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Development of probabilistic fatigue curve for asphalt concrete based on viscoelastic continuum damage mechanics

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Abstract

Due to its roots in fundamental thermodynamic framework, continuum damage approach is popular for modeling asphalt concrete behavior. Currently used continuum damage models use mixture averaged values for model parameters and assume deterministic damage process. On the other hand, significant scatter is found in fatigue data generated even under extremely controlled laboratory testing conditions. Thus, currently used continuum damage models fail to account the scatter observed in fatigue data. This paper illustrates a novel approach for probabilistic fatigue life prediction based on viscoelastic continuum damage approach. Several specimens were tested for their viscoelastic properties and damage properties under uniaxial mode of loading. The data thus generated were analyzed using viscoelastic continuum damage mechanics principles to predict fatigue life. Weibull (2 parameter, 3 parameter) and lognormal distributions were fit to fatigue life predicted using viscoelastic continuum damage approach. It was observed that fatigue damage could be best-described using Weibull distribution when compared to lognormal distribution. Due to its flexibility, 3-parameter Weibull distribution was found to fit better than 2-parameter Weibull distribution. Further, significant differences were found between probabilistic fatigue curves developed in this research and traditional deterministic fatigue curve. The proposed methodology combines advantages of continuum damage mechanics as well as probabilistic approaches. These probabilistic fatigue curves can be conveniently used for reliability based pavement design.

Keywords: Probabilistic fatigue curve; Continuum damage mechanics; Weibull distribution; Lognormal distribution

1. Introduction

Fatigue in asphalt pavement is one of the major distress mechanisms, which is primarily caused by repeated traffic loading. Initially, this damage starts with microcracks at locations of higher stress (or strain) concentration. These microcracks further coalesce into a series of interconnected macrocracks finally leading to failure of pavement. This degradation under cyclic loading has been characterized in laboratory conditions using various modes of loading including flexure [1,2], direct tension [3], indirect tension [4] and shear [5]. One common feature among all these loading modes is amplitude of strain (or stress) held constant throughout the fatigue testing and its response is recorded. The testing is continued until specimen fails completely. This process is repeated at other strain (or stress) levels to obtain relationship between strain (or stress) amplitude and number of cycles to failure. In general there are two different approaches used for fatigue life prediction i.e. phenomenological and mechanistic approaches.

In the phenomenological approach, a series of tests are performed at various conditions to capture all significant factors that contribute to the fatigue damage. Using the

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data generated, regression models are developed to relate fatigue life with other parameters like binder type, aggregate type and gradation, air voids and testing method. If one of the parameters is neglected initially, the entire set of experiments has to be repeated to capture its effect. Thus, careful attention is required while deciding on factors to be accounted, experimental plan and regression model. In this approach, fatigue damage process is considered as deterministic. Even under extremely controlled condition testing, fatigue test results developed show significant scatter/variability [6].

Alternatively, mechanistic approach is more complex which is applicable to a wide range of loading and environmental conditions. Some examples for mechanistic approach are Visco-Elastic Continuum Damage (VECD) approach, fracture mechanics, and micromechanics. VECD approach relates stress-strain using the principles of thermodynamics. In the VECD approach, a damaged body is represented by a homogeneous continuum where the scale of the test specimen is much larger than the defect size. These VECD models use macroscale observations to capture the net effect of microstructural changes. Hence, the VECD model is convenient for modeling overall behavior of the material. Present day VECD models consider material properties and damage process to be deterministic. Improper selection of input values in these VECD models will lead to significant deviation in predicted material behavior. Thus, extreme care has to be exercised while choosing input values for VECD model parameters. Probabilistic fatigue analysis can account for the variability issues like constituent materials, specimen fabrication issues, and testing practices. Therefore, there is a need of probabilistic approach for the fatigue life analysis within VECD framework.

