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Bidding behavior in a symmetric Chinese auction

Mauricio Benegas

CAEN/UFC, Brazil

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Abstract

This paper purposes a symmetric all-pay auction where the bidders compete neither for an object nor the object itself but for a lottery on receive. That lottery is determined endogenously through the bids. This auction is known as chance auction or more popularly as Chinese auction. The model considers the possibility that for some bidders the optimal strategy is to bid zero and to rely on luck. It showed that bidders become less aggressive when the lottery satisfies a variational condition. It was also shown that luck factor is decisive to determine if the expected payoff in Chinese auction is bigger or smaller than expected payoff in standard all-pay auction.

JEL classification: D01; D44; D82

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Keywords: Chinese auction; All-pay auction; Lottery; Bayesian equilibrium

Resumo

Este trabalho propõe um modelo de leilão chinês, ou seja, um leilão all-pay sobre uma loteria. A loteria é determinada endogenamente através dos lances. O modelo considera a possibilidade de que, para alguns participantes a estratégia ótima é oferecer um lance nulo e esperar pela sorte. É mostrado que a introdução de aleatoriedade no resultado do leilão pode tornar os participantes menos agressivos caso a loteria satisfaça uma dada desigualdade variacional. Também é mostrado que o fator sorte é decisivo em determinar se os ganhos esperados são maiores ou menores no leilão chinês relativamente ao leilão all-pay padrão.

Palavras-chave: leilão chinês; leilão all-pay; loteria; equilíbrio Bayesiano

1. Introduction

The design of market institutions has been considered as one of the most important research objects since the beginning of Economics. But it was when Game Theory emerged as an important analytical framework that the theme

E-mail address: mauricio_benegas@caen.ufc.br
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gained notoriety, since for the first time a series of important tools made possible a better understanding about this subject.

Mechanism design and implementation theory made possible an approach such that virtually all allocation forms could be understood through formal mathematical models. Auction theory, for example, is undisputedly considered as a successful case in designing new market institutions.[1] The applications of auction theory surpassed its basic purposes, which are the understanding of allocation rules and price formation, as examples like first- and second-price all-pay auctions,[2] double auctions and score auctions indicate.

Two key results from auction theory, the Revenue Equivalence Theorem and the Linkage Principle, connect several different types of auction mechanisms[3] through their expected revenues. Amongst the conditions that support the validity of these results, one is particularly important to this study: the auction mechanisms are standard in the sense that the winner is the person bidding the highest amount.

There are, however, countless interesting situations where agents allocate their resources and the results are not deterministically reached, i.e., a participant offers the highest bid and yet he is not the winner of the auction. This is the case, e.g., of elections and patent races. In the first example, the candidate that spends more money is not necessarily the one that is elected. In the case of patent races the prototype with the highest investment is not necessarily chosen. These situations are usually treated as a contest where the probability of winning is proportional to the participant's bid.[4]

There are other situations subtly related to auction theory. For example, in Brazil, public sector jobs are fulfilled through public contests. More specifically, a public institution announces the availability of one or more jobs which require a well-defined and non-discriminatory selective process. In general, the referred selective process includes general and specific exams according to job available. This type of mechanism has been adopted in Brazil since the 1950s and it is estimated that nowadays more than five million candidates apply for these contests every year. Generally, the classification of the candidates depends on their performances in the exams; being hired are the ones with the highest scores. Hence, an important question emerges: does higher effort lead to certain success in such mechanisms?

Suppose that all resources allocated by a candidate in a public contest (time, discipline, educational background, etc.) can be evaluated in monetary terms. This value may be interpreted as a bid that is paid at any contingency.[5] The bids do not guarantee success but only a probability of success. Therefore, a public contest may be specifically viewed as a first-price all-pay auction whose object being auctioned is not a good but a lottery where the bidder may either win or lose the good. It is reasonable to assume that this lottery establishes a given probability of winning that depends on the participant's bid (effort). More specifically, one may assume that the probability of winning follows a stochastic order where higher bids are associated with higher probabilities of winning.

This type of auction is also used in charitable events in order to raise funds and is popularly known as the Chinese auction or, more formally, a chance auction.[6]

The main objective of this paper is to build a model to represent the Chinese auction, where its characteristics are represented and its equilibrium is derived. First a general version is proposed where the probability of winning is a continuous and increasing function of the bid. Afterwards, in order to derive some additional properties of the equilibrium bid, it is supposed that this probability is a linear function of the bid. In both versions an exogenous and common-knowledge probability of winning is assumed that will be referred to as the "luck factor". This probability is positive even when the bid is zero.

In addition to the luck factor a realization defined as the threshold of the effective competitors is assumed. This realization is also common knowledge. The idea is to allow the possibility of a partial pooling equilibrium where the bidders with smaller values (below the threshold) rely on luck, choosing as an optimal strategy a zero bid.

[1] Maskin (2004).

[2] Second-price all-pay auctions are also known as "War of Attrition". In Krishna and Morgan (1997), these two types of auctions are analyzed in detail.

[3] In fact, these results are independent of the type of the auction. For further details, see Krishna (2005) or Menezes and Monteiro (2008).

[4] The literature about contests is vast and has its origins linked with the seminal works of Gordon Tullock about rent seeking. See Tullock (1967, 1980) and Konrad (2009).

[5] Another way to describe the problem is to enquire about the monetary value that the candidate would be willing to pay in order to avoid the effort, i.e., the amount that he or she will be indifferent either to pay or to effort.

[6] The origin of the term is unclear.

Among the results obtained it is shown that if the probability of winning satisfies a variational condition then the equilibrium bids in a deterministic all-pay auction dominate the equilibrium bids in the Chinese auction. It is also shown that the welfare comparison between these two types of auction depends on the magnitude of the luck factor. Finally, in the version where there is a linear probability of winning, simple comparative static exercises show that the higher the threshold of the effective competitors the lower equilibrium bids are and, among effective competitors, the response of the equilibrium bid to changes in the luck factor is asymmetric according to the proximity to the threshold.

Besides this introduction this paper is organized as follows. In Section 2 a brief literature review is performed; in Section 3 the model is formally presented; in Section 4 equilibrium is derived and its properties are analyzed; in Section 5 the model with linear probability is proposed and, finally, in Section 6 the main conclusions are presented and some extensions are suggested.

2. Related literature

In the majority of the references cited below, models are built based on the literature about contests to address situations where agents allocate their own (monetary or non-monetary) resources on the dispute for a prize where the probability of winning depends on the amount of resources allocated.

Franke et al. (2009) proposed the design of a contest and its success function through multilevel programming. The idea is to find the contest's unique Nash equilibrium and then, based on a preference parameter, obtain a success function where aggregate effort is maximized. In this study the success function depends solely on the participant's bid and is optimally determined in each agent's utility maximization problem.

Kaplan et al. (2002) proposed an all-pay auction where the prize (or ex-post utility) does not depend only on the agent's valuation but also on its bid. Although their objectives were different the model proposed in this study is isomorphic to the multiplicatively separable environment proposed by Kaplan et al. (2002).

Based on Taylor (1995), Fullerton et al. (2002) proposed a innovation competition model in two stages: first, a contest search type is conducted by a sponsor that afterwards determines the prize through a first-price auction. Hence, in Fullerton et al. (2002), the competition model occurs separately. It is worth mentioning that in the contest stage the success function is endogenously determined.

Matros (2006) extends Tullock's basic model allowing the contest's prize valuation to be asymmetric among the participants. The author concludes that the increase in equilibrium total revenues[7] with the addition of new participants persists in the asymmetric case. It is worth mentioning that this result is maintained even when the addition of new participants may eventually reduce the number of effective competitors. In summary, as the author states,

> "This result shows that the quality of the active players (i.e., their valuations of the prize) is more important than their quantity."

Maldovanu and Sela (2006) designed a competition model with elimination stages. The effort's cost function includes an ability parameter that is private information for each competitor. The authors show that when the cost is a concave (or linear) function of effort, the equilibrium total expected effort in a single contest with a unique prize is greater than in a competition scheme with elimination stages where in each stage a certain number of prizes are allocated.

