Comparison of Multi-area Reactive Power Optimization Parallel Algorithm Based on Ward and REI Equivalent

Zhi-wen Liu*,b, Ming-bo Liub, Wen-bo Xiaa

*aGuangdong Electric Power Design Institute, No.1, Tianfeng Road, Science Town, Luogang District, Guangzhou and 510663, China
bSouth China University of Technology, No.381, Wushan Road, Tianhe District, Guangzhou and 510640, China

Abstract

By the means of introducing Ward and REI equivalent partition, the solution thoughts of multi-area power system reactive power optimization parallel algorithms based on Ward and REI equivalent are analyzed and explained in this paper, and the calculation methods of external equivalent network correction and outer coordination which used in two kinds of equivalent parallel algorithm are summarized, the multi-area reactive power optimization parallel calculation are implemented by using Matlab parallel computing platform and COW/Cluster hardware platform based on master-slave parallel calculation mode. Finally testing results on IEEE 39-bus, a 538-bus practical system and a 695-bus practical systems are used to analyze and compare the computing properties of two kinds of equivalent algorithm comprehensively in comparison with the centralized optimization method.

Keywords: Ward equivalent; REI equivalent; Multi-area power system; Reactive power optimization; Parallel calculation

1. Introduction

The electric power systems have been evolving to become larger through continuing growth in load demands. Traditional central processing of reactive power optimization is time consuming and is unable to meet the demand of real-time analysis and control. If the large-scale optimization problems can be decomposed into multiple sub-problems and implemented parallel calculation by adopting decomposition and coordination algorithm for large-scale power system, it can greatly enhance the computational efficiency[1-4]. Reference [1] and [2] created a parallel computational model of multi-area reactive power optimization based on the auxiliary problem principle to speed up the computation. Reference [3] implemented a multi-process cluster calculation of distributed reactive power optimization using genetic simulated annealing algorithm and MPI parallel technique. In reference [4], a coarse-grained multiprocessor...
parallel calculation of dynamic reactive power optimization based on the block bordered diagonal model was proposed.

External network simplification with external equivalent can not only reduce the problem size, but also identify with the dispatching operation management of power systems. In recent years, the decomposition and coordination algorithms based on external equivalents were applied for distributed power flow computing and reactive power optimization\cite{5-8}. In reference\cite{5}, some computational models of decomposition and coordination algorithm based on external equivalents were proposed. Reference\cite{6} added a coordination layer to dynamically obtain the external network equivalent power injection, and investigated the dynamic decomposition and coordination computation with multi-area interconnected system. Reference\cite{7} investigated the decomposition and coordination algorithm in multi-area reactive power optimization (Ward equivalent algorithm for short) based on Ward equivalents and provided a new method for the multi-area reactive-power optimization of large scale power system. Reference\cite{8} presented a method of applying REI equivalent technology to solve reactive-power optimization problem in multi-area power system (REI equivalent algorithm for short), and coordination optimization was implemented by introducing X-REI method into outer coordinated calculation.

Ward equivalent and REI equivalent both belong to static equivalent method, but they use different equivalent ways, thus the solution thoughts are similar which apply two kinds of equivalent technology to solve reactive-power optimization problem in reference\cite{7} and\cite{8} respectively, but it is different to use specific solution methods. In this paper, the solution thoughts of Ward and REI equivalent algorithm are analyzed and explained, the calculation methods of external equivalent network correction and outer coordination which used in two kinds of equivalent parallel algorithm are summarized, on the basis of simulation, the computing properties of two kinds of equivalent algorithm are analyzed and compared comprehensively.

2. Partition Based on Two Kinds of External Equivalent

2.1. Partition based on Ward equivalent

At first, the process of Ward equivalent partition is described by using a simple two-area interconnected power system. As shown in Fig. 1(a), a two-area interconnected power system consist of area A1 and A2. The two areas are interconnected with each other by lines L_{im} and L_{jn}. In this paper, the intertie terminals n_i, n_j, n_m and n_n are defined as boundary buses and marked by set B. For area A_1, n_m and n_n are external boundary buses and marked by set B_E, their voltage magnitude and phase angle are marked by set X_{BE}, n_i and n_j are internal boundary buses and expressed by set B_I, their voltage magnitude and phase angle are denoted by set X_{BI}. Likewise, for area A_2, n_i and n_j are external boundary buses, n_m and n_n are internal boundary buses. The internal bus set of areas A_1 and A_2 are denoted by set I, their voltage magnitude, phase angle, reactive power output of generators and reactive power compensation devices are denoted by set X_i\cite{5-6}.

