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Some Blocking Semiovals which Admit a Homology Group

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The study of blocking semiovals in finite projective planes was motivated by Batten [1] in connection with cryptography. Dover in [4] studied blocking semiovals in a finite projective plane of order q which meet some line in $q-1$ points. In this note, some blocking semiovals in $PG(2, q)$ are considered which admit a homology group, and three new families of blocking semiovals are constructed. Any blocking semioval in the first or the third family meets no line in $q-1$ points.

1. INTRODUCTION

Let $\Pi = (\mathcal{P}, \mathcal{L})$ be a projective plane. A *blocking set* in Π is a set B of points such that for every line $l \in \mathcal{L}$, $l \cap B \neq \phi$, but l is not entirely contained in B. A *semioval* in Π is a set S of points such that for every point $P \in S$, there exists a unique line $l \in \mathcal{L}$ such that $l \cap S = \{P\}$. The idea of a semioval was introduced in [3] and [7]. A *blocking semioval* in Π is a set of R of points which is both a semioval and a blocking set.

At the CMS Special Session in Finite Geometry in 1997, L. M. Batten posed the problem of classifying all blocking semiovals. The study of the problem was started by Batten and Dover in [2] and Dover in [4, 5]. Known families of blocking semiovals are few. Dover in [5] constructed a family of blocking semiovals of size $3q-4$ in a projective plane Π of order q, with $q \geq 5$, if Π contains a Δ-configuration. Of course, any desarguesian plane $PG(2, q)$ has this property. The author in [6] constructed two families of blocking semiovals in $PG(2, q)$. The first one is a family of blocking semiovals of size $3q-4$, where $q \geq 5$. This family contains the one constructed by Dover as a special case. The second one is a family of blocking semiovals of size $3q-n-2$, where $q = n^e$, $n \geq 3$, n is a prime power and $e \geq 2$. Any blocking semioval in these families meets some line in $q-1$ points, and admits a nontrivial homology group if q is odd.

Let S be a blocking semioval in a projective plane Π of order q. Then let x_i denote the number of lines of Π which meet S in exactly i points. Clearly $x_0 = 0$ and $x_1 = |S|$ by the definition of S. Dover in [4] proved that $x_q = 0$ for $q > 3$.

In this note, we consider some blocking semiovals in $PG(2, q)$ which admit a nontrivial homology group, and construct the following three families of blocking semiovals:

(i) If $q = r^e$, $r \geq 3$, r is a prime power and $e \geq 2$, then there exist blocking semiovals of size $3q-4$ with $x_{q-1} = 0$ (see Theorem 3.4).
(ii) If $q = r^e$, $r \geq 3$, r is a prime power, $e \geq 2$ and $3 \leq n \leq r$, then there exist blocking semiovals of size $3q-n-2$ with $x_{q-1} = 1$ (see Theorem 4.5).
(iii) If $q = r^{e_1 e_2}$, $r \geq 3$, r is a prime power, $e_1 \neq 1$, $e_2 \neq 1$ and $3 \leq n \leq r$, then there exist blocking semiovals of size $3q-n-2$ with $x_{q-1} = 0$ (see Theorem 4.6).

All the other known families of blocking semiovals are unitals and vertexless triangles.

2. BLOCKING SEMIOVALS OF HOMOLOGY TYPE

Let $\mathcal{P}$ and $\mathcal{L}$ be the set of points and the set of lines in a desarguesian plane $PG(2, q)$ with $q \geq 7$. $PG(2, q)$ is defined on the three-dimensional vector space $V = \{(a, b, c) | a, b, c \in$