3. The model

The model encompasses an all-pay[8] auction of an indivisible object, with $N \geq 2$ risk-neutral participants who have the following characteristics:

1. the agent i submits a bid b_i and has valuation v_i that is his private information;
2. the reward in case of winning the auction is not a good but a probability distribution over events {receive, do not receive}.

[7] In Contest Theory the total revenue is referred as rent dissipation.
[8] For simplicity, is supposed that the auction is mandatory.

Define the random variable X as follows:

$$X = \begin{cases} 1 & \text{if the object is received} \\ 0 & \text{otherwise} \end{cases}$$

so that $p = \Pr[X = 1]$. Assume that individual i has a Bernoulli utility defined as:

$$u_i(x) = v_i x$$

where $v_i \in [0, \omega]$ with $\omega > 0$. His expected utility in a lottery p will be:

$$U_i(p) = p v_i$$

For any bidder i, the winning probability function[9] is defined as $p(b_i) = Pr[\, X = 1 |\, b_i]$, where b_i is a bid of agent i. It is implicit that the winning probability is anonymous, depending solely on the participant's bid. The assumption of anonymity may not be supported empirically. Castelar et al. (2009), for example, analyzed a specific public contest and concluded that there are socio-economic characteristics of individuals that interfere dramatically on the winning probability. In other words, with the same effort (bid) individuals may have different winning probabilities. On the other hand, if this supposition is withdrawn a symmetric equilibrium would not be possible, not to mention that anonymity is among the desired axioms of success functions in contests.[10]

Some assumptions are made below:

Assumption 1. $p(\cdot)$ is continuous and twice differentiable function so that $p(0) = \varepsilon > 0$ and $p' > 0$.

Assumption 2. The values of the bidders in Chinese auction correspond to a vector of iid random variables according to a distribution F with support $[0, \omega]$. F is absolutely continuous and on its differentiable points $F' = f$, and f is a positive and limited function within $[0, \omega]$.

Assumption 3. There is a realization $\tilde{v} \in [0, \omega]$ so that $[p(b_i) - \varepsilon]v_i > b_i$ for any i such that $v_i > \tilde{v}$ and $b_i \geq 0$, and $[p(b_i) - \varepsilon]v_i \leq b_i$ for any i such that $v_i \leq \tilde{v}$.

Assumption 4. All aspects considered in Assumptions 1–3 are common knowledge.

In Assumption 1 ε is an exogenous probability of winning that will be called from now on as "luck factor". The assumption that $p(\cdot)$ is strictly increasing guarantees a first-order stochastic dominance for the lotteries that are associated with higher bids. Furthermore, Supposition 3 (allow the possibility that exist) guarantees the existence of a pool of sufficiently low types that rely on luck. Based on this possibility, Definition 1, makes a distinction between bidders in the Chinese auction.

Definition 5. A bidder i will be considered an effective competitor in Chinese auction if and only if $v_i > \tilde{v}$. Otherwise i will be deemed a non-effective competitor. The realization $\tilde{v}$ is referred as a threshold of effective competitors.

For any participant i a strategy is a function $\beta_i : [0, \omega] \to [0, \overline{b}]$ such that $\beta_i(0) = 0$ and $\beta_i(\omega_i) = \overline{b} > 0$. It is assumed that β_i is a continuous and increasing function.[11] In a symmetric equilibrium it is admitted that $\beta_i = \beta$ for any i. It is worth mentioning that, implicitly, even with symmetric bidders there is a possibility of pooling for those below $\tilde{v}$.

The nature of competition requires that each effective competitor submits his bid in such a way that his expected utility is maximized, considering that he or she expects to have the maximum winning probability in the lottery that is assigned according to his bid. An effective competitor is only concerned with competition against other effective rivals, since the others (if any) receive a lottery with an exogenous winning probability ε that is common knowledge.

9 That function is analogous to success function in Contest Theory. The main difference is that in the second case the probabilities depends on all bids.

10 Skarpedas (1996) offers a detailed discussion about the axioms on success function. In Hwang (2009), the author proposes an unified form of success functions and test the empirical support to several particular forms.

11 Therefore differentiable almost everywhere.

Therefore, obtaining the maximum winning probability should be conditioned to competition solely against other effective competitors.

It is assumed that each effective competitor is uncertain about the number of existing effective rivals and, therefore, his expected gain should include this uncertainty.[12] Hence, being an effective rival or not is modeled as a binomial random variable whose probability of success (being an effective rival) is common knowledge since $Pr[v > \tilde{v}] = 1 - F(\tilde{v})$.

From the perspective of a give bidder, if the number of effective rivals is a random variable $\tilde{K}$, then the probability that there are exactly K effective rivals is given by

$$Pr[\tilde{K} = K] = \binom{N-1}{K} [1 - F(\tilde{v})]^K F(\tilde{v})^{(N-1)-K}$$

The possible values of $\tilde{K}$ lie between 0 (there are no effective rivals) and $N - 1$ (all other participants are effective rivals).

4. Equilibrium

Let a participant i with $v_i > \tilde{v}$ and define the set $N_K^i = \{j \in N; v_j > \tilde{v}, j \neq i\}$ such that $\#N_K^i = K$. Hence, if i believes that there are exactly K effective rivals, his ex-ante expected utility could be written as follows:

$$\pi_i(v_i, b_i, K) = p(b_i) v_i \mathbb{E}_{v_{-i}} \left[\mathbf{1}_{\left\{ p(b_i) \geq \max\limits_{j \in N_K^i} p(\beta(v_j)) \right\}} \middle| v_j > \tilde{v}, \forall j \in N_K^i \right] - b_i \tag{1}$$

Since $p(\cdot)$ and $\beta(\cdot)$ are increasing functions, (1) can be rewritten as

$$\pi_i(v_i, b_i, K) = p(b_i) v_i Pr\left[Y_{\max}^K \leq \beta^{-1}(b_i) \middle| Y_{\min}^K > \tilde{v} \right] - b_i$$

where $Y_{\max}^K$ e $Y_{\min}^K$ are, respectively, the highest and the lowest order statistics amongst K independent random variables. Given i's uncertainty towards the number of effective rivals, he chooses his bid so that

$$\sum_{K=0}^{N-1} \binom{N-1}{K} \pi_i(v_i, b_i, K)[1 - F(\tilde{v})]^K F(\tilde{v})^{(N-1)-K} \tag{2}$$

could be maximized.

Proposition 6. *Let $G = F^{N-1}$, then (2) is equal to:*

$$p(b_i) v_i G(\beta^{-1}(b_i)) - b_i$$

Proof. Arnold et al. (2008) show that

$$Pr\left[Y_{\max}^K \leq \beta^{-1}(b_i) \middle| Y_{\min}^K > \tilde{v} \right] = \frac{\left[F(\beta^{-1}(b_i)) - F(\tilde{v}) \right]^K}{[1 - F(\tilde{v})]^K}$$

[12] Note that the uncertainty is not about the number of participants; it is related to who are the effective competitors. A general model for the first case is presented by McAfee and McMillan (1987).

Therefore, the agent's expected payoff is

$$
\begin{aligned}
&p(b_i)v_i\sum_{K=0}^{N-1}\binom{N-1}{K}\frac{\left[F(\beta^{-1}(b_i))-F(\tilde{v})\right]^K}{[1-F(\tilde{v})]^K}[1-F(\tilde{v})]^K F(\tilde{v})^{(N-1)-K}-b_i\\
&=p(b_i)v_i\sum_{K=0}^{N-1}\binom{N-1}{K}\left[F(\beta^{-1}(b_i))-F(\tilde{v})\right]^K F(\tilde{v})^{(N-1)-K}-b_i\\
&=p(b_i)v_iG(\beta^{-1}(b_i))-b_i
\end{aligned}
$$

□

Using Proposition 6, the participant i's problem becomes[13]

$$\max_{b_i} p(b_i)v_iG(\beta^{-1}(b_i))-b_i \tag{3}$$

The first-order conditions of this problem is given by

$$p'(b_i)v_iG(\beta^{-1}(b_i))+p(b_i)v_ig(\beta^{-1}(b_i))\frac{d\beta^{-1}(b_i)}{db_i}=1 \tag{4}$$

If in equilibrium $b_i=\beta(v_i)$ then in (4) one should have that

$$p'(\beta(v_i))\beta'(v_i)G(v_i)v_i+p(\beta(v_i))g(v_i)v_i=\beta'(v_i) \tag{5}$$

Hence, participant i's equilibrium bid with value $v_i>\tilde{v}$ is obtained by the solution of the following differential equation:

$$v_i\frac{d\left[p(\beta(v_i))G(v_i)\right]}{dv_i}=\beta'(v_i) \tag{6}$$

Observe that

$$p(\beta(v_i))G(v_i)=Pr[i\ \text{ win the object}|\,i\ \text{ offer }\ \beta(v_i)]\,Pr\left[\max_{j\neq i}\beta(v_j)\leq\beta(v_i)\right]$$

that is the probability to win the object with a bid $\beta(v_i)$ and this bid is to be the highest in the auction. Hence Eq. (6) corresponds to equality between benefit and cost expected at the margin.

