



Dynamic building stock modelling: General algorithm and exemplification for Norway



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ABSTRACT

This paper presents a model based on dynamic material flow analysis, general in its principles and applied to the dwelling stock of Norway for exemplification. The algorithm at the core of the model is presented in the form of a pseudo-code and is described in detail. The driving force in the model is a population's need for housing and the necessary input are retrievable from national statistics on population, often dating back to around 1800, and its prognoses up to 2050 or beyond. Technical parameters such as the dwellings' lifetime and the renovation cycles are expressed by probability functions. Outputs of the model are the flows of construction, demolition and renovation; analysis of the renovation activity is given particular attention. The model shows how the renovation rates are a result of the need for maintenance of an ageing stock, and provides quantitative estimates of the present and future natural renovation rates, i.e. without specific incentives. The paper shows how to validate the model against statistics and other data sources, and how to use the model's future projections on construction, demolition and renovation activities in scenario based analyses of dwelling stocks' energy demand and greenhouse gases emissions.

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1. Introduction

Buildings account for 40% of energy use in the EU [1] and for about one third of both energy use and greenhouse gases (GHG) emissions in OECD countries [2,3]. The building sector is therefore important for the mitigation of climate change. The EU roadmap to a low carbon economy sets a decarbonisation target for the building sector at 88–91% by 2050, compared to 1990 values [4,5].

When forecasting energy demand or GHG emissions from a dwelling stock it is important to use a good model for the development of the dwelling stock itself, i.e. total amount of dwellings or floor area, in addition to analysing possible changes in energy or GHG emission intensities, i.e. energy or emissions per square metre of floor area.

However, it is often found that policy roadmaps and other studies use rather detailed information on energy and emission intensities, whereas the development of the dwelling stock itself – in terms of number of dwellings or floor area – is modelled using simple assumptions such as fixed rates for construction, demolition and renovation [6–17]. Some study even seems not to consider

changes in the stock composition at all [18]. Other studies make use of population forecasts to estimate the future need for dwellings, but make use of recent trends for estimating demolition and renovation activity [19–21]. Renovation rates are often assumed to increase rapidly and significantly in order to reach the energy efficiency goals for the stock. For example in Ref. [22] it is assumed that the average EU renovation rate, currently estimated at around 1%, will have to increase to more than 2.5% by 2020 and be stable thereafter in order to reach the EU 2050 roadmap decarbonisation target. Similarly, in Ref. [23] it is assumed that the renovation rate should step up to 2.3% or even 3.0% (depending on scenarios) already from 2015 in order to meet the EU policy goals by 2050.

All this may be regarded as static modelling, as opposed to dynamic dwelling stock modelling where future developments are mostly the consequence of past activities, the resulting effects of an ageing stock in need of maintenance and trends in underlying driving forces such as population development and standards of living.

A dynamic stock model was first developed by Müller [24] to analyse the Dutch dwelling stock. Based on dynamic Material Flow Analysis (MFA) principles and the underlying drivers in the dwelling stock system (population, floor area per capita, buildings' lifetime and material intensity per unit of floor area), the total floor

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area and materials demand were estimated for each year in the period 1900–2100. The floor area was modelled as the basic layer and the demand for materials as an additional layer. The demolition activity was estimated based on historical construction activity and a lifetime probability function, and the construction activity was calculated using mass balancing principles. Material flows were estimated using material intensities. The model was modified and applied to the Norwegian dwelling stock by Bergsdal et al. [25] and further developed by Sartori et al. [26] in order to model the renovation flows in an explicit way.

Similar dynamic dwelling stock models have also been used for studies of the Chinese dwelling stock [27–29]. Energy and carbon intensities were also introduced as additional layers of the models by Sandberg et al. [30,31] and by Pauliuk et al. [32]. The former analysed the long term development in energy demand and GHG emissions in the Norwegian dwelling stock; the latter studied transformations needed in the Norwegian dwelling stock to reach the 2 °C target. Further advances in the modelling consisted in segmenting the building stock in cohorts (construction periods) and building types Sandberg et al. [33,34].

The present methodology is based on the work previously done in the field, particularly on [26] though with some differences. In line to what first done in Ref. [33] the present work models the dynamics of the dwelling stock measured in number of dwellings rather than in floor area. This way one of the most uncertain input parameters, the average floor area per dwelling [25], is removed from the core of the model; while it remain simple to add it as an additional layer of the model in a later stage as it is done in Sandberg et al. [35]. Another difference is that the stock is divided in several cohorts.

This paper presents a detailed mathematical description of the dwelling stock model, showing the algorithm at the core of the model in a pseudo-code. The purpose of making the model's algorithm openly available is to provide a sound and transparent basis for applying the model in other studies and to the national dwelling stocks of other countries.

The model presented herein is general in its principles and is applied to the dwelling stock of Norway just for exemplification. This model is applied to 11 European countries in Ref. [36], where the resulting renovation rates towards 2050 are in the range of 0.6–1.6%; substantially below the level of 2.3–3.0% that should be attained already by 2020 in order to achieve the EU 2050 roadmap decarbonisation target for the building sector according to [22,23].

The model can serve as the basis for a range of applications, such as the analyses of energy use and greenhouse gas emissions, material demand and waste flows as well as market opportunities for component substitution. Many energy efficiency measures are cost efficient only if performed when a building is undergoing a deep renovation in any case. The present model aims at describing what renovation rates can be expected based on the characteristics of the dwelling stock and its need for maintenance.

The present paper does not include an energy demand or GHG emission analysis per se, as this requires a large amount of additional data and calculations. Such a study is presented in Ref. [35] where additional layers of floor area and energy intensity are added to this model in order to analyse the historical development of the energy demand in the dwelling stock of Norway from 1960 to 2015 and study phenomena and causes behind it. The following step, which is left for future work, would be to use the estimates of future construction, renovation and demolition flows from this model in order to investigate possible scenarios for the evolution of energy demand and GHG emissions from the Norwegian dwelling stock towards 2050.

2. Methodology

Dynamic Material Flow Analysis (MFA) is the methodology at the basis of the model presented here for studying the long term development of a dwelling stock. The driving force in the model is a population's need for housing and the necessary input are retrievable from national statistics on population, often dating back to around 1900 and earlier, and its prognoses up to 2100. Technical parameters such as buildings' lifetime and the frequency of renovations are expressed by means of probability functions. Outputs of the model are the flows of construction, demolition and renovation for the total dwelling stock and for each of the cohorts. Results from a detailed dynamic model like this can be used to estimate the natural turn-over and renovation rates in the dwelling stock, under different assumptions on the renovation cycle.

2.1. Mathematical model

A schematic representation of the model is given in Fig. 1, which shows how input data series (population, P , and Persons/dwelling, P_D) and parameters (demolition function, p_{DEM} , and renovation functions, p_{REN}) are combined, by means of equations, to generate output time series. The model's output time series are the flows of new construction, D_{new} , demolition, D_{dem} , and renovation, D_{ren} , activities. Several renovation functions may be considered, each with a different renovation cycle, e.g. 20, 30 and 40 years, referring to different types of renovation activities, such as replacement of equipment, envelope elements and full renovation, respectively. That explains using the plural when referring to the renovation activity, though in the following equations only one renovation flow is considered, for simplicity and readability reasons.

The equation set is explained below and the number in parenthesis in Fig. 1 is the number of the corresponding equation. Bold shapes in Fig. 1 represent variables that are obtained in an iterative process; see algorithm in Appendix .

A simpler visualization of the model's structure is given in Fig. 2 that emphasizes the connections between inputs, stock and flows while overlooking the actual equations linking the variables. It is useful to introduce this conceptual representation here since it is the same type as used in Ref. [35] (with additional layers) and [36], which are both presented in this issue and based on the model explained here.