(a) Interconnected system of two areas
The Ward equivalent model of area A2 is shown in figure 1(b), in which there are two boundary buses, an equivalent branch admittance, two ground admittances, and two equivalent injection power. Likewise, the Ward equivalent model of area A1 is similar with that of area A2, as shown in figure 1(c). If the number of boundary buses is greater than two, they are treated as combinations of the model consisting of two boundary buses. Thus the Ward equivalent model consists of branch admittances among boundary buses, ground admittance and equivalent injection power \[9\]. Figure 1 shows a partition example of two-area interconnected power system, if the interconnected power system is a multi-area system, each equivalent area will be formed by converting all the external systems of corresponding area into Ward equivalent network, the definition of boundary buses, internal boundary buses, external boundary buses are same with the two-area interconnected power system.

2.2. Partition based on REI equivalent

The process of REI equivalent partition is described by still using the two-area interconnected power system shown in figure 1(a), and the schematic diagram of REI equivalent partition of this two-area power system is shown in figure 2, the definition method of internal boundary buses and external boundary buses are same with Ward equivalent partition.
Fig. 2 Schematic diagram of REI equivalents of a two-area power system

As shown in Fig. 1(a), supposed the area A1 and A2 are the external and internal area respectively, all the active buses in area A1 are integrated into a REI bus R1, the Fig. 2(a) can be obtained. To simplify the REI network and the external area in Fig. 2(a), the simplified network can be obtained as shown in Fig. 1(b) and Fig. 1(c). To illustrate the process of the equivalent partition, suppose that there is only one REI bus in Fig.2, but numbers of REI buses can be used in REI equivalent. To improve the accuracy of REI equivalent, the buses which have the similar injected power property are always integrated into one bus. The load buses, var-compensator buses and generator buses in the external area are integrated respectively on the principles of equipment consistency in the practical progress of equivalent partition in reference [8].

3. Solution Thought of Two Kinds of Equivalent Algorithm

The independent reactive power optimization of each area can be implemented by forming external Ward and REI equivalent network. As shown in Fig.1(b), area A1 with the Ward equivalent network of external network(area A2) can simulate the whole system reactive power optimization computation. From the circuit equivalent principle, the circuit of equivalent area is equivalent to original circuit of whole network, thus reactive power optimization objective of whole network is consistent with optimization objectives of each equivalent area.

Due to treated external system by using static equivalent method, the external network will change in the course of reactive power optimization of equivalent area, the external equivalent admittance and equivalent injection power of each area can’t reflect the change of external network, especially the boundary reactive injection power can’t real-time response the needs of internal network, it is difficult to obtain approximate optimal solution that if equivalent area implements the reactive optimization independently. Therefore, in addition to inner iteration calculation of equivalent area, the external equivalent network correction and outer coordination calculation between different equivalent areas are needed in course of optimization calculation based on network equivalent. The inner iteration calculation of all equivalent areas in each round is called main cycle iteration in this paper, after several main cycle iterations and outer coordination computation, while the active power loss of each equivalent area and voltages of boundary buses of adjacent areas tend to equal each other, the optimal point is obtained, namely get the approximate optimal solution of whole network.

During these computing process, it need to give the specific methods of solving the key problems which include equivalent network initial parameters calculation, phase angles transfer, equivalent admittance matrix correction, inner iteration calculation and outer coordination calculation, which are described in reference [7] and [8] respectively. The equivalent network correction and outer coordination calculation are the key to ensure multi-area reactive power optimization convergence, the treated methods of which are in two kinds of equivalent algorithm have both similarities and differences, this paper emphatically analyzes and compares the calculation principle of outer coordination and equivalent network correction methods in two kinds of equivalent algorithm.
4. Equivalent Network Correction and Outer Coordination Calculation

On the basis of multiple optimization and coordination, the ideal approximate optimal solution of multi-area reactive power optimization can be gained by using external equivalent partition. During the independent reactive power optimization of each area, the transformer ratios will change, which will in turn lead to the change of Ward equivalent admittance corresponding to the area, fixing the transformer ratios will inevitably affect the accuracy of the equivalent. But if the equivalent admittance is recalculated after each iteration, it will need a large amount of calculation, and computation efficiency of distributed reactive power optimization is reduced, therefore it needs to find a quick and effective correction method. The Ward equivalent algorithm proposed in reference [7] use inverse matrix modification lemma to correct the external equivalent network, the REI equivalent algorithm proposed in reference [8] use X-REI equivalent thought to correct REI equivalent network, namely external equivalent network is corrected quickly by updating simple REI equivalent correction network, the simulation shows that the two kinds of methods have ideal correction effects.