Integrating (6) the equilibrium bid is given implicitly by the following integral equation:

$$\beta(v_i)=\int_{\tilde{v}}^{v_i}s\frac{d\left[p(\beta(s))G(s)\right]}{ds}ds \tag{7}$$

The following theorem establishes the sufficient conditions for the equilibrium of the Chinese auction.[14]

Theorem 7. *Consider that Assumptions 1–4 hold. Additionally assume that $1-p'(b)G(v)v>0$ for any $v\in[\tilde{v},\omega]$ and $b>0$. Hence the symmetric equilibrium strategy of the Chinese auction will be given by an increasing function $\beta(\cdot)$ defined by*

$$\beta(v_i)=\left\{\int_{\tilde{v}}^{v_i}s\frac{d\left[p(\beta(s))G(s)\right]}{ds}ds\right\}\mathbf{1}_{(\tilde{v},\omega]},\ v_i\in[0,\omega] \tag{8}$$

Proof. Initially observe that, according to Assumption 3, for every non-effective competitor, the best response to any rivals' strategy is to always submit a zero bid. Afterwards, fix some participant i and assume that all other participants in the Chinese auction follow the strategies defined by (8). Assume that i's valuation is v_i but he or she offers b different from $\beta(v_i)$. First, the bidder i never offers $b<0$ or $b>\overline{b}$. Then it must be the case that $b\in[0,\overline{b}]$. The continuity and

[13] At this point it is worth mentioning that Proposition 6 shows that the expected payoff in the Chinese auction is isomorphic to the multiplicatively separable environment proposed by Kaplan et al. (2002).

[14] Questions related to the existence and uniqueness of the equilibrium are discussed in Appendix A.

the monotonicity of $\beta(\cdot)$ in $[0, \omega]$ imply that for each $b \in [0, \overline{b}]$ there is $z \in [0, \omega]$ such that $b = \beta(z)$. Thus the deviation to b from $\beta(v_i)$ is equivalent to the deviation to z from v_i.

If $v_i \in [0, \tilde{v}]$ and $z \in [0, \tilde{v}]$ then, as it was observed initially, i's best reply is to offer $\beta(z) = 0 = \beta(v_i)$. Now assume that $v_i \in [0, \tilde{v}]$, $z \in (\tilde{v}, \omega]$ and i offers $\beta(z) > 0$. In this case the difference between the expected payoffs for $\beta(z) > 0$ and $\beta(v_i) = 0$ is

$$\begin{aligned} & p(\beta(z))v_i G(z) - \beta(z) - \varepsilon v_i G(\tilde{v}) \\ & \quad = \left[p(\beta(z))G(z) - \varepsilon G(\tilde{v})\right] v_i - \beta(z) \\ & \quad \leq \left[p(\beta(z)) - \varepsilon\right] v_i - \beta(z) \\ & \quad \quad 0 \end{aligned}$$

The last inequality is guaranteed by Assumption 3. In case $v_i \in (\tilde{v}, \omega]$ and $z \in [0, \tilde{v}]$, one should only invert the reasoning used in the previous situation.

Finally, consider the case where $v_i \in (\tilde{v}, \omega]$ and $z \in (\tilde{v}, \omega]$, and assume that i has valuation v_i and use $\beta(z)$. His expected payoff is

$$W_i(z, v_i) = v_i p(\beta(z))G(z) - \beta(z)$$

To differentiate the above function with respect to z one should have that

$$\begin{aligned} \frac{\partial W_i(z, v_i)}{\partial z} &= v_i \frac{d\left[p(\beta(z))G(z)\right]}{dz} - \beta'(z) \\ &= v_i \frac{d\left[p(\beta(z))G(z)\right]}{dz} - z\frac{d\left[p(\beta(z))G(z)\right]}{dz} \\ &= (v_i - z)\,\frac{d\left[p(\beta(z))G(z)\right]}{dz} \end{aligned}$$

therefore

$$\frac{\partial W_i(z, v_i)}{\partial z} \begin{cases} \geq 0 \text{ if } v_i \geq z \\ \leq 0 \text{ if } v_i \leq z \end{cases}$$

Thus $W_i(z, v_i)$ is maximized when $z = v_i$. Finally, the supposition that $1 - p'(b)G(v)v > 0$ guarantees that $\beta'(v) \geq 0$ for any $v \in [\tilde{v}, \omega]$. □

Integrating by parts the right-hand side of (7) it should be the case that

$$\left[v_i p(\beta(v_i))G(v_i) - \beta(v_i)\right] = \tilde{v}\varepsilon G(\tilde{v}) + \int_{\tilde{v}}^{v_i} p(\beta(s))G(s)ds \tag{9}$$

The left-hand side of (9) represents the i's expected gain when he or she offers a bid $\beta(v_i)$. The two terms of the right-hand side of the same equation are, respectively, the highest expected gain among all non-effective participants and the incremental utility gain when the probability of winning increases and becomes greater than ε by offering a positive bid. It is clear that $v_i p(\beta(v_i))G(v_i) - \beta(v_i) > \tilde{v}\varepsilon G(\tilde{v}) > 0$ as long as $p(\beta(s))G(s) > 0$ for any $s \in (\tilde{v}, v_i]$ with $v_i > \tilde{v}$.

If the auction is standard so that $p(\beta(v_i)) = 1$ for any i, then it is the case that[15]

$$\beta(v_i) = \int_0^{v_i} sg(s)ds \tag{10}$$

[15] If the auction is standard, then the supposition about the existence of a valuation $\tilde{v}$ as defined is innocuous since participants will behave in the same way for any valuation smaller or greater than $\tilde{v}$.

where $g=G'$. This is exactly the symmetric equilibrium bid of a standard all-pay auction with private and independent valuations.[16]

Notice that integrating (10) by parts it should be the case that

$$v_iG(v_i)-\beta(v_i)=\int_0^{v_i}G(s)ds \tag{11}$$

So that the left-hand side of (11) represents the expected payoff of a participant in the standard all-pay auction.

4.1. Equilibrium properties

What is the effect of the stochastic nature of the Chinese auction on the participants' bids and expected gains? The following propositions partially answer this question.

Proposition 8. *Let $B(\cdot)$ and $\beta(\cdot)$ be the equilibrium bids in an all-pay auction and in a Chinese auction, respectively, and when assumed that $B(0)=0$, one should have that*

(a) $B(v)\geq\beta(v)$ *for any* $v\in[0,\tilde{v}]$.
(b) *For all* $v\in(\tilde{v},\omega]$, $\beta(v)\leq B(v)$ *if the variational inequality* $\gamma'(b)\geq\xi(b)$ *is holding for any* $b\in(0,\overline{b}]$, *where* $\overline{b}=\beta(\omega)$, $\gamma(b)=\ln\left[1-p(b)\right]^{-1}$ *and* $\xi(b)=[H(b)\phi(b)]$ *with* $\phi=\beta^{-1}$ *and* $H=G\circ\phi$.