This paper presents a novel methodology for predicting fatigue life of asphalt concrete for a given reliability based on elastic-viscoelastic correspondence principle and VECD mechanics. The paper is divided into 6 sections of which this is the first section. Research work conducted by others in area of asphalt concrete fatigue characterization, VECD mechanics, and probabilistic fatigue modeling is discussed in the second section. Details regarding asphalt concrete mixtures, and specimen preparation and testing is presented in third and fourth sections, respectively. The next two sections deal with the proposed probabilistic fatigue analysis methodology and a case study along with statistical analysis. The last section concludes the article.

2. Previous work

Various researchers have used regression-based approach to relate fatigue life (N_f) and initial strain amplitude (ε_0) as presented in Eq. (1) [7–10]. Due to its simplicity and ease of use, this empirical model is popular among engineering community. The regression coefficients (K and n) in Eq. (1) are specific to the asphalt mixture type, volumetric composition and binder type, and the test parameters used in the laboratory evaluation. A schematic diagram of fatigue curve developed after laboratory evaluation is shown in Fig. 1. Due to mechanical compliance of machine and pneumatic control issues, strain amplitude during fatigue testing is not exactly constant. This small variation in applied strain amplitude is specific to particular equipment in use. Thus, some scatter in strain amplitude (in vertical direction) in fatigue curve is expected. Further, variability in constituent materials, aggregate microstructure, and testing procedure use will contribute to inherent variability in failure point. Hence scatter in fatigue life (in horizontal direction) is always expected.

$$N_f = k_1 \left(\frac{1}{\varepsilon_0} \right)^{k_2} \tag{1}$$

where k_i = regression coefficients.

Several studies have reported that regression coefficients in Eq. (1) are found to be very sensitive to mixture properties. Navarro and Kennedy observed that the value of K generally ranges from 5.0×10^{-20} to 6.5×10^{-5} while value of n ranges between 1.2 to 6.3 [6]. Shukla and Das [11] reported that value of K varied between 5.35×10^{-18} and 497 while value of n ranged between 2.93 and 6.17. Navarro and Kennedy [6] have reported Coefficient of Variation (CV) values of fatigue life ranged between 26 and 84%. For field-extracted specimens, Monismith et al. [12] have reported CV values between 25% and 131%. Similarly for laboratory prepared specimens, CV values ranged between 54% and 76% [12–13]. Thus, it can be concluded that CV value for the fatigue life of field specimens is more when compared to laboratory prepared specimens.

Various researchers have reported scatter in fatigue life distribution. Miura [14], Tsai et al. [15], Sun et al. [16], Klemenc et al. [17] have noted that fatigue life distribution of bituminous mixtures follows Weibull distribution. Pell and Taylor [18] found that fatigue life follows lognormal distribution. Zhao et al. [19] found that fatigue life distribution at a given strain magnitude is skewed to the right.

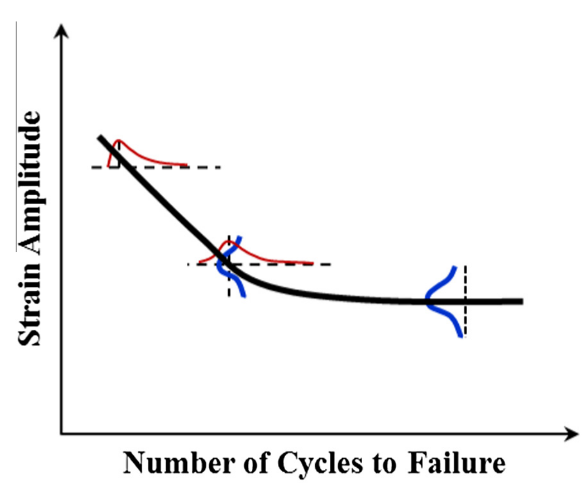


Fig. 1. Fatigue curve showing scatter in strain amplitude and fatigue life.