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$GF(q)\}$ over $GF(q)$. For $(a, b, c) \in V$, set $[a, b, c] = \{(ax, bx, cx) | x \in GF(q)\}$. Then $\mathcal{P} = \{[a, b, c] | a, b, c \in GF(q)$ and $(a, b, c) \neq (0, 0, 0)\}$. The lines of $PG(2, q)$ are the two-dimensional subspaces of V. Choose a line l_0 from $\mathcal{L}$. Let $P_0, P_1, \ldots, P_n$ be $n + 1$ distinct points on l_0, where $2 \leq n \leq q - 2$. Choose a point Q_0 off l_0. Let, for $i \in \{0, 1, \ldots, n\}$, Ω_i be a subset of $P_i Q_0 - \{Q_0, P_i\}$ with $|\Omega_i| \geq 2$. Set

$$S = (l_0) \cup \Omega_0 \cup \Omega_1 \cup \cdots \cup \Omega_n - \{P_0, P_1, \ldots, P_n\}.$$

We will derive a necessary and sufficient condition for S to be a blocking semioval. We may assume that $Q_0 = [1, 0, 0]$, $P_0 = [0, 0, 1]$, $P_i = [0, 1, a_i]$ $(i = 1, 2, \ldots, n)$. We define subsets Δ_i of $GF(q)^* = GF(q) - \{0\}$ $(i = 0, 1, \ldots, n)$ as follows:

$$\Delta_0 = \{x \in GF(q) | [1, 0, x] \in \Omega_0\},$$
$$\Delta_i = \{x \in GF(q) | [1, x, a_i x] \in \Omega_i\} \qquad (i = 1, 2, \ldots, n).$$

Clearly, $|\Delta_i| \geq 2$ $(i = 0, 1, \ldots, n)$. For $i, j, k \in \{1, 2, \ldots, n\}$ with $i \neq j$, set $\Phi_{jk} = (a_k - a_j)\Delta_k$ and $\Phi_{ijk} = \frac{a_k - a_j}{a_i - a_j}\Delta_k$.

In the rest of this section we assume the following.

ASSUMPTION 2.1. If q is odd, $-\Delta_i = \Delta_i$ for all $i \in \{0, 1, \ldots, n\}$.

LEMMA 2.2. *S is a blocking set if and only if the following hold.*

(i) $GF(q)^* = \Delta_1 \cup \Delta_2 \cup \cdots \cup \Delta_n$.
(ii) *For any $i \in \{1, 2, \ldots, n\}$, $GF(q)^* = \Delta_0 \cup \bigcup_{1 \leq j (\neq i) \leq n} \Phi_{ij}$.*

PROOF. S is a blocking set if and only if for any $i \in \{0, 1, \ldots, n\}$, any line through the point P_i except $Q_0 P_i$ and l_0 intersects $\Omega_0 \cup \Omega_1 \cup \cdots \cup \widehat{\Omega_i} \cup \cdots \cup \Omega_n$.

Any line $\{[1, b, x] | x \in GF(q)\} \cup \{P_0\} (b \in GF(q)^*)$ through the point P_0 except $Q_0 P_0$ and l_0 intersects $\Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_n$.

$\Leftrightarrow$ If $b \in GF(q)^*$, then there exists $j \in \{1, 2, \ldots, n\}$ such that $b \in \Delta_j$.
$\Leftrightarrow$ $GF(q)^* = \Delta_1 \cup \Delta_2 \cup \cdots \cup \Delta_n$.

Next let $i \in \{1, 2, \ldots, n\}$.

Any line $\{[1, x, a_i x + b] | x \in GF(q)\} \cup \{P_i\} (b \in GF(q)^*)$ through the point P_i except $Q_0 P_i$ and l_0 intersects $\Omega_0 \cup \Omega_1 \cup \cdots \cup \widehat{\Omega_i} \cup \cdots \cup \Omega_n$.