2.1.1. Equation set

Convolution is a mathematical operator between two functions f and g , denoted as $f * g$, that expresses the amount of overlap resulting as one function is shifted over the other. The convolution between f and g in the discrete domain is formally written as:

$$(f * g)(n) = \sum_m f(m) \cdot g(n - m) \quad (0)$$

Table 1 shows the equations used in the model. Eq. (1) is performed on the whole input data series; Eqs. (4) and (5) refer to input parameters (not time series). Eqs. (2), (3) and (6) refer to input and

Table 1
The model's set of equations.

$S_D = P/P_D$	(1)
$\Delta S_D(i) = S(i) - S(i - 1) = D_{new}(i) - D_{dem}(i)$	(2)
$D_{dem}(i) = D_0(i) + (p_{DEM} * D_{new})(i)$	(3)
$L = 1 - CDM(p_{DEM})$	(4)
$p_{REN_cycle} = \sum_{k=1}^K p_{REN}(k) \cdot L(\tau)$	(5)
$D_{ren}(i) = R_0(i) + (p_{REN_cycle} * D_{new})(i)$	(6)

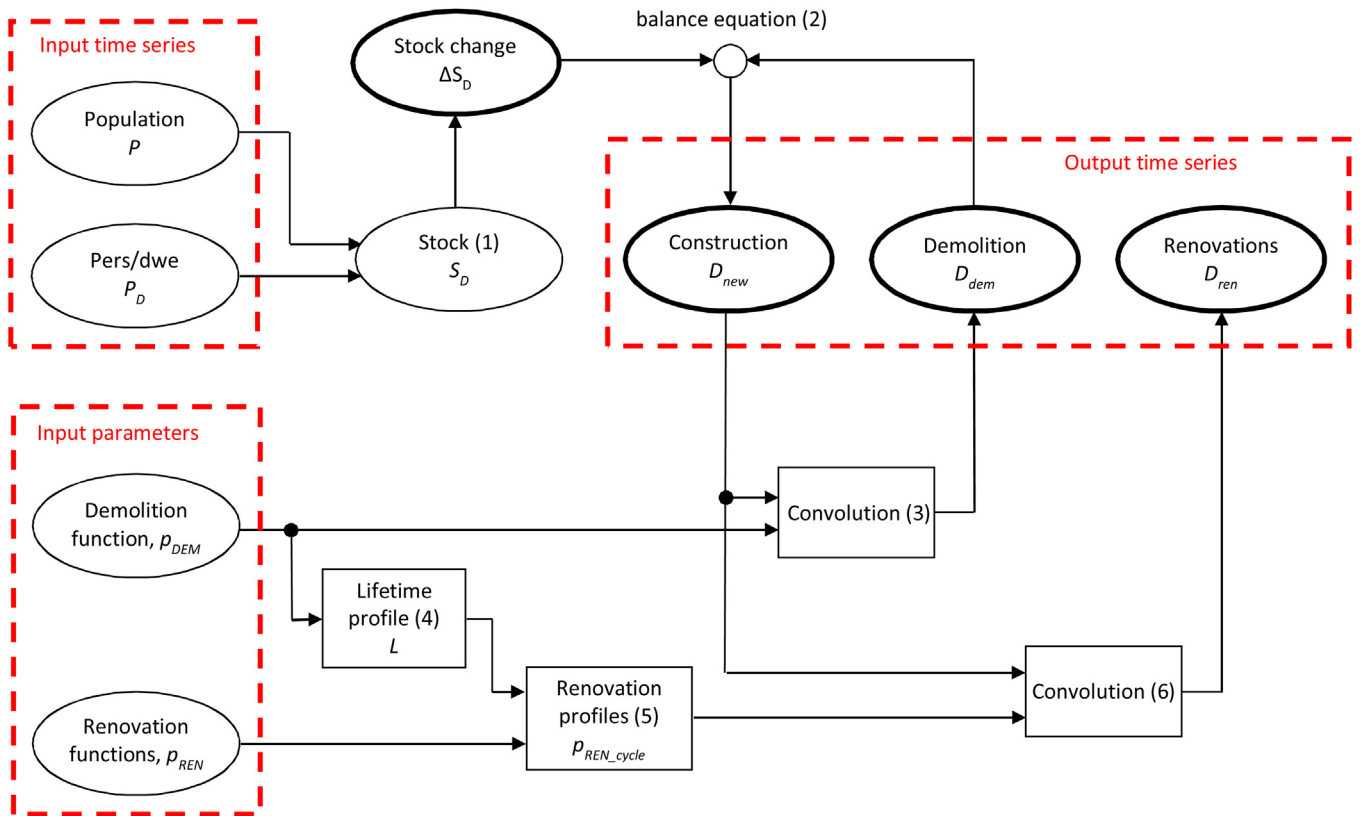


Fig. 1. Schematic of the model; equation number in parenthesis (). Shapes in bold for variables obtained/used in the iterative process.

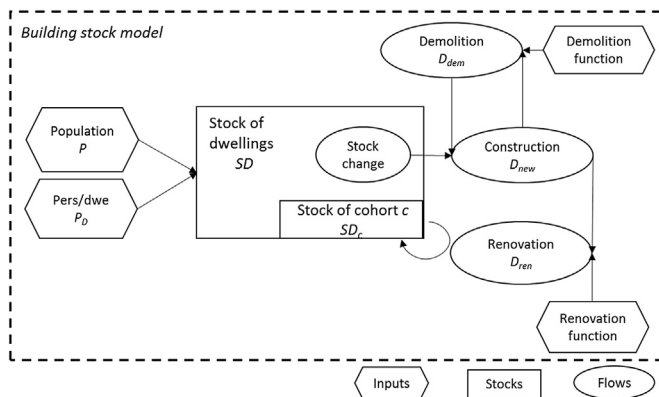


Fig. 2. Conceptual outline of the model.

output time series, but they are performed per each year, within an iterative process as shown later in the algorithm, see Appendix. Therefore, in such equations the index i is used to represent the i -th year of the iterative process. Note that Eqs. (3) and (6) make use of the convolution operator for calculation of annual demolition and renovation activities, respectively.

The first three equations govern the relation between the stock and the construction and demolition flows. With these three equations we can build up an iterative process that links together these variables and solve the equations' system for the entire period of observation and for future projections:

Eq. (1) We start from observing the total size of the stock (S_D), measured in number of dwellings. Here we need the data series in input: population (P) and household size in persons per dwelling (P_D); both the historic ones (based on census/statistics data) and

those for future projections (statistics expectation and assumptions);

Eq. (2) We look at how the total stock varies over the years (ΔS_D), hence the mass balance equation tells us the difference between new construction ($D_{new}(i)$) and demolition ($D_{dem}(i)$);

Eq. (3) Here comes in the demolition function (p_{DEM}), which is a probability function. The area under the demolition function curve is always ≤ 1 , since a dwelling can only be demolished once (and some may be preserved throughout the entire modelling period), see Fig. 4(top). The convolution between the demolition function (p_{DEM}) and the new construction (D_{new}) occurred in all previous years gives the corresponding demolition in year i ($D_{dem}(i)$). Additionally, $D_0(i)$ is the demolition of the initial stock, pre-existing the starting year of the model.

The fourth equation describes the link between the demolition function and the lifetime of a dwelling:

Eq. (4) The demolition function (p_{DEM}) is a Probability Density Function (PDF). The complementary of its Cumulative Distribution Function (1- CDF) gives the lifetime profile (L), which is the probability of a dwelling built in one year to be still in use in the future years, see Fig. 4(bottom);

It should be noted that the average lifetime refers to dwellings still in use, since the stock that is modelled is the stock of dwellings that provide housing to people; the disused dwellings are not modelled. Thus, the lifetime is the average useful lifetime, rather than the physical lifetime of a dwelling. It follows that the word 'demolition' should be interpreted in the broader sense of 'disused' dwellings, rather than "physically demolished" dwellings. Only a fraction of the dwellings in disuse is actually, physically demolished over time. Physical demolition may be due to urban renewal and densification, structural deterioration after long time of complete abandonment and accidental burning.