Zhao et al. [19] and Bucar et al. [20] showed that the Weibull or lognormal distribution could describe the statistical scatter of the fatigue life. Current practice in fatigue studies employ averaged values of strain (or stress) amplitude and fatigue life alone. Hence, this practice does not account for fatigue life variation and skewness of test data completely.

Various probabilistic approaches have been developed to account for scatter observed in fatigue life. In order to minimize the number of specimens required and laboratory testing time, Ling et al. [21] proposed a maximum likelihood method for estimating probabilistic fatigue curve parameters. Klemenc et al. [17] found that Weibull parameters are dependent on the strain amplitude through Coffin–Manson equation. Guida et al. [22] proposed Bayesian approach for fatigue data analysis that accounts for material properties and small sample size. Xiong et al. [23] proposed a single-point likelihood method technique to address the paucity of data in developing a generalized fatigue curve from small sample of test data. Zhao and Liu [24] proposed Weibull modeling of the probabilistic curves for rolling contact fatigue. Luo et al. [25] developed the probabilistic model for rubberized asphalt concrete mixtures using point estimate method. These probabilistic approaches require finding a best-fit distribution of fatigue life through statistical tests, and finally calculating the distribution parameters to develop probabilistic fatigue curves.

Use of elastic-viscoelastic correspondence principle and VECD approach in the area of asphalt concrete modeling was initiated by work of Kim [26]. Lee and Kim developed a uniaxial constitutive model of non-aging asphalt aggregate mixtures based on elastic-viscoelastic correspondence principle and Schapery's work potential theory [27]. Lee et al. [28] developed a fatigue life prediction model using the constitutive model. The expression to predict fatigue life (N_f) of asphalt concrete without rest period based on elastic-viscoelastic correspondence principle and VECD approach is given in Eq. (2). Daniel [3], and Daniel and Kim [29] found that the relationship between damage parameter (S) and pseudostiffness (C) is unique for a given mix and is commonly referred to as damage characteristic curve. Further, power law [$C = 1 - C_{11}(S)^{C_{12}}$] with C_{11} and C_{12} as regression coefficients was used to describe this damage characteristic curve.

$$N_f = \frac{f(S_f)^{p_1}}{p_1(0.125IC_{11}C_{12})^{\alpha_1}}|E|^{-2\alpha_1}\varepsilon_o^{-2\alpha_1} \tag{2}$$

$$p_1 = 1 + (1 - C_{12})\alpha_1 \tag{3}$$

$$\alpha_1 = 1 + \frac{1}{m} \tag{4}$$

where, m = slope of the linear portion of the creep compliance curve in a log-log scale, $|E^*|$ = dynamic modulus, f = frequency, S_f = damage parameter at failure, I = initial secant pseudostiffness.

VECD approach essentially requires material properties under undamaged and damaged condition to develop/calibrate constitutive model. Further, prediction at a different test conditions is made using damage characteristic curve and temperature shift factors. Hence VECD approach can be conveniently used for modeling overall behavior of the asphaltic materials under different temperature, frequency, and rate of loading. Averaged values of dynamic modulus, relaxation modulus and damage characteristic curve are used in fatigue life prediction. Due to its deterministic nature, current VECD approaches fail to capture scatter observed in actual fatigue tests.

3. Materials, mixture design and specimen preparation

Asphalt concrete mixture used in this research was designed as per New Hampshire Department of Transportation (NHDOT) specifications using Superpave mixture design approach. 12.5 mm nominal maximum aggregate size gradation along with polyphosphoric acid modified asphalt (PG 70-28) was used while designing mixture. Aggregate and binder satisfied all engineering properties specified by NHDOT. The percentage of aggregates passing 19.0, 12.5, 9.5, 4.75, 2.36, 1.18, 0.6, 0.3, 0.15, and 0.075 mm sieves are 100, 94.1, 73.2, 49.4, 37.0, 24.6, 16.3, 10.3, 6.7, and 3.6, respectively. The design binder content was 4%, and dust proportion was 0.55. Further, mixture was designed for air voids, void in mineral aggregate, and void filled with asphalt of 4%, 17.4 and 77, respectively. More details about materials and mixture can be found elsewhere [30,31].