$\Leftrightarrow$ If $b \in GF(q)^* - \Delta_0$, then there exists $j \in \{1, 2, \ldots \hat{i}, \ldots, n\}$ such that $a_i x + b = a_j x$.
$\Leftrightarrow$ If $b \in GF(q)^* - \Delta_0$, then there exists $j \in \{1, 2, \ldots, \hat{i}, \ldots, n\}$ such that $b \in (a_j - a_i)\Delta_j = \Phi_{ij}$.
$\Leftrightarrow$ $GF(q)^* = \Delta_0 \cup \bigcup_{1 \leq j (\neq i) \leq n} \Phi_{ij}$. $\square$

LEMMA 2.3. *S is a semioval if and only if the following hold.*

(i) *If $a \in GF(q) - \{a_1, a_2, \ldots, a_n\}$, then $GF(q)^* = \Delta_0 \cup \bigcup_{1 \leq i \leq n} (a_i - a)\Delta_i$.*
(ii) *If $a \in \Delta_0$, then there exists a unique element $j \in \{1, 2, \ldots, n\}$ such that $a \notin \bigcup_{1 \leq k (\neq j) \leq n} \Phi_{jk}$, but $a \in \bigcup_{1 \leq k (\neq i) \leq n} \Phi_{ik}$ for all $i \in \{1, 2, \ldots, \hat{j}, \ldots, n\}$.*
(iii) *Let $i \in \{1, 2, \ldots, n\}$ and $a \in \Delta_i$. Then one of the following occurs.*
   (a) *If $a \notin \bigcup_{1 \leq k (\neq i) \leq n} \Delta_k$, then $a \in \frac{1}{a_i - a_j}\Delta_0 \cup \bigcup_{1 \leq k (\neq i, j) \leq n} \Phi_{ijk}$ for all $j \in \{1, 2, \ldots, \hat{i}, \ldots, n\}$.*

(b) *If $a \in \bigcup_{1\le k(\ne i)\le n} \Delta_k$, then there exists a unique element $j \in \{1, 2, \dots, \hat{i}, \dots, n\}$ such that $a \notin \frac{1}{a_i-a_j}\Delta_0 \cup \bigcup_{1\le k(\ne i,j)\le n} \Phi_{ijk}$, but $a \in \frac{1}{a_i-a_s}\Delta_0 \cup \bigcup_{1\le k(\ne i,s)\le n} \Phi_{isk}$ for all $s \in \{1, 2, \dots, \hat{i}, \dots, \hat{j}, \dots, n\}$.*

PROOF. S is a semioval if and only if the following hold.
(α) Let $P \in (l_0) - \{P_0, P_1, \dots, P_n\}$. If $l \in \mathcal{L}$, $P \in l$, $Q_0 \notin l$ and $l \ne l_0$, then $(l) \cap (\Omega_0 \cup \Omega_1 \cup \dots \cup \Omega_n) \ne \phi$.
(β) For any $i \in \{0, 1, \dots, n\}$ and for any $P \in \Omega_i$, there exists a unique element $j \in \{0, 1, \dots, \hat{i}, \dots, n\}$ such that the line PP_j does not intersect $\Omega_0 \cup \Omega_1 \cup \dots \cup \widehat{\Omega}_i \cup \dots \cup \widehat{\Omega}_j \cup \dots \cup \Omega_n$.
Let $P \in (l_0) - \{P_0, P_1, \dots, P_n\}$. Then $P = [0, 1, a]$ for some $a \in GF(q) - \{a_1, a_2, \dots, a_n\}$. Let l be a line such that $P \in l$, $Q_0 \notin l$ and $l \ne l_0$. Then $l = \{[1, x, ax + b] | x \in GF(q)\} \cup \{P\}$ for some $b \in GF(q)^*$. $(l) \cap \Omega_0 \ne \phi$ if and only if $b \in \Delta_0$.
Let $1 \le i \le n$.
$(l) \cap \Omega_i \ne \phi$. $\Leftrightarrow$ There exists $x \in \Delta_i$ such that $ax + b = a_i x$. $\Leftrightarrow$ $b \in (a_i - a)\Delta_i$. Therefore, (α) is equivalent to (i).
Let $i = 0$ and $P \in \Omega_0$. Then $P = [1, 0, a]$ for some $a \in \Delta_0$. Let $1 \le j \le n$. $PP_j = \{[1, x, a_j x + a] | x \in GF(q)\} \cup \{P_j\}$. Let $k \in \{\hat{0}, 1, \dots, \hat{j}, \dots, n\}$.
The line PP_j does not intersect Ω_k.