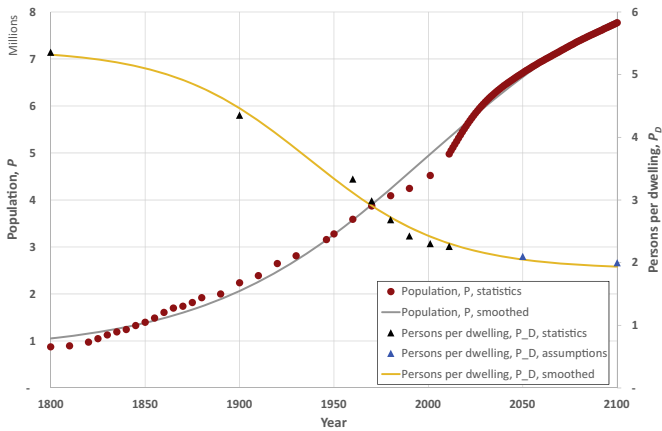


Fig. 3. Development of population P (left axis) and persons per dwelling P_D (right axis). Historical data and projections/assumptions for future development. Dots for raw input data and assumptions, line for the smoothed regression.

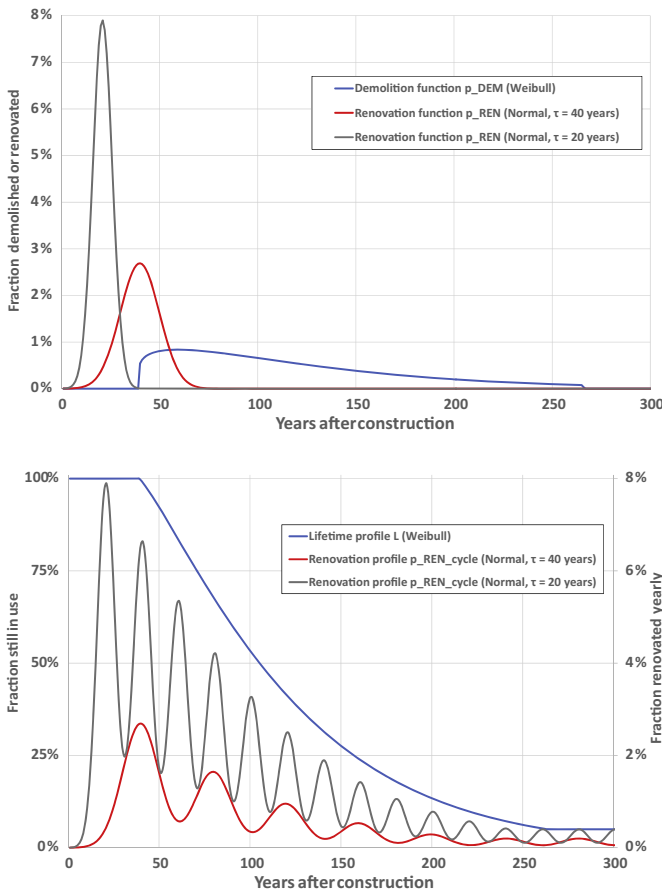


Fig. 4. Top: demolition function with Weibull probability distribution and renovation function with normal probability distribution, mean value τ of 20 and 40 years. Bottom: lifetime profile (left axis) and renovation profiles for average renovation cycles of 20 and 40 years (right axis).

The word ‘demolition’ may therefore be substituted with the word ‘disused’ for most purposes; though not for all. It depends on the application. When modelling the energy or greenhouse gases emission from the dwelling stock, what is relevant is the disused dwellings rather than the physically demolished ones. When a dwelling is left in disuse its energy use is no longer relevant. The physical demolition would have to be considered, instead, when

the object of study were the waste materials from the demolition activity.

Nevertheless, the authors prefer to adopt the word ‘demolition’ because of its more immediate understanding, leaving the further distinction between ‘disused’ and “physically demolished” dwellings to the specific application of the model.

Calculation of the renovation activity completes the model. This is a crucial part of the model, since the energy and GHG emission performance of a building stock will be significantly influenced by renovation activities, with respect to both timing and depth of the energy upgrading.

Eq. (5) Renovation is characterized by a renovation function (p_{REN}). While demolition can take place only once, renovation can take place several times during the lifetime of a building, so the renovation profile is defined (p_{REN_cycle}). This is the result of cyclic repetitions (K) of the renovation function (p_{REN}) weighed against the lifetime profile shifted by τ years ($L(\tau)$). Such shifting is done to avoid renovating a building today and demolish it tomorrow, in the model. We assume τ equal to the renovation cycle as a suitable shift; meaning that the expected lifetime of a dwelling after renovation shall be at least as long as the renovation cycle itself (e.g. $\tau = 20, 30,$ or 40 years depending on the case). The cyclic renovation profile results damped over the course of time. The area under this curve (named the renovation number N_R) is usually >1 , and tells how many times the average dwelling is renovated during its lifetime, see Fig. 3(b).

Eq. (6) The convolution between the renovation profile (p_{REN_cycle}) and the new construction (D_{new}) occurred in all previous years gives the corresponding renovation in year i ($D_{ren}(i)$). Additionally, $R_0(i)$ is the renovation of the initial stock, pre-existing the starting year of the model.

It should be noted the renovation rates calculated by this model can be seen as the “natural renovation rates” resulting from the internal dynamics of a dwelling stock, and therefore express the need for maintenance of an ageing stock. Note also that the renovation activity is calculated in parallel to the other activities, so that its value does not affect the other flows. Note that long lifetime allows for many renovation cycles (therefore high cumulative renovation activity). In “real life” one might see it the opposite way: more renovation increases the lifetime of buildings. Mathematically it does not make a difference what is the cause and what is the effect: the important is that there is the relation long lifetime equals more renovation, and vice versa.

In summary: the stock is the input (population’s need for housing) and the flows (new construction, demolition and renovation) are the outputs. That is why we need only non-technical time series data in input; because the driving force in the model is the population’s need for housing (long term characteristics, unaffected by short term market fluctuations). The technical parameters are the two probability functions for demolition and renovation.

2.2. Input and output of the model (using Norway as an example)

Long-time series are needed when modelling systems with long lifetimes, such as building stocks. A time horizon from 1800 to 2100 is used for the case study of the Norwegian residential building stock although the period 1900–2050 is of highest interest when interpreting the results. The initial stock in the starting year (cohort 0) has to be treated differently in the model, as there is no information about its age structure. Further, the dwelling stock is segmented into cohorts, according to their construction periods, as presented in Table 2. Any segmentation in cohorts is arbitrary, and the one chosen here reflects some major socio-economic changes in Europe, simply to serve the purpose of comparability between countries, as in Ref. [36]. On a national scale it would be possible to fine tune the definition of the cohorts and link it, for example,

Table 2
Definition of cohorts.

Cohort number	Start year	End year
0	–	1800
1	1801	1945
2	1946	1980
3	1981	2015
4	2016	2050

to specific changes in the prevailing construction technology in different periods. Dwellings constructed before World War II are in the first cohort, the post-war construction boom is in cohort 2, dwellings constructed in recent decades are in cohort 3 and dwellings that are expected to be constructed in the future are in cohort 4.

2.2.1. Input time series

Data on historical development in population P and persons per dwelling P_D are taken from censuses back to 1800 and projections [37,38]. The time series needed as inputs to the model are obtained using raw data and non-linear regression resulting in a smooth input curve that removes short term fluctuations from the results. Fig. 3 shows the input time series for Norway.

2.2.2. Input parameters

Buildings have a long lifetime and so it is difficult to find data that go far enough back in time to observe the entire history of a building stock. Analyses found in literature may be based on a relatively small sample of buildings or on observations limited to the oldest cohorts [39–41]. Lifetime distributions are often approximated with different functions, such as Normal, Log-Normal, Weibull, Gompertz [24,39,40].

