

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The emerging case for axion dark matter

P. Sikivie

Department of Physics, University of Florida, Gainesville, FL 32611, USA

ARTICLE INFO

Article history:

Received 7 May 2010

Received in revised form 9 November 2010

Accepted 9 November 2010

Available online 11 November 2010

Editor: S. Dodelson

Keywords:

Dark matter

Axions

Bose–Einstein condensation

ABSTRACT

Dark matter axions form a rethermalizing Bose–Einstein condensate. This provides an opportunity to distinguish axions from other forms of dark matter on observational grounds. I show that if the dark matter is axions, tidal torque theory predicts a specific structure for the phase space distribution of the halos of isolated disk galaxies, such as the Milky Way. This phase space structure is precisely that of the caustic ring model, for which observational support had been found earlier. The other dark matter candidates predict a different phase space structure for galactic halos.

© 2010 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).

One of the outstanding problems in science today is the identity of the dark matter of the universe [1]. The existence of dark matter is implied by a large number of observations, including the dynamics of galaxy clusters, the rotation curves of individual galaxies, the abundances of light elements, gravitational lensing, and the anisotropies of the cosmic microwave background radiation. The energy density fraction of the universe in dark matter is 23%. The dark matter must be non-baryonic, cold and collisionless. *Non-baryonic* means that the dark matter is not made of ordinary atoms and molecules. *Cold* means that the primordial velocity dispersion of the dark matter particles is sufficiently small, less than about $10^{-8}c$ today, so that it may be set equal to zero as far as the formation of large scale structure and galactic halos is concerned. *Collisionless* means that the dark matter particles have, in first approximation, only gravitational interactions. Particles with the required properties are referred to as ‘cold dark matter’ (CDM). The leading CDM candidates are weakly interacting massive particles (WIMPs) with mass in the 100 GeV range, axions with mass in the 10^{-5} eV range, and sterile neutrinos with mass in the keV range. To try and tell these candidates apart on the basis of observation is a tantalizing quest.

In this regard, the study of the inner caustics of galactic halos may provide a useful tool [2,3]. An isolated galaxy like our own accretes the dark matter particles surrounding it, producing discrete flows throughout its halo [4]. There is one flow of particles falling in for the first time, one flow of particles falling out for the first time, one flow of particles falling in for the second time, and so forth. The number of flows at our location in the Milky Way

halo is of order one hundred, the ratio of the age of the universe ($\sim 10^{10}$ years) to the infall time ($\sim 10^8$ years) at our location. The flows form high-density surfaces, called caustics. At a caustic the matter density diverges in the limit where the velocity dispersion of the dark matter particles vanishes. In particular, each flow in and out of the galactic gravitational potential well forms a caustic which we call ‘inner’. The inner caustic may be thought of as the envelope of the particle trajectories near their closest approach to the center. Since the accreted dark matter falls in and out of the galactic gravitational potential well many times, there is a set of inner caustics. In addition, there is a set of outer caustics, one for each outflow as it reaches its maximum radius before falling back in. We will be concerned here with the catastrophe structure and spatial distribution of the inner caustics of isolated disk galaxies.

The catastrophe structure of the inner caustics depends mainly on the angular momentum distribution of the infalling particles [3]. There are two contrasting cases to consider. In the first case, the angular momentum distribution is characterized by ‘net overall rotation’; in the second case, by irrotational flow. The archetypical example of net overall rotation is instantaneous rigid rotation on the turnaround sphere. The turnaround sphere is defined as the locus of particles which have zero radial velocity with respect to the galactic center for the first time, their outward Hubble flow having just been arrested by the gravitational pull of the galaxy. The present turnaround radius of the Milky Way is of order 2 Mpc. Net overall rotation implies that the velocity field has a curl, $\vec{\nabla} \times \vec{v} \neq 0$. The corresponding inner caustic is a closed tube whose cross-section is a section of the elliptic umbilic (D_{-4}) catastrophe [2,3]. It is often referred to as a ‘caustic ring’, or ‘tricuspid ring’ in reference to its shape. In the case of irrotational flow, $\vec{\nabla} \times \vec{v} = 0$, the inner caustic has a tent-like structure quite dis-

E-mail address: sikivie@phys.ufl.edu.

tinct from a caustic ring. Both types of inner caustic are described in detail in Ref. [3].