Proof. The part (a) is straightforward (in fact $B(\tilde{v})>0=\beta(\tilde{v})$). To prove (b), notice that from the first-order conditions for β and B, one should have that

$$\beta'=\frac{p(\beta)g(v)v}{1-p'(\beta)G(v)v} \tag{12}$$

and

$$B'=vg(v) \tag{13}$$

Together, Eqs. (12) and (13) imply that

$$\beta'=\frac{p(b)}{1-p'(b)H(b)\phi(b)}B'$$

where $H(b)$ and $\phi(b)$ are defined according to enunciate. Therefore, if

$$\frac{p(b)}{1-p'(b)H(b)\phi(b)}\leq 1\,\text{forany}\,b\in(0,\overline{b}] \tag{14}$$

then for values greater than $\tilde{v}$, the function B has more vertical tangents in all points. Notice that both functions are increasing for values greater or equal to $\tilde{v}$ and, using the first part of the proposition, the desired result is reached. Finally, after some algebraic manipulations, the condition on (14) is equivalent to the variational inequality on item (b) of Proposition 8. □

Fig. 1 illustrates the conclusions of Theorem 7.

When $\gamma'(b)<\xi(b)$ for some b, it is not straightforward to determine what the highest bid is. More specifically, there could be a pool of high values such that $B(v)\leq\beta(v)$. This possibility is illustrated by Fig. 2.

Proposition 9. *If $W^{CA}(\cdot)$ and $W^{AP}(\cdot)$ represent the expected gains in the Chinese auction and in the all-pay auction, respectively, then there is a realization $\alpha\in[0,\tilde{v}]$ such that:*

1. *If* $G(\alpha)/G(\tilde{v})<\varepsilon<1$ *then* $W^{AP}(v)\leq W^{CA}(\tilde{v})$.
2. *If* $0<\varepsilon<G(\alpha)/G(\tilde{v})\leq 1$ *then* $W^{AP}(v)\geq W^{CA}(v)$ *for any* $v\in(\tilde{v},\omega]$.

[16] See Krishna (2005).

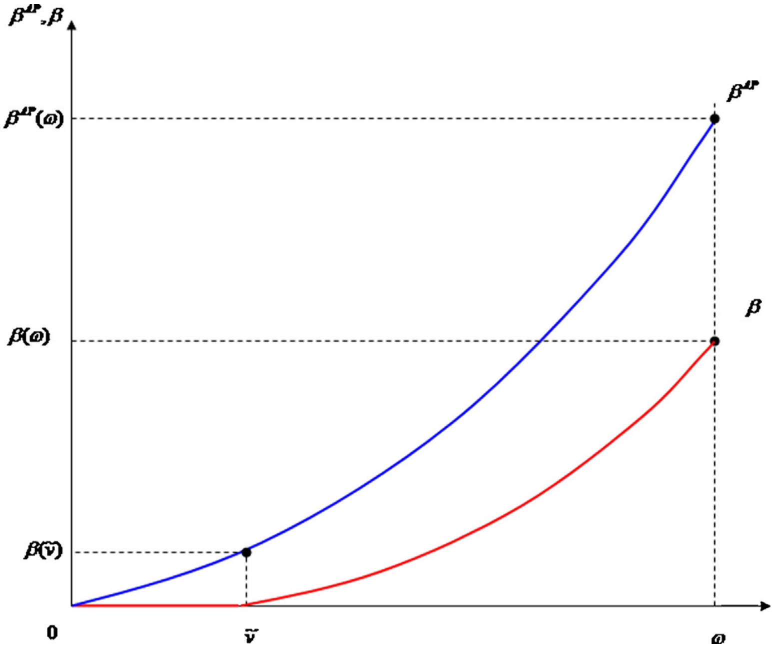

Fig. 1. Equilibrium bids in all-pay and Chinese auctions with the conditions of Proposition 8.

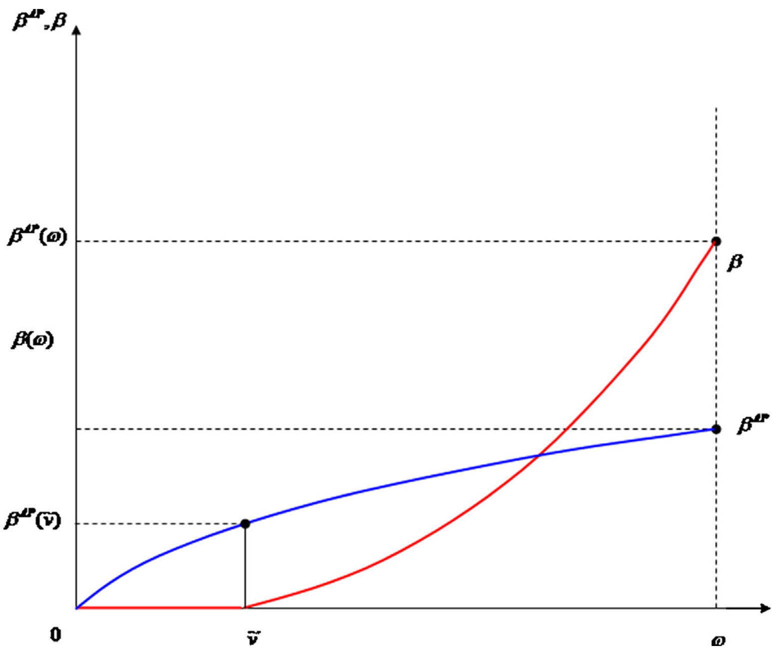

Fig. 2. Equilibrium bids in all-pay and Chinese auctions without the conditions of Proposition 8.

Proof. By definition

$$W^{AP}(v) = \int_0^v G(s)ds \,\text{for any}\, v \in [0, \omega]$$

and

$$W^{CA}(v) = \begin{cases} \tilde{v}\varepsilon G(\tilde{v}) \text{ if } v = \tilde{v} \\ \tilde{v}\varepsilon G(\tilde{v}) + \int_{\tilde{v}}^{v} s \frac{d\left[p(\beta(s))G(s)\right]}{ds} ds \text{ if } v \in (\tilde{v}, \omega] \end{cases}$$

Considering the first case where $v \in [0, \tilde{v}]$, one should have that

$$\begin{aligned} W^{AP}(v) - W^{CA}(v) &= \int_0^v G(s)ds - \tilde{v}\varepsilon G(\tilde{v}) \\ &= \int_0^v G(s)ds + \int_{\tilde{v}}^0 G(s)ds - \int_{\tilde{v}}^0 G(s)ds - \tilde{v}\varepsilon G(\tilde{v}) \\ &= \int_{\tilde{v}}^v G(s)ds + \int_0^{\tilde{v}} G(s)ds - \tilde{v}\varepsilon G(\tilde{v}) \\ &= \int_0^{\tilde{v}} G(s)ds - \tilde{v}\varepsilon G(\tilde{v}) - \int_v^{\tilde{v}} G(s)ds \end{aligned}$$

The Integral Mean Value Theorem (IMVT) guarantees that there exists $\alpha \in [0, \tilde{v}]$ such that

$$\begin{aligned} W^{AP}(v) - W^{CA}(v) &= \tilde{v}G(\alpha) - \tilde{v}\varepsilon G(\tilde{v}) - \int_v^{\tilde{v}} G(s)ds \\ &= \tilde{v}\left[G(\alpha) - \varepsilon G(\tilde{v})\right] - \int_v^{\tilde{v}} G(s)ds \end{aligned}$$

If $G(\alpha)/G(\tilde{v}) < \varepsilon < 1$ then $W^{AP}(v) \leq W^{CA}(v)$. Consider now the case where $v \in (\tilde{v}, \omega]$. Hence,

$$\begin{aligned} W^{CA}(v) - W^{AP}(v) &= \tilde{v}\varepsilon G(\tilde{v}) + \int_{\tilde{v}}^v p(\beta(s))G(s)ds - \int_{\tilde{v}}^v G(s)ds - \int_0^v G(s)ds + \int_{\tilde{v}}^v G(s)ds \\ &= \tilde{v}\varepsilon G(\tilde{v}) - \int_{\tilde{v}}^v \left[1 - p(\beta(s))\right] G(s)ds - \int_0^v G(s)ds - \int_v^{\tilde{v}} G(s)ds \\ &= \tilde{v}\varepsilon G(\tilde{v}) - \int_0^{\tilde{v}} G(s)ds - \int_{\tilde{v}}^v \left[1 - p(\beta(s))\right] G(s)ds \end{aligned}$$