The assumed average lifetime is important for the model results [25,34], but there is high uncertainty also in the assumed lifetime distribution. Previous dynamic dwelling stock models have commonly assumed a normal probability function for the lifetime. Although the normal distribution is well known, easy to implement and use, and suitable for many cases, there are some weaknesses when applied to mortality of dwelling stocks. According to [41] the Weibull distribution seems to be the most adequate distribution for demolition of dwellings. The location parameter of the Weibull distribution is a direct specification of an initial period where the probability is zero. Further, for some parameter combinations, the shape of the Weibull distribution gives a long tail – when applied for the demolition of buildings, this represents the heritage buildings that are preserved rather than demolished. Fig. 4(top) shows the demolition function when using a Weibull distribution with an average lifetime of 125 years and an initial period with no demolition of 40 years. The corresponding lifetime profile is shown in Fig. 4(bottom). This lifetime profile has a shape that is similar to the reliability function estimated for the Norwegian residential buildings in Ref. [39], which also estimated the average lifetime at 126 years based on observations of changes in the cohort composition of the stock, as reported in the censuses [37].

The demolition function is truncated in the sense that at a certain time no more dwellings from this construction year are demolished. This represents the share of dwellings that are preserved for architectural reasons or that are never demolished for other reasons.

Data availability on renovation activity is poor, and though some scattered data is reported by some EU countries [42,43] the information may be of little help because the term ‘renovation’ itself is loosely defined and data from different countries may not be easily compared. Thorough interventions on the building’s envelope, regular maintenance and simple reparation of broken elements, all

Table 3
Share of the 2011 dwelling stock in different cohorts.

Cohorts	Statistics [37]	Model
<1946	17%	22%
1946–1980	43%	34%
1981–2011	36%	44%
Unknown	4%	0%
Total	100%	100%

are reported under the general term of renovation, as well as the modernisation of bathrooms and kitchens [44]. Where figures for the single measures are available, the numbers telling the affected share of the stock per year are always with one digit. Normally it is 1–3% for single measures on the building’s envelope though in some case, e.g. outer insulation of outer walls, it may be even <1%; and higher for indoor measures like changing the heating system or modernisations, in the range 3–5%. Where aggregated numbers on any kind of renovation are given, the numbers are around or slightly above 10%. From other sources it may also be unclear whether the reported values refer to the entire stock, or only the residential or non-residential sectors, as reported in Ref. [22].

Most importantly, there are different ways of collecting the data. In Ref. [44], some countries used the number of dwellings (Sweden) while others used data on the investments made in renovation activities (Finland, France and Switzerland), or both (Austria and Germany), and thus indirectly inferred the volume of the stock being affected by renovation.

This reality opens up a window of opportunities for using model simulations to fill the gap of missing empirical data, and to explore important cause–effect relationships in the system. Three renovation cycles are explored and modelled with a normal probability function: with average time between renovations of 20, 30 and 40 years, respectively. The 20 years cycle may exemplify the replacement of the heating system; the 30 years cycle the replacement of construction elements, such as windows or roofs, and the 40 years cycle may represent the deep renovation of facades [12,44,45]. The renovation function and the renovation profiles for cycles of 20 and 40 years are shown graphically in Fig. 4(top) and (bottom), respectively. The dampening effect on the renovation profile is clearly visible. Remember from the description of Eq. (5) following Table 1 in Section 2.1.1 that the renovation function is weighed against the lifetime profile shifted by a renovation cycle. This is done to ensure that a building undergoes a renovation with cycle τ (i.e. 20, 30 or 40 years) if and only if it is expected to be still in use for at least another τ years before demolition. It can be seen in Fig. 4(bottom) that the damped renovation profiles forego the lifetime profile due to this reason. The long tail in the lifetime profile, resulting from the use of a Weibull probability function and the truncation of the demolition function, also influences the renovation cycle as the heritage buildings are renovated many times.

The renovation number R_N tells the average number of renovations for the dwellings in the stock and is equal to the area under the renovation profile curve (for the given renovation cycle). Considerations on the renovation number apply to any period since the number is purely a function of the lifetime and renovation profiles used as input parameters, and does not depend in any way from the specific input time series of a given stock.

With the given lifetime and renovation profiles for Norway, when limiting the observation to a time frame of 300 years, i.e. from 1800 to 2100, we obtain $R_N = 1.63$ for a renovation cycle of 40 years. This means that while short living dwellings are never renovated and long living dwellings are renovated several times, in average a dwelling is renovated approximately one and a half times in the course of three centuries. When limiting the observation to a time frame of 100 years, e.g. from 1950 to 2050 (which is the

period of interest for the policy horizon, and knowing that most of the stock is actually built after WWII, see Table 3), we obtain $R_N = 1.08$. Thus we may say that until 2050 we have in average (and net of the demolition effect) only one chance of renovating a post-WWII dwelling if we assume a 40 years renovation cycle. These figures should give a clear impression of the inertia in the building stock and the risk of lock-ins, and therefore a clear understanding of how precious an occasion renovation is for introducing long lasting energy conservation measures.

However, we obtain a much larger $R_N = 1.81$ when assuming a renovation cycle of 30 years for the same period 1950–2050; meaning we nearly double the average chances of renovation. With a renovation cycle of 20 years we obtain $R_N = 3.21$, a threefold increase. This should already give the insight that any policy aiming at realizing the vast energy savings potential of the building stock should consider as its highest priority to stimulate a more frequent renovation of buildings.

2.2.3. Uncertainty of inputs

The model is run using the described input data which is the best available data or qualified assumptions as gathered by the authors. There are of course uncertainties in the input data, and when later applied for energy analyses it is understood that the uncertainty in the underlying building stock model is crucial [46]. A separate paper [34] examines in-depth the sensitivity of input parameters on the final results and conclusions for the segmented dynamic dwelling stock model for the Norwegian case. The sensitivity analysis does not lead to unexpected changes in results and shows that the model is mostly sensitive to changes in population and dwellings' lifetime. Scenarios with extreme input values for population and dwellings' lifetime are considered in order to investigate the higher and lower boundaries of the resulting renovation activity. The results prove that the main conclusions regarding expected future renovation rates and activity still hold.

2.2.4. Output time series

The model's outputs are time series on historical development from year 1800 and future estimates to year 2100 of the dwelling stock size S_D and the activity flows of new construction D_{new} , demolition D_{dem} and renovation D_{ren} . Though the model considers three renovation flows of 20, 30 and 40 years respectively, as discussed in the input parameters, the outputs in Fig. 5 only show the renovation flow for the 40 years cycle for pure reason of readability. The purpose of Fig. 5 is simply to offer an understanding of what outputs are generated by the model and how they look. In the discussion of results, all three renovation flows are considered. For example, note how the demolition flow is delayed with respect to the construction flow due to the long lifetime of buildings. Note also how the renovation flow follows the same development as the total stock, though on a different scale, again with a certain delay due to the renovation cycle shifting, see Eq. (5) in Table 1. Fig. 5 also shows the flow of stock change ΔS_D , which is an internal variable in the model and is shown to offer a complete overview, together with previous Figs. 3 and 4, of all variables used in the model's equation set and presented in its schematic representation in Fig. 1.

2.3. The model's algorithm

While the mathematical description of the model as presented above remains valid, the actual implementation resorts to a slightly different algorithm. The algorithm is presented in pseudo-code in Appendix. Instead of performing directly the convolution operation, the results for future demolition and renovation activities relative to a specific year of construction are saved in column-wise vectors at each time step. So, while stock and new construction flow are represented by vectors of length n (= time steps), demoli-

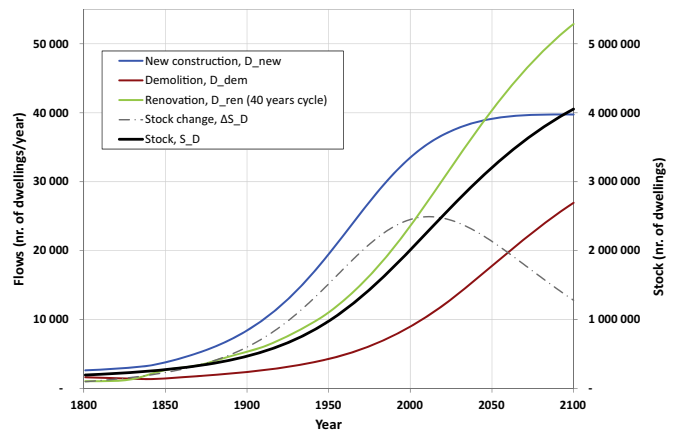


Fig. 5. Development of dwelling stock (left axis) and flows (right axis). Historical data and projections/assumptions for future development.