$\Leftrightarrow$ For any $x \in \Delta_k$ $a_j x + a \ne a_k x$.
$\Leftrightarrow$ $\frac{a}{a_k - a_j} \notin \Delta_k$.
$\Leftrightarrow$ $a \notin (a_k - a_j)\Delta_k = \Phi_{jk}$.

Therefore, when $i = 0$, (β) is equivalent to (ii).
Let $1 \le i \le n$ and $P \in \Omega_i$. Then $P = [1, a, a_i a]$ for some $a \in \Delta_i$. $PP_0 = \{[1, a, x] | x \in GF(q)\} \cup \{P_0\}$. Let $k \in \{\hat{0}, 1, \dots, \hat{i}, \dots, n\}$.
The line PP_0 does not intersect Ω_k.

$\Leftrightarrow$ For any $x \in \Delta_k$ $x \ne a$.
$\Leftrightarrow$ $a \notin \Delta_k$.

Let $j \in \{1, 2, \dots, \hat{i}, \dots, n\}$. Then $PP_j = \{[1, x + a, a_j x + a_i a] | x \in GF(q)\} \cup \{P_j\} = \{[1, x, a_j(x - a) + a_i a] | x \in GF(q)\} \cup \{P_j\}$. Let $k' \in \{1, \dots, \hat{i}, \dots, \hat{j}, \dots, n\}$.
The line PP_j intersects Ω_0.

$\Leftrightarrow$ $a_i a - a_j a \in \Delta_0$.
$\Leftrightarrow$ $a \in \frac{1}{a_i - a_j}\Delta_0$.

The line PP_j intersects $\Omega_{k'}$.

$\Leftrightarrow$ $a_j(x - a) + a_i a = a_{k'} x$ for some $x \in \Delta_{k'}$.
$\Leftrightarrow$ $\frac{a(a_i - a_j)}{a_{k'} - a_j} \in \Delta_{k'}$.
$\Leftrightarrow$ $a \in \frac{a_{k'} - a_j}{a_i - a_j}\Delta_{k'} = \Phi_{ijk'}$.

Therefore, when $1 \le i \le n$, (β) is equivalent to (iii). Thus the lemma is proved. $\square$

3. $n = 2$

In this section, under Assumption 2.1 we consider the point set S of $PG(2, q)$ defined in Section 2, when $n = 2$. We remark that from the definition of Δ's, $|\Delta_i| \ge 2 (i = 0, 1, 2)$. Then we may assume that $a_1 = 1$, $a_2 = 0$.
Lemmas 2.2 and 2.3 yield the following theorem.

THEOREM 3.1. *Let $n = 2$ and $a_1 = 1$, $a_2 = 0$. Then, S is a blocking semioval if and only if the following hold.*

(i) *If $i \neq j \in \{0, 1, 2\}$, then $GF(q)^* = \Delta_i \cup \Delta_j$.*
(ii) *If $a \in GF(q) - \{0, 1\}$, then $GF(q)^* = \Delta_0 \cup (1 - a)\Delta_1 \cup a\Delta_2$.*
(iii) *If $\{i, j, k\} = \{0, 1, 2\}$, then $\Delta_i = (\Delta_j - \Delta_k) \cup (\Delta_k - \Delta_j)$.*

From Theorem 3.1 the next result follows.