If a galactic halo has net overall rotation and its time evolution is self-similar, the radii of its caustic rings are predicted in terms of a single parameter, called j_{\max} . Self-similarity means that the entire phase space structure of the halo is time independent except for a rescaling of all distances by $R(t)$, all velocities by $R(t)/t$ and all densities by $1/t^2$ [5–8]. t is time since the big bang. For definiteness, $R(t)$ will be taken to be the turnaround radius at time t . If the initial overdensity around which the halo forms has a power law profile

$$\frac{\delta M_i}{M_i} \propto \left(\frac{1}{M_i} \right)^\epsilon, \quad (1)$$

where M_i and δM_i are respectively the mass and excess mass within an initial radius r_i , then $R(t) \propto t^{\frac{2}{3} + \frac{\epsilon}{9\epsilon}}$ [5]. In an average sense, ϵ is related to the slope of the evolved power spectrum of density perturbations on galaxy scales [9]. The observed power spectrum implies that ϵ is in the range 0.25 to 0.35 [7]. The prediction for the caustic ring radii is ($n = 1, 2, 3, \dots$) [2,8]

$$a_n \simeq \frac{40 \text{ kpc}}{n} \left(\frac{v_{\text{rot}}}{220 \text{ km/s}} \right) \left(\frac{j_{\max}}{0.18} \right), \quad (2)$$

where v_{rot} is the galactic rotation velocity. Eq. (2) is for $\epsilon = 0.3$. The a_n have a small ϵ dependence. However, the $a_n \propto 1/n$ approximate behavior holds for all ϵ in the range 0.25 and 0.35, so that a change in ϵ is equivalent to a change in j_{\max} . (ϵ, j_{\max}) = (0.30, 0.180) implies very nearly the same radii as (ϵ, j_{\max}) = (0.25, 0.185) and (0.35, 0.177).

Observational evidence for caustic rings with the radii predicted by Eq. (2) was found in the statistical distribution of bumps in a set of 32 extended and well-measured galactic rotation curves [10], the distribution of bumps in the rotation curve of the Milky Way [11], the appearance of a triangular feature in the IRAS map of the Milky Way in the precise direction tangent to the nearest caustic ring [11], and the existence of a ring of stars at the location of the second ($n = 2$) caustic ring in the Milky Way [12]. Each galaxy may have its own value of j_{\max} . However, the j_{\max} distribution over the galaxies involved in the aforementioned evidence is found to be peaked at 0.18. There is evidence also for a caustic ring of dark matter in a galaxy cluster [13]. The caustic ring model of galactic halos [8] is the phase space structure that follows from self-similarity, axial symmetry, and net overall rotation.

Self-similarity requires that the time dependence of the specific angular momentum distribution on the turnaround sphere be given by [7,8]

$$\vec{\ell}(\hat{n}, t) = \vec{j}(\hat{n}) \frac{R(t)^2}{t}, \quad (3)$$

where \hat{n} is the unit vector pointing to a position on the turnaround sphere, and $\vec{j}(\hat{n})$ is a dimensionless time-independent angular momentum distribution. In case of instantaneous rigid rotation, which is the simplest form of net overall rotation,

$$\vec{j}(\hat{n}) = j_{\max} \hat{n} \times (\hat{z} \times \hat{n}), \quad (4)$$

where \hat{z} is the axis of rotation and j_{\max} is the parameter that appears in Eq. (2). The angular velocity is $\vec{\omega} = \frac{j_{\max}}{t} \hat{z}$. Each property of the assumed angular momentum distribution maps onto an observable property of the inner caustics: net overall rotation causes the inner caustics to be rings, the value of j_{\max} determines their overall size, and the time dependence given in Eq. (3) causes $a_n \propto 1/n$.