Laboratory prepared loose mixture was compacted using Superpave gyratory compactor. After 24 h, these compacted samples were trimmed using core cutter and wet saw to obtain cylindrical specimens. Neatly cut specimens were checked for air voids using Corelok vacuum system as per AASHTO TP 69-04 standard. Specimens having $4 \pm 0.5\%$ airvoids were used for subsequent instrumentation and testing. Steel end plates, and Linearly Variable Differential Transducers (LVDTs) were glued to uniaxial specimens using plastic epoxy glue using jigs. The LVDTs were spaced 90° apart around the circumference of the specimen using a 100 mm gage length. During testing, the applied load on the specimen was measured using the load cell attached to a closed-loop servo-hydraulic system and the deformations were measured using LVDTs mounted on the specimen.

4. Test methods

The testing part included non-damage inducing tests to determine Linear Visco-Elastic (LVE) properties and damage inducing tests to determine fatigue properties of asphalt concrete. 10 specimens satisfying NHDOT specifications were tested of which 5 each were used for determining LVE properties and fatigue properties. Further, data from 3 specimens that were close to mean value were used for subsequent analysis. Here afterward, specimens used for determining LVE properties are referred to as V1, V2, and V3, respectively. Similarly, specimens used for determining fatigue properties are referred to as F1, F2, and F3, respectively.

The temperature sweep, and frequency sweep tests were conducted to determine dynamic modulus and phase angles under stress-controlled mode of loading. Temperatures of −10 °C, 0 °C, 10 °C, 20 °C, 30 °C; frequencies of 0.1 Hz, 0.2 Hz, 0.5 Hz, 1 Hz, 2 Hz, 5 Hz, 10 Hz, and 20 Hz, were used to measure dynamic moduli and phase angles. To limit specimen damage, testing was started at −10 °C and terminated at 30 °C. At a particular temperature, testing was started at a higher frequency while ending at a lower frequency. To limit specimen damage, overall strain in specimen was limited to 70 microstrain during testing.

The strain-controlled cyclic test under uniaxial mode of loading was conducted to capture material behavior at damage inducing strain levels. On-specimen displacement, load recorded by LVDT's and load cell, respectively were used for computing strain and stress experienced by the specimen. Applied crosshead strain amplitude in specimens F1, F2, F3 is 2000 $\mu\varepsilon$, 1500 $\mu\varepsilon$ and 2250 $\mu\varepsilon$, respectively. Fig. 2 shows an instrumented specimen ready for testing in the environmental chamber.

5. Methodology

This section presents the methodology for development of probabilistic fatigue curve based on VECD approach. Overall approach consisted of four major steps namely (i) Determination of linear viscoelastic properties, (ii) Determination of damage properties, (iii) Determination of fatigue failure at different levels of strain, and (iv) Development of probabilistic fatigue curve. These individual steps are explained using the flow chart (refer Fig. 3) in following paragraphs.

Fig. 2. Instrumented specimen ready for testing within environmental chamber.

The first step consisted of obtaining mastercurves for viscoelastic properties. Dynamic modulus, and phase angle mastercurves were constructed using measured stress–strain history through time-temperature superposition principle. The relaxation modulus mastercurve was obtained using storage modulus (E') data through interconversion technique proposed by Park and Kim [32]. Further, the creep compliance mastercurve was obtained from relaxation modulus mastercurve using interconversion technique [32].

In the second step, damage characteristics of mixture were evaluated using VECD approach. With the computed strain history, relaxation modulus values and reference modulus ($ER = 1$), the pseudostrain was computed. The secant pseudostiffness in any cycle i was calculated by dividing stress corresponding to maximum pseudostrain in each cycle by maximum pseudostrain in each cycle. To account for the specimen-to-specimen variation, the secant pseudostiffness history was normalized by using the initial secant pseudostiffness. Finally, the damage parameter was calculated using the pseudostrain history, normalized secant pseudostiffness values, initial secant pseudostiffness and material constant (α). The normalized pseudostiffness was cross-plotted against damage parameter to obtain the damage characteristic curve. Further, power law was fitted to this damage characteristic curve.