COROLLARY 3.2. *If S is a blocking semioval, then $|S| = 3q - 4$ and, furthermore, when q is odd, an involutory homology $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ acts on S.*

PROOF. Let S be a blocking semiovals. Since Theorem 3.1(iii) implies that the sets Δ_0, Δ_1 and Δ_2 exactly double cover $GF(q)^*$, $|S| = |\Delta_0| + |\Delta_1| + |\Delta_2| + (q-2) = 2(q-1) + (q-2) = 3q - 4$. The existence of the involutory homology follows from Assumption 2.1. □

EXAMPLE 3.3. Set $\Delta_0 = GF(q)^*$. If q is odd, let $\Delta_1 \neq \phi$ be a subset of $GF(q)^*$ such that $-\Delta_1 = \Delta_1$ and $\Delta_1 \neq GF(q)^*$. If q is even, let Δ_1 be a subset of $GF(q)^*$ such that $2 \leq |\Delta_1| \leq q - 3$. Set $\Delta_2 = GF(q)^* - \Delta_1$. Then, Δ_0, Δ_1 and Δ_2 satisfy (i), (ii) and (iii) of Theorem 3.1. Therefore, the point set S corresponding to Δ_0, Δ_1 and Δ_2 is a blocking semioval of size $3q - 4$ such that $x_{q-1} = 1$. This is a blocking semioval as defined in Theorem 4.2 of [5].

THEOREM 3.4. *Let $q = r^e$, $r \geq 3$, r be a prime power and $e \geq 2$. Set $\Delta_2 = GF(r)^*$. Let Φ be a nonempty subset of $GF(r)^*$ such that $-\Phi = \Phi$ and $\Phi \neq GF(r)^*$. Here, if r is even, let $2 \leq |\Phi| \leq r - 3$. Set $\Delta_1 = (GF(q)^* - GF(r)^* \cup \Phi$ and $\Delta_0 = GF(q)^* - \Phi$. Then Δ_0, Δ_1 and Δ_2 satisfy (i), (ii) and (iii) of Theorem 3.1. Therefore, the point set S corresponding to Δ_0, Δ_1 and Δ_2 is a blocking semioval of size $3q - 4$ such that $x_{q-1} = 0$ and $x_{q-2} \neq 0$.*

PROOF. It is clear that Δ_0, Δ_1 and Δ_2 satisfy (i) and (iii) of Theorem 3.1. We will show that Δ_0, Δ_1, and Δ_2 satisfy (ii) of Theorem 3.1. Let $a \in GF(q) - \{0, 1\}$. We want to show that $GF(q)^* = \Delta_0 \cup (1 - a)\Delta_1 \cup a\Delta_2$. Let $x \in GF(q)^* - \Delta_0$. Then $x \in \Phi \subseteq GF(r)^*$. If $a \notin GF(r)^*$, then $\frac{x}{1-a} \notin GF(r)^*$, and hence $x \in (1 - a)\Delta_1$. If $a \in GF(r)^*$, then $\frac{x}{a} \in GF(r)^* = \Delta_2$ and therefore $x \in a\Delta_2$. Thus $GF(q)^* = \Delta_0 \cup (1 - a)\Delta_1 \cup a\Delta_2$. □

The author proved that the following conjecture is true for $q \in \{7, 11, 13, 17\}$ using a computer.

CONJECTURE 3.5. *If q is a prime with $q \geq 7$, then any blocking semioval S defined in Theorem 3.1 is one of the blocking semiovals constructed in Example 3.3.*

4. $n \geq 3$

In this section we consider the set S of points of $PG(2, q)$ defined in Section 2 for $n \geq 3$.

ASSUMPTION 4.1. (i) $q = r^e$, $r \geq 3$, r is a prime power and $e \geq 2$.
(ii) $3 \leq n \leq r$ and $GF(r) \supseteq \{a_1, a_2, \ldots, a_n\}$.
(iii) For any $i \in \{0, 1, \ldots, n\}$ and for any $x \in GF(r)^*$, $x\Delta_i = \Delta_i$.

Lemmas 2.2 and 2.3 yield the following theorem.