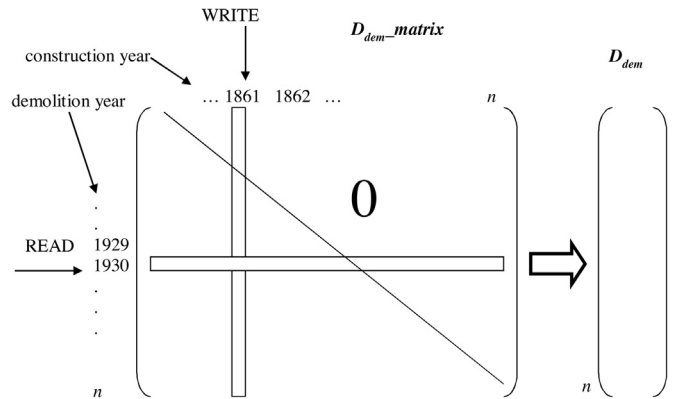


Fig. 6. The two-dimensional matrix and the corresponding vector; example for the demolition matrix.

tion and renovation flows are stored as two-dimensional matrices of dimension $n \times n$. The matrices are written in columns, each column corresponding to the activity derived by the construction in a single year. It follows that these are triangular matrices, filled with '0' in the upper-right part. Subsequently, reading such matrices row-wise (= summing up all the elements in a row) results again in vectors of length n , containing per each year the amount of demolition or renovation activities that are due to constructions made in all the previous years, see Fig. 6. At the end the results are exactly the same as if calculated by means of the convolution operations. Indeed, the operations performed correspond exactly to the definition of the convolution operation in the discrete domain, see Eq. (0) before Table 1.

The reason for adopting this algorithm has to be addressed to its more intuitive approach as well as the fact that the two-dimensional matrices for demolition and renovation contain additional information on "where" (= which year of construction) the demolished and renovated buildings come from. This information is indispensable for future analysis of material and energy flows.

Finally, it should be reminded that the algorithm is implemented with a subdivision of the stock and flows in cohorts and using three renovation flows, but the explanation reported here treats the entire stock as a single cohort and uses only one renovation flow, for reasons of simplicity and readability.

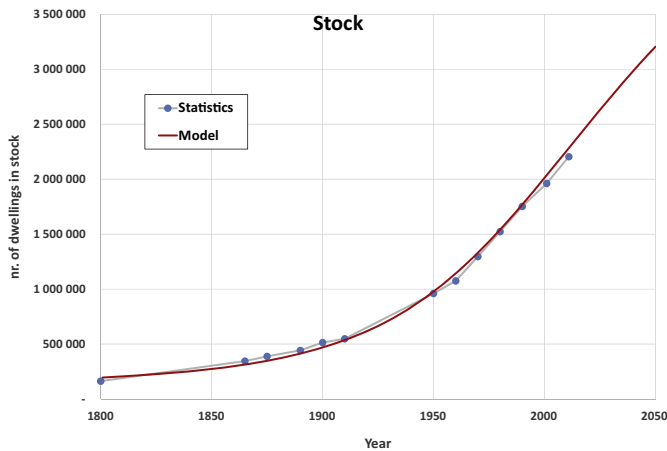


Fig. 7. Comparison of the model results with data from statistics: dwelling stock.

3. Results and discussion

This section discusses how the model can be validated against existing statistical and other data sources, and how the model's future projections may be used to gain insights into possible evolutions of the national dwelling stock under analysis. The purpose is to discuss the two aspects of model validation and future projections in general, while the data from the Norwegian case are used purely as an exemplification.

3.1. Model validation

The model's output time series can be compared against available data from statistics. However, it is often found [36] that only statistical data on the total number of dwelling in the stock are available in long time series, from censuses dating back to the beginning of the 20th or 19th century. Time series data on new construction are also generally available, though only for the most recent decades; in general after WWII only. Data on demolition and renovation in the form of time series are almost never available. Point data or estimates may be found in some case for some specific year (e.g. linked to an occasional survey) and/or some time series data may exist on investments in renovation rather than directly on the number of dwellings or square metres, as discussed in Section 2.2.2 Input parameters.

3.1.1. Dwelling stock

For Norway, time series data are available for the dwelling stock and the construction flow. Dwelling stock data from censuses [37] are actually an input to the model, so the comparison is done to check that the model is set up correctly and that the smoothing of input data does not lead to large errors in the results, as seen in Fig. 7.

Although the total dwelling stock (population's need for housing) is actually an input to the model, its distribution in age cohorts is not. Census data can be used for comparing the stock composition in given years and hence evaluate how well the model fits with reality. In Table 3 the results of the model are compared with the dwelling stock composition described in the 2011 census [37].

In the Norwegian statistics data, 4% of the dwellings are of unknown construction period. Most of these dwellings are probably in the first cohort (<1946), since the reporting systems have improved over time. The share of dwellings reported to be constructed before 1946 plus the share of unknown age is 21% which is close to the modelled share of 22%. The modelled share of the stock is somewhat lower than statistics for the period 1946–1980 and equivalently higher for the period 1981–2011. This is the effect

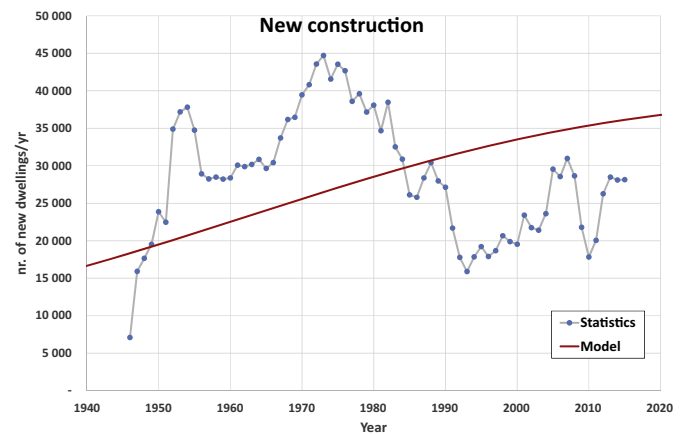


Fig. 8. Comparison of the model results with data from statistics: new construction.

of the modelled construction activity being below the statistics values until the mid '80s and above thereafter, as shown in Fig. 8 (see related discussion below in Section 3.1.2 New construction). Altogether, the differences in the cohort composition between the model results and the statistics are deemed acceptable, being below 10%. A similar pattern was observed and conclusion made for the other countries studied in Ref. [36].

3.1.2. New construction

In Fig. 8, the model results for construction activity are compared with statistics [47]. There have been large variations in the number of dwellings constructed each year. It shall be noted that new construction is sensitive to mid- and short-term socio-economic factors that are not considered in the model, such as the post-war construction boom, variations in the general economic situation of the country, as well as fluctuations in the estate market. The statistics therefore present variations that our model is not able to – and neither is meant to – capture.

However, the model's under-estimation in the period 1950–80s and over-estimation thereafter may be explained by different modes of urbanisation [38]. During the period 1950–80s urbanisation was more pronounced and happened mainly by expansion of the cities' footprint, accompanied by massive diffusion of automobiles within a general tendency towards increased urban sprawl. This created a demand for new dwellings "by migration"; that is additional to the demand "by increasing housing need" of a population (as a result of both increase in population and decrease in persons per dwelling) or "by substitution" of demolished dwellings, which are the phenomena modelled in Eqs. (1) and (2) in Table 1. In the following period from the 1990s to present, urbanisation in Norway has been less pronounced and has happened, at least in major cities, in a context of city densification rather than expansion. This means that new dwellings may be created by down-sizing existing ones, i.e. during renovation work, and therefore part of the increase in the population's need for housing is covered without resorting to new physical constructions. However, the censuses will then register an increased number of dwellings that the model tries to explain as new built, see Eq. (2) in Table 1.