In the third step, the number of cycles to failure for a given strain level was predicted. Using viscoelastic properties and damage properties determined in previous steps, fatigue life was predicted for all possible combinations of individual specimen properties. For example, testing 3 specimens each for viscoelastic properties and damage properties resulted in 9 combinations. All these combinations will lead to 9 fatigue life prediction at a particular strain level. This process of fatigue life prediction was repeated at several strain levels.

In the fourth step, fatigue life at a particular strain level for a given probability was calculated. Initially, lognormal and Weibull distributions (2-parameter, 3-parameter) were fitted to fatigue lives using graphical approach. Graphical analysis (through coefficient of determination, R^2) and statistical test (Kolmogorov-Smirnov) were performed to compare the fatigue life distribution fits. Preliminary analysis indicated that Weibull distribution describes fatigue life better when compared to lognormal distribution. Hence further analysis was carried out using Weibull distribution. Further, Weibull distribution parameters were obtained by Maximum Product of Spacing (MPS) method. This was based on previous observation that MPS method can be efficiently used with small sample sizes [33]. Fatigue

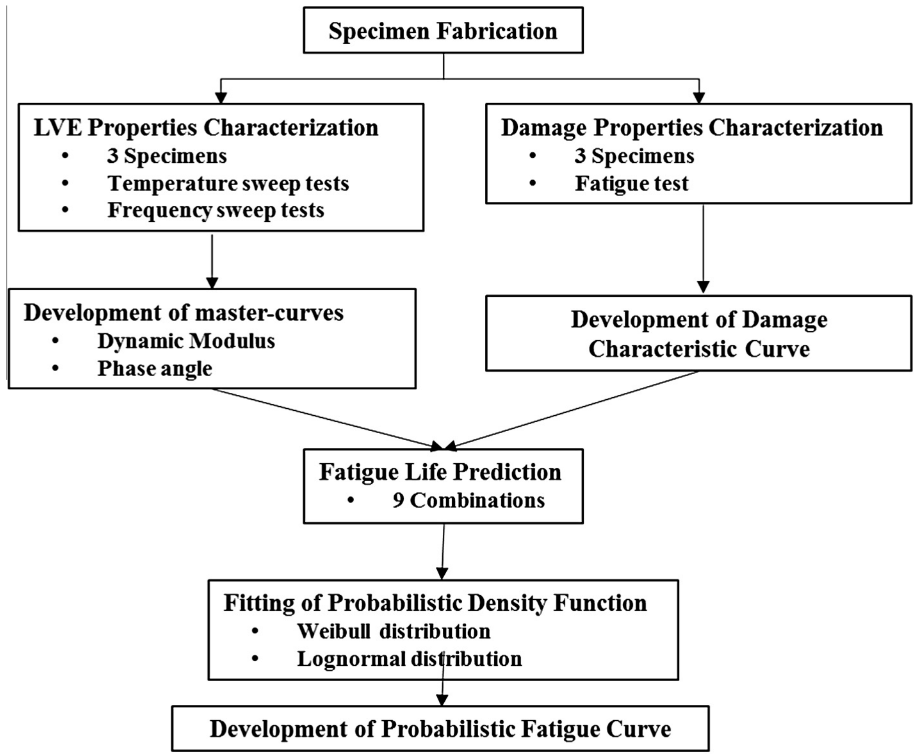


Fig. 3. Flowchart of proposed methodology.

life for a given probability at a given strain (S) for 3-parameter Weibull distribution was then obtained using the Eq. (5). When location parameter is equal to zero then Eq. (5) changes into 2-parameter Weibull distribution. Finally, fatigue curve at a probability level was obtained.

$$P(X/S) = \exp\left\{-\left[\left[\frac{(X-\gamma)}{\eta}\right]^{\beta}\right]\right\} \tag{5}$$

where, γ = location parameter, η = scale parameter, and β = scale parameter.