The angular momentum distribution assumed by the caustic ring halo model may seem implausible because it is highly organized in both time and space. Numerical simulations [14] suggest

that galactic halo formation is a far more chaotic process. However, since the model is motivated by observation, it is appropriate to ask whether it is consistent with the expected behavior of some or any of the dark matter candidates. In addressing this question we make the usual assumption, commonly referred to as ‘tidal torque theory’, that the angular momentum of a galaxy is due to the tidal torque applied to it by nearby protogalaxies early on when density perturbations are still small and protogalaxies close to one another [15,16]. We divide the question in three parts: 1. Is the value of j_{\max} consistent with the magnitude of angular momentum expected from tidal torque theory? 2. Is it possible for tidal torque theory to produce net overall rotation? 3. Does the axis of rotation remain fixed in time, and is Eq. (3) expected as an outcome of tidal torque theory?

1. Magnitude of angular momentum

The amount of angular momentum acquired by a galaxy through tidal torquing can be reliably estimated by numerical simulation because it does not depend on any small feature of the initial mass configuration, so that the resolution of present simulations is not an issue in this case. The dimensionless angular momentum parameter

$$\lambda \equiv \frac{L|E|^{\frac{1}{2}}}{GM^{\frac{3}{2}}}, \quad (5)$$

where G is Newton’s gravitational constant, L is the angular momentum of the galaxy, M its mass and E its net mechanical (kinetic plus gravitational potential) energy, was found to have median value 0.05 [17]. In the caustic ring model the magnitude of angular momentum is given by j_{\max} . As mentioned, the evidence for caustic rings implies that the j_{\max} -distribution is peaked at $j_{\max} \simeq 0.18$. Is the value of j_{\max} implied by the evidence for caustic rings compatible with the value of λ predicted by tidal torque theory?

The relationship between j_{\max} and λ may be easily derived. Self-similarity implies that the halo mass $M(t)$ within the turnaround radius $R(t)$ grows as $t^{\frac{2}{3\epsilon}}$ [5]. Hence the total angular momentum grows according to

$$\frac{d\vec{L}}{dt} = \int d\Omega \frac{dM}{d\Omega dt} \vec{\ell} = \frac{4}{9\epsilon} \frac{M(t)R(t)^2}{t^2} j_{\max} \hat{z}, \quad (6)$$

where we assumed, for the sake of definiteness, that the infall is isotropic and that $\vec{j}(\hat{n})$ is given by Eq. (4). Integrating Eq. (6), we find

$$\vec{L}(t) = \frac{4}{10 + 3\epsilon} \frac{M(t)R(t)^2}{t} j_{\max} \hat{z}. \quad (7)$$

Similarly, the total mechanical energy is

$$E(t) = - \int \frac{GM(t)}{R(t)} \frac{dM}{dt} dt = - \frac{3}{5 - 3\epsilon} \frac{GM(t)^2}{R(t)}. \quad (8)$$

Here we use the fact that each particle on the turnaround sphere has potential energy $-GM(t)/R(t)$ and approximately zero kinetic energy. Combining Eqs. (5), (7) and (8) and using the relation $R(t)^3 = \frac{8}{\pi^2} t^2 GM(t)$ [5], we find

$$\lambda = \sqrt{\frac{6}{5 - 3\epsilon} \frac{8}{10 + 3\epsilon} \frac{1}{\pi}} j_{\max}. \quad (9)$$

For $\epsilon = 0.25, 0.30$ and 0.35 , Eq. (9) implies $\lambda/j_{\max} = 0.281, 0.283$ and 0.284 respectively. Hence there is excellent agreement between $j_{\max} \simeq 0.18$ and $\lambda \sim 0.05$.

The agreement between j_{\max} and λ gives further credence to the caustic ring model. Indeed if the evidence for caustic rings were incorrectly interpreted, there would be no reason for it to produce a value of j_{\max} consistent with λ . Note that the agreement is excellent only in Concordance Cosmology. In a flat matter dominated universe, the value of j_{\max} implied by the evidence for caustic rings is 0.27 [2,8].