What matters for the validation of the model is its ability to capture long-term trends that are the consequence of a population's need for housing, net of the 'noise' introduced by transitory effects. The model seems to be able to reproduce the long-term trend of the construction activity. The total number of dwellings constructed over the entire period 1946–2013, thus also the average number of dwellings constructed per year, is just 1.5% lower in the model than it is in the statistics.

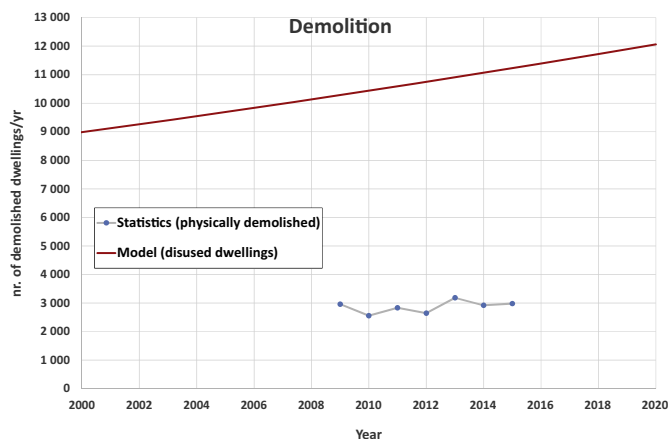


Fig. 9. Comparison of the model results with data from statistics: demolition.

Taking the stock in year 2010 as a reference¹ – which is an arbitrary but useful choice to compare past trends and future projections – we see that the rate of yearly new construction from statistics oscillated between 0.7 and 2.0% in the period 1946–2015 (being at around 1.2–1.3% in the last decade, as reported also in other national studies [18–10]). In the model, the rate increased from 0.8 to 1.5% during the same period, following the same pattern as shown in Fig. 8.

3.1.3. Demolition

Demolition time series data for Norway are available only from 2009 [47], showing a slightly increasing trend – though it is not possible to say anything about long-term behaviour with such a short series of data – averaging at about 3000 dwellings/year; which gives a share of 0.13%. The model's result for the same period is three times as high at about 10,000 dwellings/year, as shown in Fig. 9.

One obvious explanation might be the average lifetime, which should be longer in order to cause less demolition activity in the model. However, changing the lifetime would affect also stock composition, new construction and renovation because all variables are linked together. The model match with statistics is satisfactory for all other variables, suggesting that the assumed lifetime of 125 years, interpreted as useful lifetime as discussed in Section 2.2.2 Input parameters, as well as the assumed Weibull profile should be reasonable estimates.

The demolition over-estimate can rather be explained by the difference between disuse and physical demolition of a building (and the dwellings in it). In the model, when a dwelling reaches the end of its useful life it is considered as demolished, in the meaning of “dwellings exiting the model's stock of inhabited buildings” as discussed in Section 2.2.2 Input parameters. However, in reality this is not necessarily the case, since buildings may be simply left uninhabited rather than physically demolished. This may be especially true in a country still subject to some urbanisation process like Norway [38], because people moving to urban areas leave behind uninhabited buildings that are not necessarily demolished.

If we were to force a match between the model's result on demolition (or disused dwellings) and the statistics (physical demolition) for the years that the latter are available, this would result in a share of 27%. Meaning that only 27% of disused dwellings are physically demolished in any given year (regardless of how long they may have been in disuse). Although this is a forced interpretation, it is interesting to explore how it may help answer the questions: how many are these uninhabited buildings? And what happen to them?

¹ For the model, while for the statistics it is 2011, the year of the last census.

Two factors should be considered. First, while the censuses only report the number of inhabited dwellings, other national registers of property account also for uninhabited ones. The number of inhabited dwellings in 2011 was ca. 2.2 million, with an additional 210,000 uninhabited dwellings [47]: a non-negligible amount of ca. 10% of the inhabited stock. Dwellings counted as unoccupied in the census because occupied by persons registered on another address, e.g. students, may only partly explain such large difference.

Second, one should take a look at vacation homes. In the last decades vacation homes have been built on purpose, showing a steady increase from about 2000 units per year in the early 1980s to about 5 000 units per year in the early 2010s, for a total of ca. 100,000 built in the period covered by the statistics (which began in 1983). Additionally, one may estimate approximately another 50,000 built previously – and still standing – considering an average lower rate of construction at about 1000 units per year for the previous 50 years (according to the Norwegian Tourist Association, construction of vacation homes for other than the elites began in the 1930s [48]). Nevertheless, the amount of vacation homes existing per 2011 was just above 400,000. The difference between existing and built-on-purpose vacation homes is thus 250,000 units, which may therefore be former houses (in coastal or rural areas) that have been disused as permanent housing but are still in use as vacation homes. It should further be noted that for this type of houses (mostly former farms or fishermen's houses) it is not improper to assume that one house/building equals one dwelling.

This gives us a total of ca. 460,000 dwellings, summing uninhabited ones and non-built-on-purpose vacation homes. Now, if we consider that only a fraction of 27% of the model's demolition flow, or disused dwellings, is actually demolished every year (the forced interpretation mentioned above) we can count the remainder part as dwellings that should be still standing though not in use as permanent housing. If we do this for the whole modelled period, i.e. from 1800, we get a total of about 550,000 dwellings. If we limit the counting from 1850 – approximately the period when both urbanisation and emigration began to take on a major character in Norway – the count goes to about 500,000 dwellings.

At the minimum, these numbers show that it is possible that the relatively large discrepancy between the model and the statistics on demolished dwellings is explained by the large amount of dwellings still standing but not in permanent use in Norway.

The demolition/disuse rate from the model for recent years is about 0.4–0.5%, close to the assumed 0.6% in national studies [9,10]; while in Ref. [8] a rate of 0.2% was used, taking it directly from estimates of physical demolition [6].

3.1.4. Renovation

Data on renovation are scarce in Norway, and are indirect data based on economic indicators. For example, it is estimated that renovation work in average – per square metre of floor area – costs 72.5% of the cost of building new [49]. Knowing the total investment that nation-wide goes into renovation work one can infer the amount of square metres renovated. However, time series statistics on investments in buildings normally do not discern between new construction and renovation, and only point data are available such as from Ref. [50] for 1996, in which year the renovation activity would then result in about 2.51 mm² compared to about 2.67 mm² of new construction.

The production index for construction is available as a time series from the mid 1990s, with split between new and renovation work. It is then possible to extrapolate time series data on renovation activity starting from point data, such as the one from 1996, and the result so obtained are shown in Fig. 10.

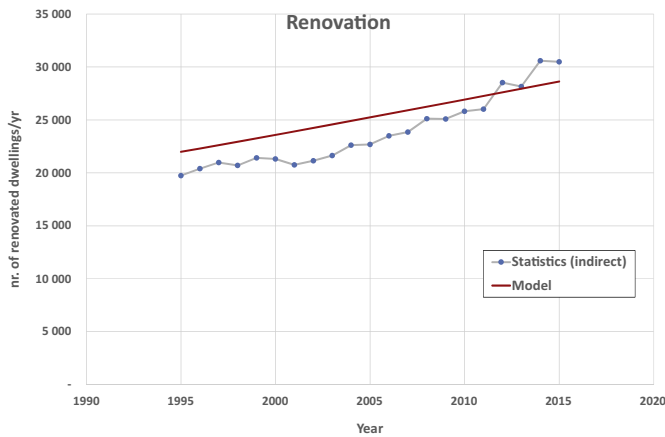


Fig. 10. Comparison of the model results with data from statistics: renovation (40 years cycle).