THEOREM 4.2. *Under Assumption* 4.1*, S is a blocking semioval if and only if the following hold.*

(i) *For any $i \in \{0, 1, \dots, n\}$, $GF(q)^* = \cup_{0 \le j(\neq i) \le n} \Delta_j$.*
(ii) *For any $a \in GF(q) - \{a_1, a_2, \dots, a_n\}$, $GF(q)^* = \Delta_0 \cup \bigcup_{1 \le i \le n} (a_i - a)\Delta_i$.*
(iii) *For any $i \in \{0, 1, \dots, n\}$ and for any $a \in \Delta_i$, there exists a unique element $j \in \{0, 1, \dots, \hat{i}, \dots, n\}$ such that $a \in \Delta_j$.*

PROOF. Assume that S is a blocking semioval. It is clear that (i) and (ii) hold. Let $i \in \{0, 1, \dots, n\}$ and $a \in \Delta_i$. Then, by (ii) and (iii) of Lemma 2.3, there exists a unique element $j \in \{0, 1, \dots, \hat{i}, \dots, n\}$ such that $a \notin \bigcup_{0 \le k(\neq i, j) \le n} \Delta_k$. However, since $GF(q)^* = \bigcup_{0 \le k(\neq i) \le n} \Delta_n$, $a \in \Delta_j$. Thus we obtain (iii).
The converse is clear. □

THEOREM 4.3. *Under Assumption* 4.1*, S is a blocking semioval if and only if for all distinct $i, j \in \{0, 1, \dots, n\}$ there exists a subset Δ_{ij} of $GF(q)^*$ which satisfies the following.*

(i) *Each Δ_{ij} is closed under multiplication by $GF(r)^*$.*
(ii) *For all distinct $i, j \in \{0, 1, \dots, n\}$, $\Delta_{ij} = \Delta_{ji}$.*
(iii) *For any $i \in \{0, 1, \dots, n\}$, $\Delta_i = \bigcup_{0 \le j(\neq i) \le n} \Delta_{ij} \neq \phi$.*
(iv) *$GF(q)^* = \bigcup_{0 \le i < j \le n} \Delta_{ij}$ is a disjoint union.*
(v) *For any $a \in GF(q) - \{a_1, \dots, a_n\}$, $GF(q)^* = \Delta_0 \cup \bigcup_{1 \le i \le n} (a_i - a)\Delta_i$.*

PROOF. Assume that S is a blocking semioval. For all distinct $i, j \in \{0, 1, \dots, n\}$, set $\Delta_{ij} = \Delta_i \cap \Delta_j$. Then clearly (i), (ii) and (v) hold.
Let $0 \le i \le n$. From Theorem 4.2(i) it follows that $\Delta_i = \Delta_i \cap GF(q)* = \Delta_i \cap \left(\bigcup_{0 \le j(\neq i) \le n} \Delta_j\right) = \bigcup_{0 \le j(\neq i) \le n} (\Delta_i \cap \Delta_i) = \bigcup_{0 \le j(\neq i) \le n} \Delta_{ij}$. Therefore we obtain (iii).
Let $0 \le i \neq j \le n$, $0 \le i \neq j' \le n$, and $j \neq j'$. If $b \in \Delta_{ij} \cap \Delta_{ij'}$, then $b \in \Delta_i$, $b \in \Delta_j$ and $b \in \Delta_{j'}$ which are contrary to (i) and (iii) of Theorem 4.2. Therefore $\Delta_{ij} \cap \Delta_{ij'} = \phi$. Furthermore, let $0 \le i' \neq i \le n$ and $i' \le j'$. Since Δ_{ij} and $\Delta_{ij'}$ are mutually disjoint, and $\Delta_{ij'} = \Delta_i \cap \Delta_{j'} = \Delta_{j'i}$ and $\Delta_{i'j'}$ are mutually disjoint, $\Delta_{ij} \cap \Delta_{i'j'} = \phi$. Thus, by (i) of Theorem 4.2, we have (iv).
The converse is clear. □