2. Net overall rotation

Next we ask whether net overall rotation is an expected outcome of tidal torquing. The answer is clearly no if the dark matter is collisionless. Indeed, the velocity field of collisionless dark matter satisfies

$$\frac{d\vec{v}}{dt}(\vec{r}, t) = \frac{\partial \vec{v}}{\partial t}(\vec{r}, t) + (\vec{v}(\vec{r}, t) \cdot \vec{\nabla})\vec{v}(\vec{r}, t) = -\vec{\nabla}\phi(\vec{r}, t) \quad (10)$$

where $\phi(\vec{r}, t)$ is the gravitational potential. The initial velocity field is irrotational because the expansion of the universe caused all rotational modes to decay away [18]. Furthermore, it is easy to show [3] that if $\vec{\nabla} \times \vec{v} = 0$ initially, then Eq. (10) implies $\vec{\nabla} \times \vec{v} = 0$ at all later times. Since net overall rotation requires $\vec{\nabla} \times \vec{v} \neq 0$, it is inconsistent with collisionless dark matter, such as WIMPs or sterile neutrinos. If WIMPs or sterile neutrinos are the dark matter, the evidence for caustic rings, including the agreement between j_{\max} and λ obtained above, is purely fortuitous.

Axions [19–22] differ from WIMPs and sterile neutrinos in this respect. Axions are not collisionless in the sense of Eq. (10) because they form a rethermalizing Bose–Einstein condensate [23]. Bose–Einstein condensation (BEC) may be briefly described as follows: if identical bosonic particles are highly condensed in phase space, if their total number is conserved and if they thermalize, most of them go to the lowest energy available state. The condensing particles do so because, by yielding their energy to the remaining non-condensed particles, the total entropy is increased. In the case of cold dark matter axions, thermalization occurs because of gravitational interactions between the low-momentum modes in the axion fluid. This process is quantum mechanical in an essential way and not described by Eq. (10).

Axions form a rethermalizing Bose–Einstein condensate when the photon temperature reaches of order 100 eV ($f/10^{12}$ GeV) $^{1/2}$ [23] where f is the axion decay constant. By *rethermalizing* we mean that thermalization rate remains larger than the Hubble rate so that the axion state tracks the lowest energy available state. The compressional (scalar) modes of the axion field are unstable and grow as for ordinary CDM, except on length scales too small to be of observational interest [23]. Unlike ordinary CDM, however, the rotational (vector) modes of the axion field exchange angular momentum by gravitational interaction. Most axions condense into the state of lowest energy consistent with the total angular momentum, say $\vec{L} = L\hat{z}$, acquired by tidal torquing at a given time. To find this state we may use the WKB approximation because the angular momentum quantum numbers are very large, of order 10^{20} for a typical galaxy. The WKB approximation maps the axion wavefunction onto a flow of classical particles with the same energy and momentum densities. It is easy to show that for given total angular momentum the lowest energy is achieved when the angular motion is rigid rotation. So we find Eq. (4) to be a prediction of tidal torque theory if the dark matter is axions.

Thermalization by gravitational interactions is only effective between modes of very low relative momentum [23]. After the axions fall into the gravitational potential well of the galaxy, they form multiple streams and caustics like ordinary CDM [24]. The momenta of particles in different streams are too different from each other for thermalization by gravitational interactions to oc-

cur across streams. The wavefunction of the axions inside the turnaround sphere is mapped by the WKB approximation onto the flow of classical particles with the same initial conditions on that sphere. The phase space structure thus formed has caustic rings since the axions reach the turnaround sphere with net overall rotation. The axion wavefunction vanishes on an array of l lines, which may be thought of as the vortices characteristic of a BEC with angular momentum. However, the transverse size of the axion vortices is of order the inverse momentum associated with the radial motion in the halo, $(mv_r)^{-1} \sim 20$ meters for a typical value (10^{-5} eV) of the axion mass. In a BEC without radial motion the size of vortices is of order the healing length [25], which is much larger than $(mv_r)^{-1}$.