6. Case study and discussion

This section elaborates the methodology presented in previous section with help of a case study. As mentioned previously, 3 specimens were used for determining LVE properties. The dynamic modulus and phase angle master-curves obtained for all three specimens (V1, V2, and V3) are presented in Fig. 4a and b, respectively. The relaxation modulus and creep compliance mastercurves obtained using appropriate inter-conversion technique are presented in Fig. 4c and d, respectively. Mixture averaged sigmoidal curves for relaxation modulus mastercurve, and creep compliance mastercurve are shown in Fig. 4c and d, respectively. Damage characteristic curves obtained for all three specimens (F1, F2, F3) along with mixture averaged power law fit are presented in Fig. 5. From the non-damage inducing and damage inducing test results it is clear that samples prepared from the same mixture having similar volumetric properties exhibit significant variation in their response. As seen in Figs. 4 and 5, these variations are dependent on reduced frequency, applied strain amplitude, and loading history.

In the next stage, fatigue life was estimated using Eq. (2). It is evident from Eq. (2) that predicted fatigue life depends on numerical values of viscoelastic and damage properties. In this research 3 specimens each were used for non-damage and damage analysis. Thus, each combination would result in one fatigue life estimation. All 9 combinations used in this research and corresponding ID are presented in Table 1. Fatigue curves obtained for all combinations of specimens are presented in Fig. 6. Fatigue curves obtained using the VECD model for all combinations shows significant scatter at all strain levels. Fatigue curve obtained using mixture-averaged values (both viscoelastic properties and damage characteristic curve) is also presented in Fig. 6. As seen in Fig. 6, depending on the availability of specimen properties, fatigue curve will be positioned within certain band with respect to fatigue curve predicted with mixture-averaged values. Use of any combination will either underestimate or overestimate the fatigue life. This necessitates the requirement of probabilistic approach for efficient utilization of fatigue damage information.

Statistical analysis of fatigue life obtained (using all 9 combinations) at different strain amplitude levels were carried out. The results are presented in Table 2. Table 2 indicates that standard deviation increases with decreasing strain amplitude. Physically this implies that scatter in