Outer coordination computation is main means to ensure external equivalent accuracy, whether can fully reflect the power flow change of external system is the key to determine the coordination effects. Such as the two areas shown in Fig. 1, during the reactive power optimization, the reactive power output of generator $Q_G$, transformer ratio $T_B$ and output of reactive compensator $Q_C$ are control variables, while voltage $U$ is state variable. The control variables determine the state variables. Likewise, the voltages of internal boundary buses $U_{BI}$ and the voltages of external boundary buses $U_{BE}$ map one-to-one with the control variables of $Q_G$, $T_B$ and $Q_C$ in each area which are determined by the control variables of $Q_G$, $T_B$ and $Q_C$ respectively.

As the cross distribution of boundary buses in two areas [5-6], namely the external boundary buses in area A1 is the internal boundary buses in area A2; the external boundary buses in area A2 is the internal boundary buses in area A1. If the bus voltages at boundary of each area are exchanged, the initial voltage value $U_{BE}$ of external boundary buses in each area can be obtained at the $k+1$ th loop of the main program loop. That is $U_{BE}^{(k+1)} = U_{BE}^{(k)}$ and $U_{BE}^{(k+1)} = U_{BE}^{(k)}$. $U_{BI}$ maps one-to-one with the control variables $Q_G$, $T_B$ and $Q_C$ in each area, so the following equations can be obtained:

\[ \{Q_{G1}^{(k)}, T_{B1}^{(k)}, Q_{C1}^{(k)}\} = \phi_1(U_{BE2}^{(k+1)}) \]  
(1)

\[ \{Q_{G2}^{(k)}, T_{B2}^{(k)}, Q_{C2}^{(k)}\} = \phi_2(U_{BE1}^{(k+1)}) \]  
(2)

From equations (1) and (2), the voltages of new external boundary in each area obtained from the exchange maps with control variables of external network one by one, thus the changes of power flow in external network can be considered.

In Ward equivalent algorithm, the external boundary injection power recalculated by using new voltages will have good coordination effects [7]. In the course of outer coordination in REI equivalent algorithm, after gained the new boundary voltages by using same methods, the effective coordination between REI equivalent partitions is realized by updating REI equivalent correction network and correction injection power based on X-REI equivalent thought, and the coordination process just is the correction process of external equivalent network [8].

5. Implementation Methods of Parallel Calculation

According to the features of the Ward and REI equivalent algorithm, the implement of parallel calculation can be applied easily if master-slave mode is adopted. Master-slave mode includes one master process and several slave processes. Master process get in charge of the division, assignment of jobs and the collection of results. Slave processes response for the reception, computation and the results sending of the sub-task [10]. In the process of multi-area reactive power optimization parallel calculation based on
Ward and REI equivalent, master process mainly responds for outer coordination calculation, parameter transfer and so on, slave process mainly corresponds to the inner optimization iteration process of each equivalent partition.

According to the features of master-slave parallel calculation mode, this paper uses Matlab R2009a as parallel calculation software platform and chooses COW/Cluster as parallel calculation hardware platform. The Matlab R2009a parallel calculation platform includes parallel computing toolbox and distributed computing server, the parallel computing toolbox is installed on the computer developed program, which is used to develop parallel computing program and decompose algorithm into several parallel computing subtasks, all the distributed computing servers include a job manager, and are installed on cluster computers which are responsible for parallel computing tasks, users can distribute parallel computing tasks to each cluster computer and collect calculation results by using job manager \[11-12\]. The parallel calculation process based on Matlab is shown in figure 3.

COW/Cluster parallel system is composed of high-grade computers connected by high-speed interconnection network. Each node of COW/Cluster is a computer with complete operating system \[13\]. COW/Cluster is good of scalability, computing ability and performance price ratio, and is widely used. The structure of COW/Cluster is shown in Fig. 4.

In Fig. 4, MB represents memory bus, LD represents local disc, bridge presents interface between memory bus and I/O bus, IOB represents I/O bus. From figure 3 and 4, the parallel calculation process based on Matlab is suitable to master-slave parallel mode, and parallel work mode coincides with COW/Cluster parallel computer system structure, these are beneficial to implement multi-area reactive power optimization parallel calculation used external equivalent technique.
6. Test cases

6.1. Partition of test systems

The Ward and REI equivalent algorithm proposed in reference [7] and [8] are verified in IEEE 39-bus system, a 538-bus practical system and a 695-bus practical system respectively, and the a detail comparison and analysis on two kinds of equivalent algorithms is given. IEEE 39-bus system is divided into two areas, such as shown in Fig. 5, where A1 is the master area, bus numbers of A1 and A2 are 25 and 14 respectively, generation numbers are 6 and 4, capacitor numbers are 5 and 3, and transformer numbers are 9 and 3.

