been performed during five months on a crane working on a construction site. Figure 6 and Figure 7 present the recorded data versus the load chart respectively at the beginning and at the end of crane cycles. On the latters, horizontal tendencies can be observed at different levels of load. It corresponds to different natures of hoisted loads which can be identified separately. Three types of work were distinguished on the jobsite. The first one corresponds to the concrete pouring cycles where the concrete bucket is moved from the concrete batching plant to the wall or the floor to be fabricated. The second one corresponds to the positioning cycles. The loads lifted then are forms, walkways or prefabs. Finally, all the cycles that do not match the previous definitions are brought together in the category "other cycles".

Figure 8(a), (b) and (c) present the radii histograms respectively at the beginning and at the end of the crane cycles. Figure 8(a) shows that all concrete pouring cycles start at a radius of 50 meters. That corresponds to the position of the concrete batching plant on the studied construction site. All the other pictures associated to the radii present quite similar profiles. The mode is more or less situated around the mean radius of the jib.

Figure 9(a), (b) and (c) depict the maximum hoisted load histograms. Figure 9(a) depicts the histogram of the maximum load recorded during the concrete pouring cycles. The mean of the distribution is about 26 kN and corresponds to the load of the concrete bucket with the dynamic overload. Concerning the positioning cycles, the distribution of maximum load lifted is much more widespread. It can be explained by the fact that there are a lot of possible combinations of forms, walkways and prefabs. Most of the time, there are three standardized sizes of form on a construction site (2.5 m, 1.25 m and 0.625 m). An equipped form of 2.5 m width weighs 17 kN, dynamic overload included. It could explain the peaks situated approximately at 34 kN, 17 kN and 8.5 kN (see Figure 9(b)). The rest of the variability is probably due to the lifting of walkways or prefabs (balconies, stairs, etc.). Figure 9(c) shows the distribution of the load lifted during the other cycles where two main modes can be identified. The peak around 17 kN seems to correspond to the ballasts of the forms lifted very often on the construction site. The rest of
the variability of the distribution comes from the lifting of iron frameworks, junk buckets, etc.

![Figure 9](image)

Figure 9 – Load histograms for (a) concrete pouring cycles (b) positioning cycles (c) other cycles.

3.3 Load and radius random variables

The assessment of jib element use for each construction site can be made through the modeling of a set of random variables. The choice of parametric radius and load distributions is derived from the studied jobsite, considered as representative.

Concerning concrete pouring cycles, it was chosen to define a normal distribution for $R_i$ (see Figure 8(a)). The mean of the distribution is defined by the location of the concrete batching plant and the standard deviation is fixed by expert opinion. All the other distributions associated to the radii of the cycles are modeled thanks to triangular distributions (see Figure 8(a), (b) and (c)). Thus, for each triangular distribution, it is necessary to define three parameters (two bounds and one mode).

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Possible concrete buckets.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (L)</td>
<td>800</td>
</tr>
<tr>
<td>Weight (kN)</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Possible form configurations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N°</td>
<td>1</td>
</tr>
<tr>
<td>Weig.</td>
<td>w/4</td>
</tr>
<tr>
<td>Prob.</td>
<td>$\mu$</td>
</tr>
<tr>
<td>$R_i$</td>
<td>$R_i$</td>
</tr>
<tr>
<td>$w = 15$ kN, $\mu = 0.7$ and $q = 1-p$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Parameters for the fitted distributions for $R_i$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_i$</td>
<td>Estimation</td>
</tr>
<tr>
<td>Concrete pouring</td>
<td>$\mathcal{N}$</td>
</tr>
<tr>
<td>Positioning</td>
<td>$\mathcal{F}$</td>
</tr>
<tr>
<td>Other</td>
<td>$\mathcal{F}$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{F}$</td>
</tr>
</tbody>
</table>

Concerning the modeling of the load random variables, it was chosen to define a normal variable for the concrete pouring cycles with a mean equal to the weight of the filled concrete bucket plus a dynamic overload. The possible concrete buckets that can be randomly chosen are given in Table 2. Figure 9(b) shows that it is not possible to define a simple model for the lifted load during the positioning cycles. Instead, it was considered that no more than 5 linear meters of forms can be lifted in one time. Then, knowing that for a given radius of the trolley, the load chart constrains the maximum possible lifted load (see Figure 5), a configuration of forms is randomly chosen between the possible ones at each cycle. Table 3 gives the weight of all the possible form configurations and their associated probability. Note that a difference is made in term of probability between configurations 1, 2, 4, and 7 and the others. The implicit assumption is made that it is more likely to hoist these four configurations than the others. Concerning the other cycles, it was chosen to consider that all construction sites use the same ballasts for the forms. It implies to consider a linear combination of lognormal and normal distributions to model the other lifted...
loads (see figure 9(c)). The mean of the normal distribution corresponds to the load of a ballast. The mean and the standard deviation of the lognormal distribution are randomly chosen for each construction site. The implicit hypothesis here is to consider that all the randomness of the construction sites can be modeled by varying the parameters of the previous distributions. All the estimated parameters and the assumptions made on these parameters are summarized in tables Table 4, Table 5 and Table 6 where \( U, \mathcal{F}, \mathcal{N} \) and \( \mathcal{LN} \) are respectively the uniform, triangular, normal and lognormal probability laws.