Applying once again the IMVT one should have that

$$\begin{aligned} W^{CA}(v) - W^{AP}(v) &= \tilde{v}\varepsilon G(\tilde{v}) - \tilde{v}G(\alpha) - \int_{\tilde{v}}^v \left[1 - p(\beta(s))\right] G(s)ds \\ &= -\tilde{v}\left[G(\alpha) - \varepsilon G(\tilde{v})\right] - \int_{\tilde{v}}^v \left[1 - p(\beta(s))\right] G(s)ds \end{aligned}$$

Therefore, if $0 < \varepsilon < G(\alpha)/G(\tilde{v}) \leq 1$ then $W^{CA}(v) \leq W^{AP}(v)$. □

The first part of the proposition above establishes that if the exogenous winning probability is reasonably high, then, on average, participants of a Chinese auction will obtain higher payoffs than the participants of an all-pay auction. This result is strongly based on Assumption 3. In other words, this result states that whenever there are good chances to get the good auctioned for free in the Chinese auction, agents will be better off comparing to the situation where they pay some positive amount to receive the good with probability one.

In the second part the reasoning is inverted. If there are only effective competitors in both types of auctions, a low exogenous winning probability increases the winning probability that depends on the bid boosting competition in the Chinese auction, inducing participants to raise their bids which reduce the expected payoff. Finally, since there is no effect on competition in the all-pay auction the participants will be, on average, better off than in the Chinese auction.

In order to illustrate the conclusions from Proposition 9, consider the following example. Suppose that F is uniform in $[0, \omega]$. Then, it should be the case that

$$\int_0^{\tilde{v}} \left(\frac{s}{\omega}\right)^{N-1} ds = \frac{\tilde{v}^N}{N\omega^{N-1}}$$

Hence, it is possible to notice that $\tilde{v}G(\alpha) = \tilde{v}^N / N\omega^{N-1}$ implying $G(\alpha) = N^{-1}(\tilde{v}/\omega)^{N-1} = N^{-1}G(\tilde{v})$ such that $G(\alpha)/G(\tilde{v}) = 1/N$. Hence, according with Proposition 9, if $\varepsilon > 1/N$ then the expected payoffs will be greater in the Chinese auction for values in $[0, \tilde{v}]$ and if $\varepsilon < 1/N$ then the expected payoffs will be greater in the all-pay auction for values in $(\tilde{v}, \omega]$.

5. Model with linear probability

Suppose that $p(\cdot)$ is defined as

$$p(b) = (1 - \varepsilon)\theta b + \varepsilon \tag{15}$$

The linear probability may be seen as a general case of success function in its ratio form. In fact, with perfect information about the bids it is possible to define, for every i

$$p(b_i) = \frac{b_i}{\sum_{j \neq i} b_j + b_i} \tag{16}$$

so that if $\theta = 1/\left(\sum_{j \neq i} b_j + b_i\right)$ and $\varepsilon = 0$ (15) and (16) are equal.

Eq. (15) indicates that $p(0) = \varepsilon$ and $p(b) = \theta b$ if $\varepsilon = 0$. Besides, the slope coefficient of $p(b)$ reflects how much the winning probability is affected by the bid so that, for a given θ, the smaller the luck factor the greater will be the bid's relevance. On the other hand, for a given ε, greater relevance of the bid is obtained for higher values of θ. Hence, the lower the values of θ greater influence other random factors will have on the winning probability, besides the participant's bid.[17]

The following result is actually an explicit solution of the equilibrium bid when the winning probability function is linear.

Proposition 10. *In the Chinese auction, if the lottery specifies a winning probability according to (12), then the equilibrium bid will be given by*

$$\beta(v_i) = \frac{\varepsilon}{(1 - \varepsilon)\theta} \left[\exp\left((1 - \varepsilon)\theta \int_{\tilde{v}}^{v_i} h(s)ds\right) - 1\right] \mathbf{1}_{(\tilde{v},\omega]}, \; v_i \in [0, \omega] \tag{17}$$

where

$$h(s) = \frac{g(s)s}{[1 - (1 - \varepsilon)\theta G(s)s]} \tag{18}$$

Proof. For all i, if $v_i \in [0, \tilde{v}]$ it was already shown in Theorem 7 that the best response is $\beta(v_i) = 0$. Then, consider that $v_i \in (\tilde{v}, \omega]$ for i fixed. In this case, first-order conditions of i's expected payoff maximization problem are given by

$$(1 - \varepsilon)\theta\beta'(v_i)G(v_i)v_i + [(1 - \varepsilon)\theta\beta(v_i) + \varepsilon]\, g(v_i)v_i = \beta'(v_i) \tag{19}$$

Rearranging (19) leads to the following differential equation:

$$\beta'(v_i) - (1 - \varepsilon)\theta h(v_i)\beta(v_i) = \varepsilon h(v_i) \tag{20}$$

[17] In most works about contests, θ is determined endogenously by construction of the success function. An exception is the work of O'Keeffe et al. (1984) where the authors propose a rationale for the determination of θ. This rationale implies in raising the influence of random factors when one seeks to minimize an inefficiently high level of effort.

Define

$$\mu(v_i) = \exp\left(-\int_{\tilde{v}}^{v_i} (1-\varepsilon)\theta h(s)ds\right) \tag{21}$$

that is, $\mu(v_i)$ is the integrating factor of the differential equation in (20). Observe that

$$\mu'(v_i)\theta(1-\varepsilon) = -h(v_i)\mu(v_i) \tag{22}$$

Hence, multiplying both sides of (20) by (21) and using (22) it should be the case that

$$\frac{d\,[\beta(v_i)\mu(v_i)]}{dv_i} = -\frac{\varepsilon}{(1-\varepsilon)\theta}\mu'(v_i) \tag{23}$$

Integrating (23) from $\tilde{v}$ to v_i one should have that

$$\beta(v_i) = \frac{\varepsilon}{(1-\varepsilon)\theta}\left[\exp\left(\int_{\tilde{v}}^{v_i} (1-\varepsilon)\theta h(s)ds\right) - 1\right] \tag{24}$$

□

Example 11. In this example, the equilibrium bid is obtained assuming that v is uniformly distributed in $[0, \omega]$. Then, for $v \in (\tilde{v}, \omega]$

$$\beta(v_i) = \frac{\varepsilon}{(1-\varepsilon)\theta}\left\{\exp\left[(N-1)\int_{\tilde{v}}^{v_i} \frac{s^{N-1}}{a^N - s^N}ds\right] - 1\right\} \tag{25}$$

where $a = \omega/[\omega(1-\varepsilon)\theta]^{1/N}$. The solution in (25) is explicitly given as

$$\beta(v_i) = \frac{\varepsilon}{(1-\varepsilon)\theta}\left\{\left(\frac{\omega^{N-1} - (1-\varepsilon)\theta\tilde{v}^N}{\omega^{N-1} - (1-\varepsilon)\theta v_i^N}\right)^{(N-1)/N} - 1\right\} \tag{26}$$

A more convenient way to represent the solution in (26) can be obtained by substituting the effective competitors threshold for the quantile function of the distribution.[18] The quantile function of the uniform distribution is given by $Q(t) = t\omega$ with $0 < t < 1$. In this case, (26) becomes

$$\beta(v_i) = \frac{\varepsilon}{(1-\varepsilon)\theta}\left\{\omega^{1/N}\left(\frac{1 - (1-\varepsilon)\theta t^N\omega}{\omega^{N-1} - (1-\varepsilon)\theta v_i^N}\right)^{(N-1)/N} - 1\right\} \tag{27}$$

The winning probability function may be expressed in terms of competitor i's valuation as follows:

$$q(v_i) = p(\beta(v_i)) = \varepsilon\omega^{1/N}\left(\frac{1 - (1-\varepsilon)\theta t^N\omega}{\omega^{N-1} - (1-\varepsilon)\theta v_i^N}\right)^{(N-1)/N}$$

Notice that the solution in (27) satisfies the conditions of Theorem 7 if the parameters of the model satisfy the restriction that $(1-\varepsilon)\theta\omega < 1$, which is trivially satisfied if ω is normalized to one. Suppose in this case there are 5 competitors, $\theta = 0.75$, $\varepsilon = 0.01$ and the effective competitors threshold is the distribution's median ($t = 0.5$). For this numerical example in particular Fig. 3 depicts both the graphs of the equilibrium bid and the winning probability as functions of the competitors' valuations.