It must be said that this approach is subject to three types of error. First, the production index is a volume index,² thus not a perfect proxy for determining the quantity of an activity because it reflects changes in both quantity and quality. For example, if there is a fall in the total quantity of goods produced or goods consumed, volume data will decrease. But volume data will also decrease where the production or consumption pattern is changing towards cheaper goods (even if total quantity is not decreasing). Second, converting an index time series into an activity time series is going to be biased by the data point used for the conversion, from year 1996 in this case. Finally, it requires differentiating between the average size of new built dwellings in 1996, 149 m² [47], and the average size of renovated dwellings in 1996, assumed to be equal to the stock average, 123 m² [12].

Despite these limitations, we can get a sense of the goodness of the indirect time series data so obtained for renovation by repeating the same exercise for new construction, for which we also have direct data. When doing so we obtain a new construction data series that is as ‘bumpy’ as the one shown in Fig. 8 (since the production index itself is almost equally ‘bumpy’ too) but a total over-estimation of 12% for the entire period 1995–2015. This is mainly due to the bias of using a single data point for converting the index time series into the activity time series,³ as discussed. In particular, the year 1996 happens to be at the lower end of the new construction range for the period 1995–2015 (see Fig. 8), thus causing the over-estimation.

The production index for renovation is more even than the one for new construction, suggesting that extrapolating a time series from a single data point is less subject to error. The model results in Fig. 10 show a good fit with the indirect data from statistics. Expressed in percentage over the stock size in year 2010, the rate of yearly renovation from statistics increased nearly linearly from 0.9 to 1.4% over the period 1995–2015, while the model results show a more moderate increase from 1.0 to 1.3%. These values are in line with other estimates from both national studies [9,10] and [8] (reporting 1.5% and a range from 0.6 to 1.3%, respectively) and international ones focusing on North-Western European countries [44] (reporting 1.2%).

² Meaning it monitors volume movements by holding the price constant. By keeping fixed the prices, period to period changes reflect changes in the quantities and/or the quality of the different products.

³ Had we used the period's average – which we know from statistics only for new construction but not for renovation – as basis for the conversion, we would have obtained a time series slightly different in each year's value but with the same total for the entire period.

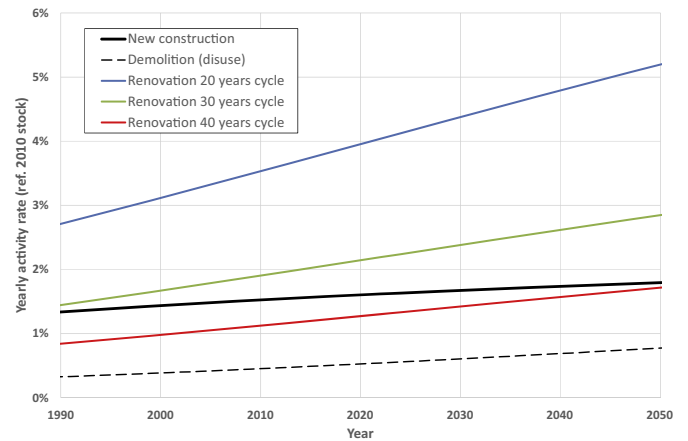


Fig. 11. Yearly activity rate for new construction, demolition and renovation, relative to the 2010 stock size. Model results for the recent past and future projections.

3.2. Future projections

The model's output time series can be used as future projections of how construction, demolition and renovation activities may evolve in a given dwelling stock. The focus in this section is mainly on the renovation rates. Recalling the discussion on the renovation activity in Section 2.2.2 the renovation rates calculated by this model can be seen as the “natural renovation rates” resulting from the internal dynamics of a dwelling stock, and therefore express the need for maintenance of an ageing stock.

A sensitivity analysis in the case of Norway [34] showed that the model is most sensitive to variations in the population projections and the average lifetime of dwellings, which are also the inputs with highest uncertainty. However, these input variations affect mainly the future projections of construction and demolition activities. Renovation activity towards 2050 mainly depends on the current stock size and composition and is not significantly sensitive to future population developments. In Ref. [34] it is shown that even when changing population projections and the average lifetime of dwellings to extreme and unrealistic values, the main conclusions regarding future renovation rates remain unchanged.

Fig. 11 shows the model's result for the Norwegian dwelling stock in the recent past and future projections for construction, demolition and renovation rates, all relative to the 2010 dwelling stock size; see Refs. [33] and [34] for a detailed analysis.

The construction rate is expected to be rather stable, increasing from 1.5% in 2010 to 1.8% by 2050. Since data on construction activity are normally available, it would be advisable always to consider these and eventually adjust the model's projection accordingly. For example, as discussed in Section 3.1.2 New construction, the model's under- and over-estimation of actual construction activity in different periods may be due to the model's inability to capture urbanisation trends that swing from sprawling to densification. Since it is known that the recent trend is towards densification – and this is expected to continue – it would be convenient in any future analysis to assume a construction rate somewhat lower than what estimated by the model. In other national studies the construction rate for the coming decades is assumed, based on recent trends from statistics, at around 1.0–1.3% [8–10].

The demolition rate is expected to increase due to the growing and ageing stock. As discussed in the previous Section 3.1.3 Demolition, it would sound more reasonable to consider the demolition rate from the model – more properly representing the disused dwellings – rather than values from statistics on physical demolition activity when analysing future scenarios on energy demand. While [8] had considered estimates of physical demolition of 0.2%,

Table 4
Renovation activity (40 years cycle) in different years broken down per cohorts of the stock.

Year	Cohort 0: ->1800	Cohort 1: 1801–1945	Cohort 2: 1946–1980	Cohort 3: 1981–2015	Cohort 4: 2016–2050	Total [dwe/y]
2010	4%	29%	54%	13%	0%	26,919
2030	3%	16%	26%	54%	1%	33,769
2050	3%	11%	22%	44%	20%	40,366

[9] and [10] used a rate of 0.6%, which is within the range of the increase from 0.5 to 0.8% described by the model for the period from 2010 to 2050.

The model considers different renovation cycles of 20, 30 and 40 years, which can be used in future analysis to model the effect of different renovation measures. As discussed in Section 2.2.2 Input parameters, the 40 years cycle may be suitable to represent deep renovation of the facade, while the 30 years cycle may represent the replacement of other parts of the envelope (e.g. windows, roof), and the 20 years cycle may represent replacement of the heating system. Alternatively, analysis of the renovation rates resulting from different renovation cycles may suggest the need for energy policies to stimulate more frequent renovation of buildings.

Looking at renovations that take place with a 40 years cycle, Fig. 11 shows that an increase in the renovation rate is expected as the stock ages and more dwellings face the need for maintenance, moving from just above 1% in 2010 to ca. 1.8% in 2050; the average for the period 2020–2050 being about 1.5%. This is also the renovation rate assumed in the national studies [9] and [10] for major renovations of buildings, while [8] had considered a range from 0.7 to 1.4%. Additionally, [9] and [10] considered a further activity of single energy saving measures on the envelope and estimated it at a rate of 2%, close to today's value for the 30 years renovation cycle in the model. Nevertheless, it is disputable whether the effect of single measures should be added to the effect of major renovations.

In fact, the 40 years cycle may be assumed to be representative of major renovations but it shall be reminded that in literature this value is reported as the frequency of one type of single measure on the building's envelope – perhaps the most difficult to implement: deep renovation of external walls. Taking this renovation flow as representative of a full deep renovation the entire building's envelope may be a suitable approximation in the domain of scenario making. It is the equivalent of lumping together the benefits of several single measures – that have taken place at different times – into a single event, which has the same frequency of the least frequent measure. When this “single event” is further spread in time by means of a probability distribution function, as is done in this model, it seems altogether a reasonable approximation of a complex phenomenon such as the deep renovation of buildings.

The authors' recommendation is therefore to consider the 40 years renovation cycle to represent deep renovation of the envelope and/or the lumped effect of subsequent single measures (as done in Refs. [8,33] in literature and in Refs. [35,36]). Alternatively, one may consider a combination of 30 and 40 years renovation cycles to represent the different single envelope measures, such as new windows/roof and facade renovation, respectively. Adding the effect of both would amount to double counting. On the contrary, the 20 years renovation cycle would be suitable to model the shift in the energy carriers mix (sometimes called fuel switch), i.e. due to changes in the heating or cooling system. Since the substitution of the heating or cooling system can normally happen regardless of other measures on the building's envelope,⁴ it would be correct to

add the effect of both types of renovation when modelling changes in a dwelling stock's energy demand.