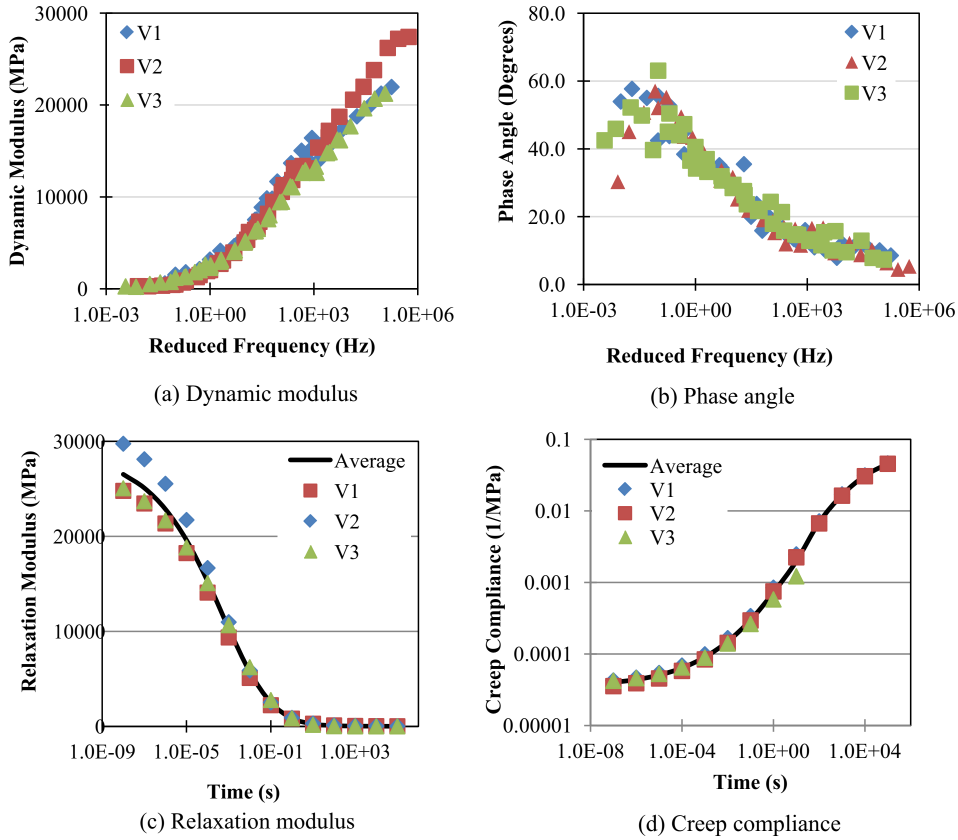


Fig. 4. Mastercurves of viscoelastic properties.

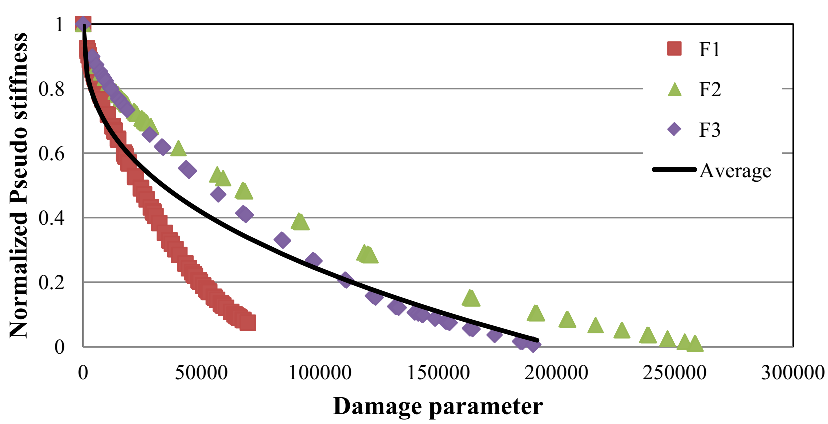


Fig. 5. Damage characteristic curves obtained from damage inducing cyclic tests.

Table 1
Combination of Specimens Used for Fatigue Life Estimation

		Specimen ID used for LVE material characterization		
		V1	V2	V3
Specimen ID used for damage characterization	F1	Combination 1	Combination 2	Combination 3
	F2	Combination 4	Combination 5	Combination 6
	F3	Combination 7	Combination 8	Combination 9

fatigue life is more at lower strain levels. The skewness of the distribution at all strain levels was found to be positive. This implies that the majority of the data values falls to the left of the mean and cluster at the lower end of the distribution; the “tail” is to the right. i.e., the distribution is skewed to the right [34]. The low value of kurtosis indicates

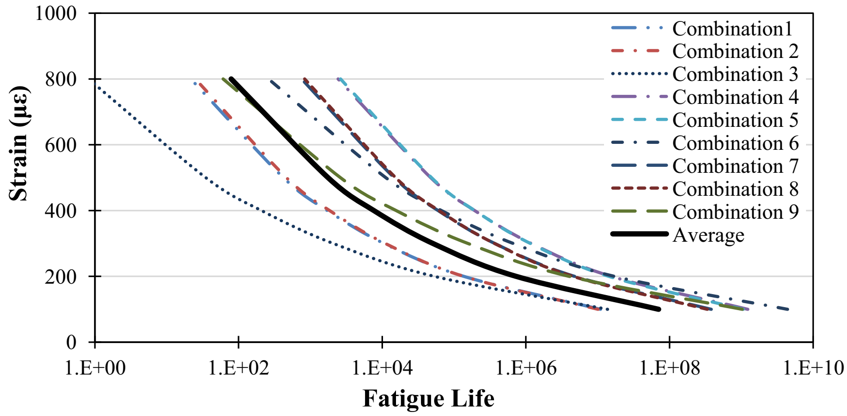


Fig. 6. Fatigue lives obtained for all combinations of material properties.