In Fig. 4, the threshold is now the first quartile of the distribution, the value of θ is reduced to 0.5 and the luck factor is raised to $\varepsilon = 0.3$.

5.1. Parameters' effects on competition in the Chinese auction

In this section some comparative statics results are presented aiming to evaluate how the parameters of the model affect competition in the Chinese auction. Propositions 12 and 13, below, show how the value of $\tilde{v}$ affects the probability of occurrence of effective competitors as well as the equilibrium bid.

[18] The quantile functions of F is defined as $Q(t) = \inf\{z\,;F(z) \geq t\}$. See Severini (2005) for details.

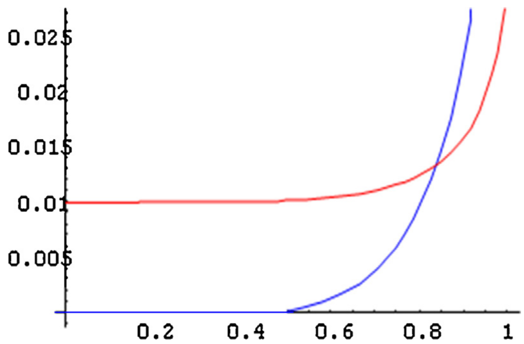

Fig. 3. Equilibrium bid (blue line) and winning probability function (red line) for $N=5$, $\theta=0.75$, $\varepsilon=0.01$, $t=0.5$ and $\omega=1$.

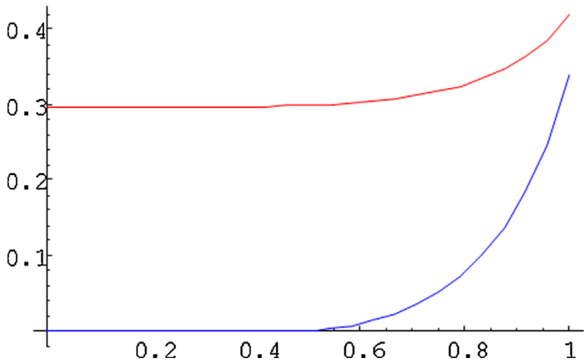

Fig. 4. Equilibrium bid (blue line) and winning probability function (red line) for $N=5$, $\theta=0.9$, $\varepsilon=0.001$, $t=0.25$ and $\omega=1$.

Proposition 12. *For any $\tilde{v} \in (0, \omega)$, define the function $g_K : (0, \omega) \to [0, 1]$ as*

$$g_K(\tilde{v}) = Pr[\tilde{K} = K \big| \tilde{v}]$$

Additionally denote by $\lfloor (N-1)/2 \rfloor$ and $\lceil (N-1)/2 \rceil$ the smaller and larger integers closest to $(N-1)/2$, respectively. Then, one should have that

1. *For $K = 0, 1, ..., \lfloor (N-1)/2 \rfloor$ if $F(\tilde{v}) < \frac{1}{2}$ then $g'_K(\tilde{v}) > 0$;*
2. *For $K = \lceil (N-1)/2 \rceil, \lceil (N-1)/2 \rceil + 1, ..., N-1$ if $F(\tilde{v}) > \frac{1}{2}$ then $g'_K(\tilde{v}) < 0$*

Proof. It is straightforward to show that

$$g'_K(v) \propto \frac{[1 - F(v)]}{F(v)} - \frac{K}{[(N-1) - K]} \text{ for any } K = 0, 1, \ldots, N-1 \tag{28}$$

By definition $\lfloor (N-1)/2 \rfloor \leq (N-1)/2$, then $K = 0, 1, \ldots, \lfloor (N-1)/2 \rfloor$ if and only if $K \leq (N-1)/2$ implying that $K/[(N-1)-K] \leq 1$. However if $F(\tilde{v}) < \frac{1}{2}$ then $[1 - F(v)]/F(v) > 1$ which concludes the proof of 1. The proof of 2 follows the same reasoning. □

This result establishes that if $\tilde{v}$ is smaller than the median of the distribution F, then the probability of occurrence of effective competitors until approximately half of the participants increases with $\tilde{v}$. On the other hand, if the probability of occurrence of values smaller than $\tilde{v}$ is high (and, therefore, the probabilities above $\tilde{v}$ is low) then the probability of occurrence of effective competitors until approximately half of the participants decreases with $\tilde{v}$.

Suppose, for example, that the distribution of values is uniform in [0, 1] and that $N=4$, so that $\lfloor (N-1)/2 \rfloor = 1$ and $\lceil (N-1)/2 \rceil = 2$. Notice that

$$g'_K(\tilde{v}) = \binom{3}{K} [1 - \tilde{v}]^K \tilde{v}^{3-K} (3-K) [1 - \tilde{v}]^{-1} \left\{ \frac{1 - \tilde{v}}{\tilde{v}} - \frac{K}{(3-K)} \right\}$$

Assume initially that $\tilde{v} < 1/2$. Hence,

$$g_0'(\tilde{v}) = \tilde{v}^3 3[1-\tilde{v}]^{-1}\left\{\frac{1-\tilde{v}}{\tilde{v}}\right\} > 0$$

$$g_1'(\tilde{v}) = 3\,[1-\tilde{v}]\,\tilde{v}^2 2[1-\tilde{v}]^{-1}\left\{\frac{1-\tilde{v}}{\tilde{v}} - \frac{1}{2}\right\} > 0$$

The signal of $g_2'(\tilde{v})$ is ambiguous since $(1-\tilde{v})/\tilde{v}$ may be greater or smaller than 1/2. Assume now that $\tilde{v} > 1/2$. In this case, the sign of $g_1'(\tilde{v})$ is also ambiguous. On the other hand,

$$g_2'(\tilde{v}) = 3[1-\tilde{v}]^2\tilde{v}[1-\tilde{v}]^{-1}\left\{\frac{1-\tilde{v}}{\tilde{v}} - 2\right\} < 0$$

$$g_3'(\tilde{v}) = -\infty < 0$$

The following result indicates how chances in the threshold of effective competitors affect the equilibrium bids.

Proposition 13. *For any effective competitor, the equilibrium bid decreases with $\tilde{v}$.*

Proof. For simplicity, define $b_i = \beta(v_i)$. In this case, after some algebraic manipulations, one should have that

$$\frac{\partial b_i}{\partial \tilde{v}} = -\varepsilon h(\tilde{v})\exp\left((1-\varepsilon)\theta\int_{\tilde{v}}^{v_i} h(s)ds\right) < 0$$

□

The proposition above indicates that an increase in $\tilde{v}$ makes effective competitors less aggressive. There are two explanations that can shed a light on this result. First, according to the conditions of Proposition 12, if $\tilde{v}$ is greater than the distribution's median then an increase in $\tilde{v}$ reduces the belief that there is a large number of effective competitors. Thus, each effective competitor starts to believe that there are fewer effective competitors, increasing his chances of winning even with his bid remaining unchanged. And, second, an increase in $\tilde{v}$ boosts the pool of values to which the optimal strategy is to bid zero or, in other words, the occurrence of competitors relying on luck becomes more likely.

Another interesting exercise is to evaluate the effect of changes in the luck factor on the equilibrium bid of the effective competitors. Actually, changes in ε modify the format of the winning probability function. More specifically, in the model with linear probability, an increase in ε raises the intercept but lowers the slope of the success function. There are two possible settings in this case. First, it may be the case that the increase in the intercept is proportionally smaller than the decrease in the slope of the winning probability function so that this function after an increase in ε crosses at some point the function before ε changes. The second possibility is the case where the winning probability increases uniformly with the luck factor. These two possibilities are depicted below.