COROLLARY 4.4. *If S is a blocking semioval, then $|S| = 3q - n - 2$ and the homology $\begin{pmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix}$ $(s \in GF(r)^*)$ acts on S.*

PROOF. $|S| = \sum_{0 \le i \le n} |\Delta_i| + (q - n) = \sum_{0 \le i \le n} \sum_{0 \le j(\neq i) \le n} |\Delta_{ij}| + (q - n) = 2 \sum_{0 \le i < j \le n} |\Delta_{ij}| + (q - n) = 2(q - 1) + (q - n) = 3q - n - 2$. Since each Δ_i is closed under multiplication by $GF(r)^*$, the homologies do exist. □

THEOREM 4.5. *Assume Assumption* 4.1*. Set $\Delta_0 = GF(q)^*$ and let $GF(q)^* = \Delta_1 \cup \cdots \cup \Delta_n$ be a mutually disjoint union. Then $\Delta_0, \Delta_1, \dots, \Delta_n$ satisfy (i), (ii) and (iii) of Theorem* 4.2*, and the size of the blocking semioval corresponding to $\Delta_0, \Delta_1, \dots, \Delta_n$ is $3q - n - 2$ and $x_{q-1} = 1$. In particular, when $n = r$, the blocking semioval is one of the blocking semiovals defined in Theorem* 5.2 *of* [6].

THEOREM 4.6. *Let* $q = r^e = r^{e_1 e_2}$, r *be a prime power,* $e_1 \neq 1$, $e_2 \neq 1$ *and* $3 \leq n \leq r$. *Set* $\Delta_{01} = GF(q) - GF(r^{e_1})$. *Let* $GF(r^{e_1})^* = \bigcup_{0 \leq i < j \leq n, (i,j) \neq (0,1)} \Delta_{ij}$ *be a partition such that each* Δ_{ij} *is closed under multiplication by* $GF(r)^*$. *For distinct* $i, j \in \{0, 1, \ldots, n\}$ *with* $i < j$, *set* $\Delta_{ji} = \Delta_{ij}$. *Then,* $\Delta_{ij} (0 \leq i \neq j \leq n)$ *satisfy (i)–(v) of Theorem* 4.3. *Therefore, the set S of points corresponding to* $\Delta_{ij} (0 \leq i \neq j \leq n)$ *is a blocking semioval of size* $3q - n - 2$ *with* $x_{q-1} = 0$ *and* $x_{q-n} \neq 0$.

PROOF. We must show that $\Delta_{ij} (0 \leq i \neq j \leq n)$ satisfy (i)–(v) of Theorem 4.3. It is clear that (i), (ii), (iii) and (iv) hold. We will show that (v) holds. Let $a \in GF(q) - \{a_1, a_2, \ldots, a_n\}$. Let $x \in GF(q) - \Delta_0$. Then $x \in GF(r^{e_1})^*$.

Assume that $a \notin GF(r^{e_1})^*$. Then for any $i \in \{1, 2, \ldots, n\}$, $\frac{x}{a_i - a} \notin GF(r^{e_1})^*$ and hence $\frac{x}{a_i - a} \in \Delta_{01} \subseteq \Delta_1$. In particular, let $i = 1$. Then $x \in (a_1 - a)\Delta_1$.

Assume that $a \in GF(r^{e_1})^*$. Then, for any $i \in \{1, \ldots, n\}$, $\frac{x}{a_i - a} \in GF(r^{e_1})^* = \bigcup_{0 \leq i < j \leq n, (i,k) \neq (0,1)} \Delta_{ij}$. Therefore there exists $j \in \{1, 2, \ldots, n\}$ such that $\frac{x}{a_i - a} \in \Delta_j$. In particular, let $i = j$. Then $x \in (a_j - a)\Delta_j$. $\square$

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