![Fig.5 Schematic diagram of 2 areas in 39-bus system](image)

538-bus system includes 593 lines, 48 generators, 98 capacitor banks and 409 transformer branches (64 OLTCs participated in the optimization calculation). The system is divided into 4 areas according to region management. The schematic diagram is shown in Fig. 6.

![Fig.6 Schematic diagram of 4 areas in 538-bus system](image)

As shown in Fig. 6, among the four areas, A1 presents the master area. The basic information of the areas is shown in Tab. 1.

<table>
<thead>
<tr>
<th>Area</th>
<th>Bus</th>
<th>Generator</th>
<th>Qc</th>
<th>Transformer</th>
<th>Tie line</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>364</td>
<td>32</td>
<td>68</td>
<td>43</td>
<td>9</td>
</tr>
<tr>
<td>A2</td>
<td>71</td>
<td>8</td>
<td>13</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>A3</td>
<td>64</td>
<td>3</td>
<td>12</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>39</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

695-bus system includes 1107 lines, 138 generators, 67 capacitor banks or reactor banks and 360 transformer branches (31 OLTCs participated in the optimization calculation). The system is divided into 5 areas according to region management. The schematic diagram is shown in Fig. 7.
As shown in Fig. 7, among the five areas, A1 presents the master area. The basic information of the areas is shown in Tab. 2.

**Tab. 2 Basic data of four areas in 695-bus system**

<table>
<thead>
<tr>
<th>Area</th>
<th>Bus</th>
<th>Generator</th>
<th>Qc</th>
<th>Transformer</th>
<th>Tie line</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>168</td>
<td>47</td>
<td>12</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>A2</td>
<td>119</td>
<td>23</td>
<td>9</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>133</td>
<td>21</td>
<td>12</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>A4</td>
<td>149</td>
<td>32</td>
<td>12</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>A5</td>
<td>126</td>
<td>15</td>
<td>22</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

6.2. Computation results

During the parallel simulation computation, the independent optimization calculation of each area is assigned to one computer based on the workstation cluster parallel platform. Namely the simulation computation with IEEE 39-bus system, 538-bus system and 695-bus system use 2, 4 and 5 computers respectively. The computers participated in parallel simulation computation are connected to each other by network exchanger. Each computer is configured to Pentium(R) 4 (2.8 GHz) with 512 MB. The algorithm simulation is implemented by using Matlab R2009a platform. The results of parallel simulation platform. The results of parallel simulation computation are shown in Tab. 3. In Tab. 3, nonlinear primal-dual interior point algorithm proposed in reference [14] and [15] is adopted in inner iteration computation of each equivalent area and centralized optimization algorithm.

**Tab. 3 Comparison of results obtained from three kinds of reactive optimization algorithm**

<table>
<thead>
<tr>
<th>Test system</th>
<th>Centralized optimization algorithm</th>
<th>Ward equivalent algorithm</th>
<th>REI equivalent algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Active power loss/pu</td>
<td>Computing time/s</td>
<td>Active power loss/pu</td>
</tr>
<tr>
<td>39-bus system</td>
<td>0.4034</td>
<td>0.782</td>
<td>0.4188</td>
</tr>
<tr>
<td>538-bus system</td>
<td>1.6237</td>
<td>21.178</td>
<td>1.6406</td>
</tr>
<tr>
<td>695-bus system</td>
<td>5.7345</td>
<td>30.301</td>
<td>5.7563</td>
</tr>
</tbody>
</table>

6.3. Comparison and analysis of two kinds of equivalent algorithm

6.3.1 Comparative analysis on computational errors

From Tab. 3, the active power loss calculated respectively by the two equivalent algorithms has a little deviation compared with the centralized optimization algorithm, thus the results obtained by the two kinds of equivalent algorithms are not exact optimal solution, but approximate optimal solution. In the same test system, the active power loss obtained by REI equivalent algorithm is more close to the active power loss obtained by centralized optimization algorithm than Ward equivalent algorithm. Taking 695-bus system for example, the active power loss calculated by REI equivalent algorithm has a 0.16%
deviation from centralized optimization algorithm, while Ward equivalent algorithm has a 0.38% deviation. Thus REI equivalent algorithm has higher calculation accuracy than Ward equivalent algorithm.

