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## Discussion

### Response to discussion on “An extended strain energy density failure criterion by differentiating volumetric and distortional deformation”

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I thank in advance Dr. Andrianopoulos for his interest in the work. The discussion enables me to clarify some of the points and also recognize the related existing discussions on this topic. The extension to the strain energy density (SED) failure criterion developed by Sih (1974) was motivated by recognizing that volumetric and distortional deformations may be weighed differently when determining the failure of materials (Wei, 2012), and the necessity of differentiating these two deformation modes has indeed discussed by Andrianopoulos and coauthors (1982, 1984, 1988, 2012). Being an applied mathematic model, the original SED (Sih, 1974) assumed a core radius  $r_0$  by taking  $r_0$  – constancy. We note that this assumption is indeed not necessary, but the accuracy of stress fields at a crack tip does rely on the region away from the crack-tip plastic zone size, where the applicability of Williams' equations for the stress field at a crack tip is valid. This point has been discussed by Theocaris and Andrianopoulos (1982). The author found it essential but natural for any applied mathematical model to be applicable.

In the extended SED model, we need two material fracturing properties to characterize crack propagation. In the paper (Wei, 2012), I discussed the ways to obtain the two parameters (Page 1125 in Section 6). It is important to know that we do not need two ‘pure’ experiments to identify  $S_v$  and  $S_d$ . In addition to a pure mode I or II test, we need one mix-mode fracture test. The combination of these two can determine  $S_v$  and  $S_d$ . Essentially, any two tests with different mix-mode angles would serve the purpose of determining both  $S_v$  and  $S_d$ . If one really wants to do the ‘two ‘pure’ experiments to identify  $S_v$  and  $S_d$ , he/she may resort to current atomic scale simulations or density functional theory calculations for  $S_v$ . In the paper, I have used normalized parameters for the application to PMMA and Beryllium. In that case, we only need to define the ratio of  $S_v$  over  $S_d$ , i.e.,  $\beta$ . The detailed numbers for  $\beta$  are given in the paper, with  $\beta = 0.2$  for Beryllium and  $\beta = 1.4$  for PMMA.

The extended SED model assumes that the direction of the crack extension is governed by the direction with minimum strain energy density, and it requires  $S$  reaching a critical value  $S_{crit}$ . There is no single mechanism which determines the direction of crack

extension in the extended strain energy density failure criterion. Both the loading conditions and the parameter  $\beta$  govern the direction of crack extension. When the stress increase from  $\sigma_\infty^*$  to  $\sigma_\infty$ , as shown in Fig. 1(a) in Andrianopoulos (2012), the conditions for crack propagation is not fully satisfied based on the criterion. The strain energy density model hypothesizes that the crack will extend along the direction where  $S$  is minimized, and it requires that  $S$  reaches a critical value  $S_{crit}$  in that direction. So the strain energy density criterion is distinct from those based on a single critical number.

At the end, I would like to thank Dr. Andrianopoulos again for his valuable discussion on the problem. While the extension to the SED failure criterion considers different weights for volumetric and distortional deformations when accounting for failure (Wei, 2012), further improvement based on the SED concept is certainly desired as we bring more physics in the applied mathematic model. Indeed, there is a very nice thread in iMechanica discussing the advantages and disadvantages of the strain energy density model and its derivatives for material failure, which could be of the interest of IJSS readers (Suo et al., 2010).

## References

- Andrianopoulos, N.P., 2012. Discussion on ‘An extended strain energy density failure criterion by differentiating volumetric and distortional deformation’.
- Andrianopoulos, N.P., Manolopoulos, V.M., 2012. Can Coulomb criterion be generalized in case of ductile materials? An application to Bridgman experiments. *Int. J. Mech. Sci.* 54, 241–248.
- Andrianopoulos, N.P., Theocaris, P.S., 1988. LEFM brittle and ductile fracture as described by the T criterion. *Eng. Fract. Mech.* 30, 5–12.
- Sih, G.C., 1974. Strain-energy-density factor applied to mixed mode crack problems. *Int. J. Fract.* 10, 305–321.
- Suo, Z.G. et al., 2010. Mixed-mode fracture. Curved crack path. <http://imechanica.org/node/8036>.
- Theocaris, P.S., Andrianopoulos, N.P., 1982. The Mises elastic–plastic boundary as the core region in fracture criteria. *Eng. Fract. Mech.* 16, 425–432.
- Theocaris, P.S., Andrianopoulos, N.P., 1984. Authors' closure on the discussion by G.C. Sih, E.T. Moyer Jr. and E.E. Gdoutos. *Eng. Fract. Mech.*, 691–694.
- Wei, Y., 2012. An extended strain energy density failure criterion by differentiating volumetric and distortional deformation. *Int. J. Solids Struct.* (2012 4), 1117–1126.

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