In Fig. 5, the two possible changes in the winning probability function due to an increase in the luck factor from ε to ε' are shown. In (a) the case where the new winning probability function intercepts the previous one at some bid associated to a value v_ε is depicted. And, in (b), on the other hand, it is assumed that the winning probability increases uniformly with ε.

In order to make the analysis more intuitive, consider the case where for any changes in the luck factor, effective competitors will simply adjust their bids so that their winning probabilities remain unchanged. Note that, in the first case, effective competitors' behavior is asymmetric in respect to an increase in the luck factor; competitors with valuations below a certain v_ε will reduce their bids and competitors with valuations above v_ε will increase their bids. Before analyzing these comments, it is important to assess if this intersection is actually possible given the model's conditions. In other words, is there v_ε such that $(1-\theta\beta(v_\varepsilon))d\varepsilon = 0$ for $d\varepsilon \neq 0$? If this is the case, then $\theta\beta(v_\varepsilon) = 1$ which would imply that $p(\beta(v_\varepsilon)) = 1$ contradicting the supposition that $p(b) < 1$ for any b. Hence, the only plausible case is the one represented in Fig. 5(b), i.e., the winning probability increases uniformly with the luck factor.

Notice that in the case depicted in Fig. 5(b) all effective competitors respond in the same way to an increase in the luck factor; in order to keep their winning probabilities unchanged, they all will reduce their bids. However, these reductions will be proportionally uneven. As suggested by the figure in consideration, effective competitors with low valuations

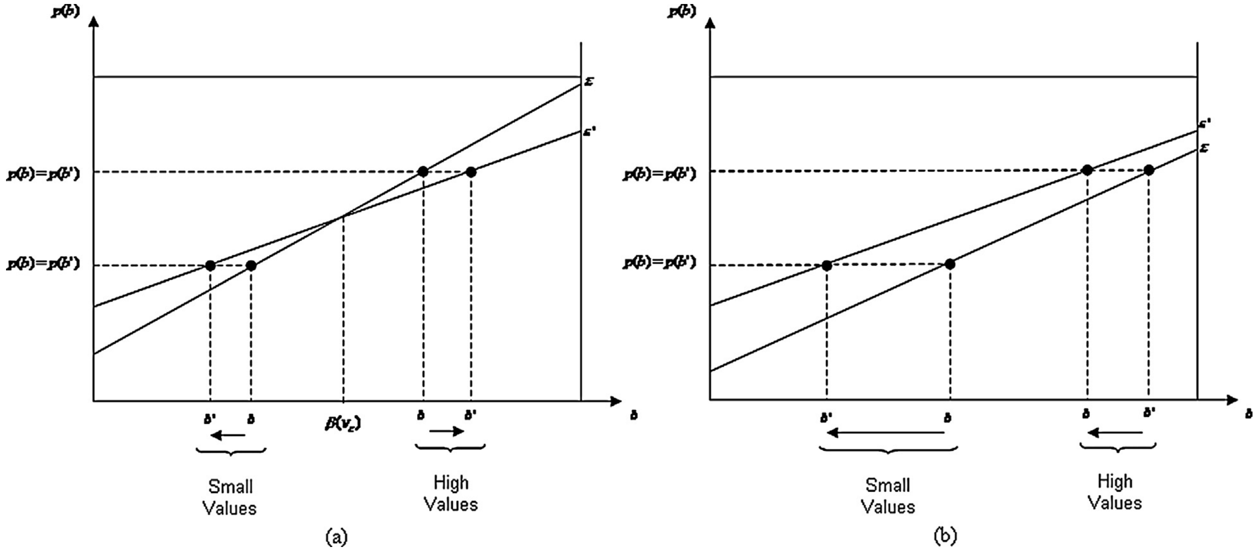

Fig. 5. Exogenous change in winning probability.

(close to $\tilde{v}$) will decrease their bids proportionally more than those with high valuations (close to ω). This could be seen analytically by noticing that for any $b > 0$ and a fixed $p(b)$ it should be the case that $db/d\varepsilon = -(1-\theta b)/(1-\varepsilon)\theta < 0$ and $|db/d\varepsilon|$ is decreasing in b and, consequently, in v.

The intuition behind this result becomes clearer by considering a reduction in the luck factor instead of an increase. This reduction makes all effective competitors more aggressive in terms of keeping their winning probabilities unchanged. However, this aggressiveness is uneven. Those competitors with low valuations would increase their bids more proportionally than those with high valuations. This result suggests that low valuations are proportionally more affected by a reduction in the luck factor because their winning probabilities are very close to the luck factor and, therefore, a reduction in this factor requires a proportionally higher effort by these participants in order to keep their winning probabilities unaltered.

Effective competitors with high valuations, on the other hand, will have winning probabilities well above the luck factor and, therefore, the necessary adjustments to maintain their winning probabilities unchanged need not be so great (in fact, on the limit, for sufficiently small reductions these adjustments will be close to zero).

As an illustration, consider two public contests where everything is the same except for the luck factor. In contest 1 the luck factor is ε_1 and in contest e the luck factor is ε_2 such that $\varepsilon_1 > \varepsilon_2$. One possible interpretation is that the first contest's exam is easier than second contest's exam.[19]

The linear probability model suggests that individuals with low valuations will provide higher levels of effort in exam 2 than those with high valuations, as they aim to keep the winning probability unaltered in both exams.

The fundamental question is that if effective competitors will really adjust their bids in order to keep their winning probabilities unchanged. In order to provide an answer to this question, payoffs in the case that the winning probability do not change should be compared to the gains resulting from an adjustment, from below or from above, on the winning probability.

[19] This may seem unreasonable at first. However, it is possible to provide two different justifications. First, imagine that in each contest there are two types of tests: one that is difficult to associate ε_2, and an easier one, associated to ε_1. The exams are randomly chosen before being applied and participants do not know about this randomization mechanism and, therefore, they have no beliefs about the type of exam. Another justification is that the luck factor may be used to "calibrate" the level of competition when it is affected by an exogenous interference. As an example, a public placement may not specify ex-ante in which location in the country the selected participants will actually work. Compared to another contest that specifies the location, the organizers of the contest may deal with the lack of interest of potential participants by applying an easier exam, i.e., increasing the luck factor.

Defining W as the equilibrium expected payoff of an effective competitor with valuation v and bid $b = \beta(v)$. Then, according to Eq. (9), it should be the case that

$$\frac{dW}{d\varepsilon} = (1-\theta b)vG(v) - [1-(1-\varepsilon)\theta vG(v)]\frac{db}{d\varepsilon} \tag{29}$$

Hence,

$$\frac{dW}{d\varepsilon} \propto \left[\frac{(1-\theta b)vG(v)}{1-(1-\varepsilon)\theta vG(v)} - \frac{db}{d\varepsilon}\right] \tag{30}$$

The first term on the right-hand side of Eq. (30) is positive and the second term, as assumed before (see Theorem 7), is also positive, i.e., $1-(1-\varepsilon)\theta vG(v) > 0$. Thus

$$\frac{dW}{d\varepsilon}\begin{cases} \geq 0 \text{ if } \dfrac{db}{d\varepsilon} \leq \dfrac{(1-\theta b)vG(v)}{1-(1-\varepsilon)\theta vG(v)} \text{ forany } v \in (\tilde{v}, \omega] \\ < 0 \text{ if } \dfrac{db}{d\varepsilon} > \dfrac{(1-\theta b)vG(v)}{1-(1-\varepsilon)\theta vG(v)} \text{ forany } v \in (\tilde{v}, \omega] \end{cases} \tag{31}$$

Note that expression (31) establishes that if the bid is reduced because of an increase in the luck factor, then expected payoff will be bigger. But, this expression also shows that the payoffs increase due to a larger luck factor if the bid increases by no more than $(1-\theta b)vG(v)/[1-(1-\varepsilon)\theta vG(v)]$ for any $v \in (\tilde{v}, \omega]$.