<table>
<thead>
<tr>
<th>( L )</th>
<th>Estimation</th>
<th>Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete pouring</td>
<td>( \mathcal{N} )</td>
<td>( \mu )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma )</td>
</tr>
<tr>
<td>Positioning</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Other</td>
<td>( f_x(x) ) ( \alpha )</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_2 )</td>
</tr>
</tbody>
</table>

* Radius-dependent: \( \mu = L_{\text{max}} \) \footnote{For } \( R = R^{(i)} \), \( L \leq L_{\text{max}}^{(i)} \) (see Figure 5) \( \circ \) \( f_x(x) = \alpha \mathcal{N}(\mu_x, \sigma_x) + (1-\alpha)\mathcal{LN}(\mu_x, \sigma_x) \)

3.4 Equivalent number of cycles per year

Once the loading history of the jib element is established through Monte Carlo simulations based on the previous probability laws, an equivalent number of cycles at the reference load range \( \Delta F_{\text{ref}} \) must be determined (see Figure 2). This equivalent number of cycles \( N_{\text{eq}(\text{use})} \) at reference load range \( \Delta F_{\text{ref}} \) is calculated based on the Miner linear damage accumulation rule \( N_{\text{eq}(\text{use})} = \sum \left( \frac{\Delta F_{\text{ij}}}{\Delta F_{\text{ref}}} \right) \).

4. Calculation of fatigue damage probability

4.1 Monte Carlo Simulations

MC simulations are performed in order to assess the distribution of \( N_{\text{eq}(\text{use})}(x,t) \) where \( x \) is the vector of all random variables and \( t \) is the time in years. The steps are explained below:

For each crane simulation (i.e. for each MC iteration):
- Random sampling of a number of construction sites with their duration (\( U \) (4,24) months) and with the time between two construction sites (\( U \) (0.5,2) months).
- For each construction site:
  - Transformation of the construction site duration into a number of cycles assuming the following conditions:
    - the median number of working days per month is equal to 22.
    - the median number of crane cycle per hour is equal to 12.
    - the number of working hours per day is randomly chosen in the normal distribution \( \mathcal{N} \) (9.1,5) hours.
  - The number of cycles per type of work for the construction site is then inferred taking into account the following rates:
    - 33% for concrete pouring cycles, 17% for positioning cycles, 50% for other cycles.
  - Random sampling of jib length for the construction site according to the possible configurations from 25 m to 65 m.
  - Random sampling of the parameters associated to \( R_1, R_2 \) and \( L \).
  - Generation of the corresponding time-histories.
  - Cycle counting.
  - Calculation of an equivalent number of cycles for year \( t \).
- Registering of \( N_{\text{eq}(\text{use})} \).

4.2 Modeling of the equivalent number of cycles per year

Time-histories of 40 years have been generated 5000 times following the method presented above. The distributions of \( N_{\text{eq}(\text{use})}(x,t) \) found after 10, 20, 30 and 40 years are depicted in Figure 10. The distributions are modeled by lognormal distributions as the theoretical fitted curves show.
4.3 Reliability index calculation

To calculate the fatigue damage risk of the jib element, it is necessary to define a performance function. Due to the nature close to lognormal of the resistance and use distributions, it was chosen to define it using the logarithm of the number of cycles for both the resistance ($N_{res}$) and the use ($N_{eq(use)}(x,t)$) leading to an analytical expression of the reliability index.

$$G(x,t) = \ln N_{res} - N_{eq(use)}(x,t) \to \beta(t) = \frac{\lambda_{N_{res}} - \lambda_{N_{eq(use)}}(t)}{\sqrt{\sigma_{N_{res}}^2 + \sigma_{N_{eq(use)}}^2}(t)}$$

(8)

$\lambda_x$ and $\sigma_x$ represent respectively the mean and the standard deviation of the random variable ln($\lambda$). Figure 11 depicts the reliability index versus the time in years. Note that the value of the reliability index of the jib element is high regardless the number of years of work of the crane (more than 5 even after 40 years), compared to the target $\beta$ values proposed by recommendations such as [14].

5. Conclusions

In this work, a probabilistic approach has been developed to model tower cranes use and the reliability of jib elements was assessed. For 40 years of lifetime, the reliability index of these elements was found to be higher than 5, which is large compared to recommendations.

Two main perspectives were identified concerning crane use modeling. It is planned to include more data records coming from other jobsites, on one hand, and to develop a model allowing transformation of any jobsite drawing into crane loading time-histories, on the other hand. Relatively large statistics of crane use could be produced in limited time thanks to this technique.

6.References