Nevertheless, the trend of the “natural renovation flow” with a 40 years cycle will not be sufficient to reach the range of 2.3–3.0% that in Refs. [22,23] is deemed necessary, in a EU-wide context, to achieve the EU policy targets on decarbonisation by 2050. Policy measures are needed to increase the number of dwellings going through deep renovation, or when using the terms of this model: policy measures are needed to reduce the average time between deep renovations of dwellings. For the Norwegian case reaching an average renovation rate of about 2.5% in the period 2020–2050 means that the average interval between deep renovations should be approximately 30 years. In order to reach a 3% renovation rate by 2030 the average time between deep renovations of a dwelling would need to be reduced to about 25 years.

We argue that the natural renovation rates should always be considered, at least as a baseline scenario, in the analysis of future scenarios on a dwelling stock's energy demand and related greenhouse gases emissions. As a matter of fact, and net of all the input uncertainties and within the limits of the model's validation, we deem it methodologically more consistent to consider the natural renovation rates rather than other ones exogenously assumed, extrapolated based on recent years' trends or imposed as a precondition for achieving policy goals. In particular, we notice the substantial difference between the slow and progressive change in the natural renovation rates compared to the ambitious step-change deemed indispensable to achieve the policy decarbonisation target [22,23].

Table 4 shows how the renovation activity with a cycle of 40 years is distributed to dwellings constructed in the different periods. It can be seen how the total amount of expected renovation will increase over time along with how the importance of different cohorts vary over time. While per today (2010) the total amount of (40 years cycle) renovation is about 27,000 dwellings/year and more than half of it affects the cohort 1946–1980, in 2030 the total amount is expected to rise to about 34,000 dwellings/year with more than half of it affecting the cohort 1981–2015. This cohort will remain dominant up to 2050 when the total amount will be about 40,000 dwellings/year and 20% of the renovation will be on buildings built after 2015.

4. Conclusions

The dynamic dwelling stock model presented here provides a deep understanding of the dynamics driving the developments in the dwelling stock. The model allows detailed analysis of the dwelling stock's size and composition, and of the construction, renovation and demolition activities: how they developed over time and how the future projections look like. The model's description is fully transparent and the core algorithm is described in detail and presented in the Appendix in pseudo-code. The model is of general validity and relies on input data from population statistics available in principle in any country. The model thus provides a solid basis for studying the related impacts on energy demand, GHG emissions, materials demand and waste flows.

The dynamic dwelling stock model aims at describing the long term development of the stock. It has been shown how to validate the model's results against available statistics and other data

⁴ Though the overall system's efficiency might depend on both the heating/cooling system itself and the envelope's thermal properties, e.g. with heat pumps.

sources; where the results for Norway show a satisfactory match with the statistics. It has also been discussed how the model's result can be used for future projections of construction, demolition and renovation activities.

The model considers different renovation cycles of 20, 30 and 40 years as representative of different types of interventions, respectively: the substitution of heating/cooling systems, replacement of windows or roof, deep renovation of facades. The resulting renovation rates can be seen as the natural renovation rates of the stock, representing the need for maintenance of an ageing stock. The authors argue that such natural renovation rates should always be considered, at least as a baseline, in the analysis of future scenarios on a dwelling stock's energy demand and related greenhouse gases emissions; before considering other ones exogenously assumed, extrapolated based on recent years' trends or imposed as a precondition for achieving policy goals.

In particular, it has been discussed how the 40 years renovation cycle may be representative of major renovations of a building's envelope. This is the equivalent of lumping together the effect of several single measures – that have taken place at different times – into a single event, which has the same frequency of the least frequent measure, namely the deep renovation of facades. In the domain of scenario making, this should be a suitable approximation of a complex phenomenon such as the deep renovation of buildings, especially since this “single event” is further spread in time by means of a probability distribution function. Furthermore, it has been discussed how the praxis of adding the effect of single measures on the envelope to that of deep renovation leads to double counting when the rate of deep renovation corresponds to the one of a 40 years cycle, as in Refs. [9] and [10].

It has been shown how, given the lifetime and renovation profiles assumed for Norway, until 2050 there is in average only one chance of renovating a post-WWII dwelling when assuming a 40

years renovation cycle. Roadmaps and scenario analyses for future developments in building stocks commonly assume – or hope for – a strong and rapid increase in the renovation rate from today's level of ca. 1% to a desired level of 2.3–3.0%, in order to achieve EU policy targets by 2050, as in Refs. [22] and [23]. This model shows that the natural renovation rate is likely to increase slowly from just above 1% in 2010 to ca. 1.8% in 2050, in the case of Norway. Policy measures would be needed to reduce the average time between deep renovations of dwellings and therefore increase the deep renovation rate. For Norway, reaching an average renovation rate of about 2.5% in the period 2020–2050 means that the average interval between deep renovations should be approximately 30 years. In order to reach a 3% renovation rate by 2030 the average time between deep renovations of a dwelling would need to be reduced to about 25 years.

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Appendix.

Algorithm in pseudo-code.

```

%--- ALGORITHM ---%

% Note: the index operator ':' means "all the vector's length"
%
% for the 2-D matrices:
%     column = year of construction
%     row     = year of demolition/renovation

% calculate dwelling stock from population and persons/dwelling
 $S_D[:,] = P[:,] / P_D[:,]$ 

% calculate lifetime profile from demolition function
% CDF = Cumulative Distribution Function
 $L[:,] = \mathbf{1}[:,] - CDF(p_{DEM}[:,])$ 

% calculate cyclic renovation profile (K cycles, with K big enough)
% from renovation function & lifetime shifted  $\tau$  years (e.g. with  $\tau$  = renovation cycle)
 $p_{REN\_cycle}[:,] = \sum_{k=1}^K p_{REN}[:,](k) \cdot L[:,](\tau)$ 

% initializations (e.g.  $p_{DEM\_hist}$  &  $p_{REN\_hist}$  = functions known/assumed a priori)
 $D_{dem}[:,] = D_{dem\_matrix}[:, \mathbf{1}] = p_{DEM\_hist}[:,] \cdot S_D[\mathbf{1}]$  % demolition of initial stock,  $D_0$ 
 $D_{ren}[:,] = D_{ren\_matrix}[:, \mathbf{1}] = p_{REN\_hist}[:,] \cdot S_D[\mathbf{1}]$  % renovation of initial stock,  $R_0$ 
 $\Delta S_D[\mathbf{1}] = \mathbf{0}; D_{new}[\mathbf{1}] = D_{dem}[\mathbf{1}]$  % initial conditions

% iterate for total nr. of years
For  $i = 2 : n$ 

    % stock change
     $\Delta S_D[i] = S_D[i] - S_D[i - 1]$ 

    % balance equation
     $D_{new}[i] = \Delta S_D[i] + D_{dem}[i]$ 

    % write in 2-D matrix column  $i$  the future demolition of  $D_{new}(i)$ 
     $D_{dem\_matrix}[i + 1 : n, i] = p_{DEM}[:,] \cdot D_{new}[i]$ 

    % read from 2-D matrix row  $i+1$  the demolition  $D_{dem}(i+1)$  (for next iteration)
     $D_{dem}[i + 1] = \sum_{j=1}^i D_{dem\_matrix}[i + 1, j]$ 

    % write in 2-D matrix column  $i$  the future renovation of  $D_{new}(i)$ 
     $D_{ren\_matrix}[i + 1 : n, i] = p_{REN\_cycle}[:,] \cdot D_{new}[i]$ 

    % read from 2-D matrix row  $i$  the renovation  $D_{ren}(i)$ 
     $D_{ren}[i] = \sum_{j=1}^i D_{ren\_matrix}[i, j]$ 

End For

%--- END ALGORITHM ---%

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