One might ask whether there is a way in which net overall rotation may be obtained other than by BEC of the dark matter particles. I could not find any. General relativistic effects may produce a curl in the velocity field but are only of order $(v/c)^2 \sim 10^{-6}$ which is far too small for the purposes described here. One may propose that the dark matter particles be collisionfull in the sense of having a sizable cross-section for elastic scattering off each other. The particles then share angular momentum by particle collisions after they have fallen into the galactic gravitational potential well. However, the collisions fuzz up the phase space structure that we are trying to account for. The angular momentum is only fully shared among the halo particles after the flows and caustics of the model are fully destroyed. Axions appear singled out in their ability to produce the net overall rotation implied by the evidence for caustic rings of dark matter.

3. Self-similarity

The third question provides a test of the conclusions reached so far. If galaxies acquire their angular momentum by tidal torquing and if the dark matter particles are axions in a rethermalizing Bose–Einstein condensate, then the time dependence of the specific angular momentum distribution on the turnaround sphere is predicted. Is it consistent with Eq. (3)? In particular, is the axis of rotation constant in time?

Consider a comoving sphere of radius $S(t) = Sa(t)$ centered on the protogalaxy. $a(t)$ is the cosmological scale factor. S is taken to be of order but smaller than half the distance to the nearest protogalaxy of comparable size, say one third of that distance. The total torque applied to the volume V of the sphere is

$$\vec{\tau}(t) = \int_{V(t)} d^3r \delta\rho(\vec{r}, t) \vec{r} \times (-\vec{\nabla}\phi(\vec{r}, t)) \quad (11)$$

where $\delta\rho(\vec{r}, t) = \rho(\vec{r}, t) - \rho_0(t)$ is the density perturbation. $\rho_0(t)$ is the unperturbed density. In the linear regime of evolution of density perturbations, the gravitational potential does not depend on time when expressed in terms of comoving coordinates, i.e. $\phi(\vec{r} = a(t)\vec{x}, t) = \phi(\vec{x})$. Moreover $\delta(\vec{r}, t) \equiv \frac{\delta\rho(\vec{r}, t)}{\rho_0(t)}$ has the form $\delta(\vec{r} = a(t)\vec{x}, t) = a(t)\delta(\vec{x})$. Hence

$$\vec{\tau}(t) = \rho_0(t)a(t)^4 \int_V d^3x \delta(\vec{x}) \vec{x} \times (-\vec{\nabla}_x\phi(\vec{x})). \quad (12)$$

Eq. (12) shows that the direction of the torque is time independent. Hence the rotation axis is time independent, as in the caustic ring model. Furthermore, since $\rho_0(t) \propto a(t)^{-3}$, $\tau(t) \propto a(t) \propto t^{2/3}$ and hence $\ell(t) \propto L(t) \propto t^{5/3}$. Since $R(t) \propto t^{2/3 + \frac{2}{9\epsilon}}$, tidal torque theory predicts the time dependence of Eq. (3) provided $\epsilon = 0.33$. This value of ϵ is in the range, $0.25 < \epsilon < 0.35$, predicted by the evolved spectrum of density perturbations and supported by the evidence

for caustic rings. So the time dependence of the angular momentum distribution on the turnaround sphere is also consistent with the caustic ring model.

In conclusion, if the dark matter is axions, the phase space structure of galactic halos predicted by tidal torque theory is precisely, and in all respects, that of the caustic ring model proposed earlier on the basis of observations. The other dark matter candidates predict a different phase space structure for galactic halos. Although the QCD axion is best motivated, a broader class of axion-like particles behaves in the manner described here.

Acknowledgements

I am grateful to Ozgur Erken, James Fry, Qiaoli Yang and the members of the ADMX Collaboration for useful discussions. This work was supported in part by the U.S. Department of Energy under contract DE-FG02-97ER41029.