The calculation error of the proposed two equivalent algorithms in this paper depends on the accuracy of the external network during the optimization process, namely depends on calculation accuracy of external equivalent admittance and injection power of the boundary buses. Although REI and Ward equivalent are both using the external network topology data to simply external system, REI equivalent merges and retains the PV buses of external network according to a certain rule, which can better satisfy the requirement of accurate reactive power response of external system when inner network has been disturbed, so it has better calculation accuracy. Although equivalent admittance calculation reflected the transformer ratio change of external network is taken into account in Ward equivalent algorithm, and the injection power calculation method of boundary buses sufficiently reflects the variation of external system power flow, however, the external PV buses are removed during the Ward equivalent process, the reactive power response accuracy of external network has been effected, thus the calculation accuracy has a little lower than REI equivalent algorithm.

6.3.2 Comparative analysis on computational speed

From Tab. 3, comparing with the centralized optimization algorithm, Ward and REI equivalent algorithm improve the calculation speed in different extend in the three test systems, and the speedup ratio gradually increases as the expansion of the system scale. However, the calculation speed in small system using the two equivalent algorithms is not significant as the large system. The main reason is that small system has a small parallel granularity and the time cost which is paid by the coordinated computation and the data communication during the parallel computation has a higher proportion than large system. Therefore large system has a faster calculation speed obviously.

In Tab.3, although 538-bus system and 695-bus system both belong to large-scale and complicate system, the difference of their calculation speedup ratios is relatively great, taking the REI equivalent algorithm for example, the speedup ratio of 538-bus system is 1.958, and speedup ratio of 695-bus system reaches to 5.114, the main reason is that speedup ratios of two kinds of equivalent parallel algorithm relate to the size degree of task which are assign to computers of cluster, the bus numbers of maximum area account for 67.6% of total bus numbers in 538-bus system, this proportion is 24.2% in 695-bus system, therefore if the assignments are allocated uniformly with the same hardware configuration, namely loads are under balance, the calculation speed will be improved effectively.

Through comparison on Tab. 3, the calculation speed of REI equivalent algorithm is faster than Ward equivalent algorithm in the same test system. The computational speedup ratio of Ward equivalent algorithm is 1.328 in IEEE 39-bus system and 4.635 in 695-bus system respectively, while the computational speedup ratio of REI equivalent algorithm is 1.417 and 5.114 respectively. Except for the difference of the coordinate modified method and simulation program design, the main reason of the calculation speed of REI equivalent algorithm faster than Ward equivalent algorithm is that REI equivalent algorithm merges generator buses, load buses and var-compensator buses of the external system respectively based on equipment consistency principle. Comparing with Ward equivalent algorithm, REI algorithm has better external reactive response accuracy and less main loop number during the computational process, hence has faster calculation speed.

6.3.3 Comparative analysis on application of algorithms

With comparative analysis on the calculation efficiency and deviation of the two kinds of equivalent algorithms, it can be found that the multi-area reactive power optimization based on REI equivalent has faster computing speed and smaller calculation error due to the mergence of the load buses, var-compensator buses and generator buses in the external system. But the prerequisite is that details of all the parameters of lines and buses in the external area can be obtained, this is a relatively harsh condition for the current domestic and international power management system. Requirements of the parameters in the external area based on Ward equivalent are relatively at a lower level, and the application progress of the equivalent is easy. Therefore, it is widely used in the equivalent calculation of power systems. On the basis of Ward equivalent, the reservations of the PV buses which have the minimum electrical distance to the inner area or have the largest reactive power reserves in the external area may lead to the
improvement on the ability of reactive power response in the external system due to the disturbance in the inner system, thus a further study on the combination of the advantages of the two algorithms is needed.

7. Conclusions

(1) The large-scale optimization problem is decomposed into multiple sub-problems by adopting Ward and REI equivalent, which can reduce the problem scale and is suitable for both strong coupling and weak coupling power systems.

(2) Ward and REI equivalent both belong to static equivalent, two kinds of equivalent parallel algorithm have same solution thoughts, and use similar outer coordination modes, but the specific solution methods used in two kinds of algorithm are different.

(3) Two kinds of equivalent parallel algorithm can improve the calculation speed of reactive power optimization significantly, and the acceleration is more obviously for large systems, it avoids the curse of dimensionality problem in optimization computation due to large-scale problem size with low computational efficiency.

(4) Comparing with Ward equivalent algorithm, REI equivalent parallel algorithm can better meet reactive power response need when internal network is disturbed, and has better computation speed and accuracy, but the application of Ward equivalent algorithm is more widely.

Reference