The question of interest here is that if it is advantageous for an effective competitor to maintain his winning probability unaltered or modify it in response to a change in the luck factor. In order to investigate this problem, consider the following approach. Define

$$\frac{db}{d\varepsilon}|_{W(\varepsilon)=const.} = \frac{(1-\theta b)vG(v)}{1-(1-\varepsilon)\theta vG(v)}$$

Then, Eq. (30) may be written as

$$\frac{dW}{d\varepsilon} \propto \left[\left.\frac{db}{d\varepsilon}\right|_{W(\varepsilon)=const.} - \frac{db}{d\varepsilon}\right] \tag{32}$$

If the bid is adjusted so that the winning probability is unaltered, then $db/d\varepsilon = -(1-\theta b)/(1-\varepsilon)\theta$ and Eq. (32) results in

$$\frac{dW}{d\varepsilon} \propto \frac{(1-\theta b)}{(1-\varepsilon)\theta} > 0 \tag{33}$$

Hence, if the bid is adjusted so that the winning probability is unaltered, the effective competitors' payoffs increase with the luck factor. This result was also reached by Amegashie (2006). The difference is that, in his study, the impact of the luck factor on bids is symmetrical and welfare gains due to an increase in the luck factor occur directly (and proportionally), independent of the effect over the bids.[20]

6. Conclusions and extensions

This paper proposed to model the Chinese auction where risk-neutral participants compete, through an all-pay auction, for a lottery where each participant may or may not win an indivisible object.

A symmetric equilibrium was obtained and some of its properties were studied. More specifically, it was shown that when uncertainty about the result of the auction is introduced in the analysis, participants become less aggressive on their bids when the winning probability satisfy a variational condition. It was also shown that the luck factor is decisive in respect to the expected payoff when the Chinese and the standard all-pay auctions are compared. As established by Proposition 9, among non-effective competitors, average payoffs will be higher in the Chinese auction if the luck

[20] It is important to point out that in the paper of Amegashie (2006), luck is modeled in a different way. Specifically, the author uses a parameter that measures the sensitivity of the success function with respect to the residual probability that does not depend on effort.

factor is reasonably high. On the other hand, effective competitors will be, on average, worse off in the Chinese auction when the luck factor is small.

In Section 5, the model was specified assuming that the winning probability is linear on bids. Considering this specification, it was shown that bids decrease with the effective competitors' threshold. Finally, the effect of the luck factor over effective competitors' bids was discussed. Although it was not possible to reach a conclusion about the direction of the change of the bids in respect to changes in the luck factor, it was shown that, if effective competitors adjust their bids to keep the winning probability unaltered, a reduction in the luck factor will result in an increase in the bids. This increase, however, occurs in greater proportions for effective competitors with valuations closer to the threshold.

There are several possible extensions to the original model. As mentioned before in the introduction, the public contest mechanism may be thought as a form of the Chinese auction. But, in many cases, a public contest offers several placements (as in the case of university applications). Hence, the model could be extended, in this case, to situations where there are multiple identical goods (with unit demand).

Another possible extension would be not to assume that winning probabilities are anonymous. A way to do this would be to admit that the luck factor depends on each participant. As a consequence, an immediate complication will emerge: equilibrium will not be symmetrical.

Finally, perhaps the most interesting modification that can be proposed would be to allow agents to be risk-averse. This would certainly enrich the model as considerations towards their attitudes of risk in respect to the lottery would change dramatically their behavior in the Chinese auction.

Appendix A. Existence and uniqueness of the equilibrium

Theorem 14. *Under the Assumptions 1 and 2 and if* $1 - p'(b)G(v)v > 0$ *for any* $b \geq 0$ *and* $v \in [\tilde{v}, \omega]$*, then there exists an unique equilibrium* $b(\cdot)$ *in the Chinese auction for* $v \in [0, \omega]$ *such that* $b(0) = 0$ *and* $b(\omega) = \bar{b}$*.*

Proof. The proof of this theorem is basically the proof of uniqueness and existence of the solution for the initial value problem[21]

$$b'(v) = h(v, b), \quad b(\tilde{v}) = 0 \tag{34}$$

where

$$h(v, b) = \frac{p(b)g(v)v}{1 - p'(b)G(v)v} \tag{35}$$

The Theorem of existence and uniqueness for the solution of ordinary differential equations[22] establishes that if a function $h(v, b)$ is uniformly Lipschitzian in the second variable, then there exists a solution to (19) and it is unique. Let $|\cdot|$ be the usual norm in $\mathbb{R}$. It should be shown that for any $b_1, b_2 \in [0, \bar{b}]$ there is a constant $L \geq 0$ such that

$$|h(v, b_1) - h(v, b_2)| \leq L\,|b_1 - b_2| \text{ for } \text{ any } v \in [\tilde{v}, \omega]$$

For that, notice that suppositions 1 and 2 guarantee that there are nonnegative constants[23] M, L_1 and L_2 such that

$$g(v) \leq M, \quad \forall v \in [0, \omega] \tag{36}$$

$$|p(b_1) - p(b_2)| \leq L_1, \forall b_1, b_2 \in [0, \bar{b}] \tag{37}$$

$$\left|p'(b_1) - p'(b_2)\right| \leq L_2, \forall b_1, b_2 \in [0, \bar{b}] \tag{38}$$

After some algebraic manipulations it is possible to conclude that

$$|h(v, b_1) - h(v, b_2)| = g(v)v\frac{\left|\left[p(b_1) - p(b_2)\right] + G(v)v\left[p(b_2)p'(b_1) - p(b_1)p'(b_2)\right]\right|}{\left|\left[1 - p'(b_1)G(v)v\right]\left[1 - p'(b_2)G(v)v\right]\right|} \tag{39}$$

[21] Note that for any $v \in [0, \tilde{v}]$, $b(v) = 0$ is a solution.

[22] See Agarwal and O'Regan (2008).

[23] The assumption that $p(\cdot)$ is twice differentiable guarantee that $p(\cdot)$ and $p'(\cdot)$ are Lipschitzian functions.

Using the triangular inequality in (37) one should have that

$$\begin{aligned} |h(v, b_1) - h(v, b_2)| &\leq g(v)v \frac{|p(b_1) - p(b_2)| + G(v)v \left|p(b_2)p'(b_1) - p(b_1)p'(b_2)\right|}{\left|\left[1 - p'(b_1)G(v)v\right]\left[1 - p'(b_2)G(v)v\right]\right|} \\ &\leq g(v)v\,|p(b_1) - p(b_2)| + g(v)v^2 \left|p(b_2)p'(b_1) - p(b_1)p'(b_2)\right| \end{aligned} \tag{40}$$

The second inequality in (25) results from the supposition that $1 - p'(b)G(v)v > 0$ for any $b \geq 0$ and $v \in [\tilde{v}, \omega]$. Using (21) and (22) and the fact that $v \leq \omega$ it should be the case that

$$|h(v, b_1) - h(v, b_2)| \leq M\omega L_1\,|b_1 - b_2| + M\omega^2 \left|p(b_2)p'(b_1) - p(b_1)p'(b_2)\right| \tag{41}$$

Note that

$$\left|p(b_2)p'(b_1) - p(b_1)p'(b_2)\right| = \left|\left[p(b_2) - p(b_1)\right] p'(b_1) + p(b_1)\left[p'(b_1) - p'(b_2)\right]\right| \tag{42}$$

Using the triangular inequality and (21) and (22) in (27) and then the result in (26) one should have that

$$\begin{aligned} |h(v, b_1) - h(v, b_2)| \quad \leq \quad & M\omega L_1\,|b_1 - b_2| + \\ & M\omega^2 L_1\,|b_2 - b_1|\,p'(b_1) + M\omega^2 p(b_1) L_2\,|b_1 - b_2| \end{aligned} \tag{43}$$

Finally, considering the fact that[24] $p'(b) \leq N$ for some $N \geq 0$ and $p(b) \leq 1$ it should be the case that

$$|h(v, b_1) - h(v, b_2)| \leq C\,|b_1 - b_2|$$

where $C = M\omega\,[L_1\,(1 + \omega N) + \omega L_2]$. □

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[24] Every Lipschitzian function has limited derivative.