References

- [1] For a recent review, see, in: Gianfranco Bertone (Ed.), *Particle Dark Matter*, Cambridge Univ. Press, 2010.
- [2] P. Sikivie, *Phys. Lett. B* 432 (1998) 139;
P. Sikivie, *Phys. Rev. D* 60 (1999) 063501.
- [3] A. Natarajan, P. Sikivie, *Phys. Rev. D* 73 (2006) 023510.
- [4] P. Sikivie, J. Ipser, *Phys. Lett. B* 291 (1992) 288.
- [5] J.A. Fillmore, P. Goldreich, *Ap. J.* 281 (1984) 1.
- [6] E. Bertschinger, *Ap. J. Suppl.* 58 (1985) 39.
- [7] P. Sikivie, I. Tkachev, Y. Wang, *Phys. Rev. Lett.* 75 (1995) 2911;
P. Sikivie, I. Tkachev, Y. Wang, *Phys. Rev. D* 56 (1997) 1863.
- [8] L.D. Duffy, P. Sikivie, *Phys. Rev. D* 78 (2008) 063508.
- [9] A.G. Doroshkevitch, *Astrophysics* 6 (1970) 320;
P.J.E. Peebles, *Ap. J.* 277 (1984) 470;
- Y. Hoffman, J. Shaham, *Ap. J.* 297 (1985) 16.
- [10] W. Kinney, P. Sikivie, *Phys. Rev. D* 61 (2000) 087305.
- [11] P. Sikivie, *Phys. Lett. B* 567 (2003) 1.
- [12] A. Natarajan, P. Sikivie, *Phys. Rev. D* 76 (2007) 023505.
- [13] V. Onemli, P. Sikivie, *Phys. Lett. B* 675 (2009) 279.
- [14] J.F. Navarro, C.S. Frenk, S.D.M. White, *Ap. J.* 462 (1996) 563;
B. Moore, et al., *Ap. J. Lett.* 499 (1998) L5;
M. Vogelsberger, S.D.M. White, R. Mohayaee, V. Springel, *MNRAS* 400 (2009) 2174.
- [15] G. Stromberg, *Ap. J.* 79 (1934) 460;
F. Hoyle, in: J.M. Burgers, H.C. van de Hulst (Eds.), *Problems of Cosmical Aerodynamics*, Central Air Documents Office, Dayton, OH, 1949, p. 195.
- [16] P.J.E. Peebles, *Ap. J.* 155 (1969) 2;
P.J.E. Peebles, *Astron. Ap.* 11 (1971) 377.
- [17] G. Efstathiou, B.J.T. Jones, *MNRAS* 186 (1979) 133;
J. Barnes, G. Efstathiou, *Ap. J.* 319 (1987) 575;
B. Cervantes-Sodi, et al., *Rev. Mex. AA.* 34 (2008) 87.
- [18] S. Weinberg, *Gravitation and Cosmology*, Wiley, 1973;
S. Dodelson, *Modern Cosmology*, Academic Press, 2003.
- [19] R.D. Peccei, H. Quinn, *Phys. Rev. Lett.* 38 (1977) 1440;
R.D. Peccei, H. Quinn, *Phys. Rev. D* 16 (1977) 1791;
S. Weinberg, *Phys. Rev. Lett.* 40 (1978) 223;
F. Wilczek, *Phys. Rev. Lett.* 40 (1978) 279.
- [20] J. Kim, *Phys. Rev. Lett.* 43 (1979) 103;
M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys. B* 166 (1980) 493;
A.P. Zhitnitskii, *Sov. J. Nucl.* 31 (1980) 260;
M. Dine, W. Fischler, M. Srednicki, *Phys. Lett. B* 104 (1981) 199.
- [21] J. Preskill, M. Wise, F. Wilczek, *Phys. Lett. B* 120 (1983) 127;
L. Abbott, P. Sikivie, *Phys. Lett. B* 120 (1983) 133;
M. Dine, W. Fischler, *Phys. Lett. B* 120 (1983) 137.
- [22] Reviews of axions include: J.E. Kim, *Phys. Rep.* 150 (1987) 1;
M.S. Turner, *Phys. Rep.* 197 (1990) 67;
G.G. Raffelt, *Phys. Rep.* 198 (1990) 1;
P. Sikivie, *Lect. Notes Phys.* 741 (2008) 19.
- [23] P. Sikivie, Q. Yang, *Phys. Rev. Lett.* 103 (2009) 111301.
- [24] L.M. Widrow, N. Kaiser, *Ap. J.* 416 (1993) L71.
- [25] T. Rindler-Daller, P. Shapiro, arXiv:0912.2897.