Table 2
Descriptive statistics of predicted fatigue lives at different strain levels

Strain (με)	Mean fatigue life	Standard deviation	Skewness	Kurtosis
800	780	1042	1.278	0.09
500	15645	19334	1.217	0.01
400	66221	77443	1.117	−0.14
300	437387	472584	0.850	−0.77
200	6735763	6984958	0.624	−1.55

a relatively flat distribution when compared to normal distribution. Absolute value of kurtosis value is more at lower strain levels indicating more flat distribution at lower strain levels.

Further, fatigue lives computed at a particular strain level were checked for Weibull distribution and lognormal distribution. To check the accuracy of Weibull distribution and lognormal distribution fits, a straight line was fit to probability plots. Further, R^2 was computed using a straight-line fit to probability plot. Kolmogorov–Smirnov test was conducted to check goodness of fit. Summary of both tests are presented in Table 3. Table 3 clearly indicates that Weibull distribution describes fatigue life better than lognormal distribution.

Further, fatigue lives computed at a particular strain level were fitted with 2-parameter and 3-parameter Weibull distribution using MPS method. It was observed that with increasing strain levels, numerical value of scale parameter exhibited decreasing trend. This implies scatter is more at lower stain levels. Physically this implies spread in fatigue life is increasing with decreasing strain amplitudes. The shape parameter was found to be less than one indicating that the distribution is skewed to the right. Similar observations regarding Weibull parameters were made by Zhao [24] and Hwang et al. [35]. Location parameter exhibited decreasing trend with increasing strain level. Physically this implies that the distribution is shifting toward origin. In general, Weibull distribution parameters were found to vary with the strain magnitude. Similar conclusions regarding Weibull distribution dependency on applied stress (or strain) levels have been made by Zhao and Liu [24]. Oh [36], and Tsai et al. [15].

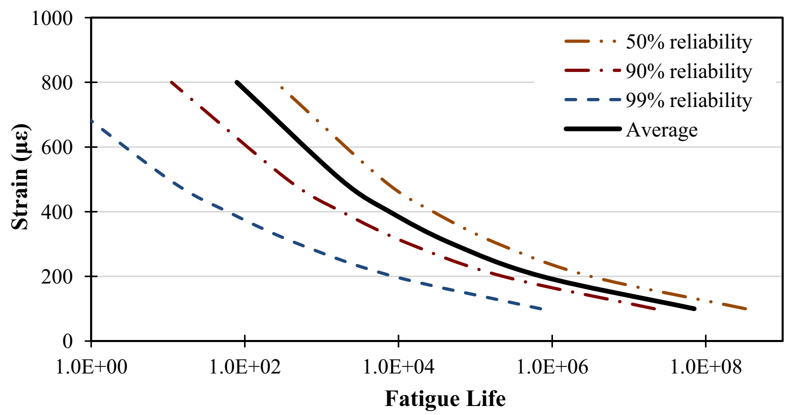
Statistical analysis during this study indicated Weibull distribution described fatigue life well when compared to lognormal distribution. Thus, it was decided to use Weibull distribution for further analysis. More details regarding the statistical analysis can be found elsewhere [37]. The fatigue life at various reliability levels was calculated to obtain probabilistic fatigue curves using survival probability function (Eq. (5)). Finally, Eq. (1) was fitted to calculated fatigue lives using ordinary least squares approach. Probabilistic fatigue curves obtained using 2-parameter

Table 3
Results of statistical analysis of fatigue life distribution

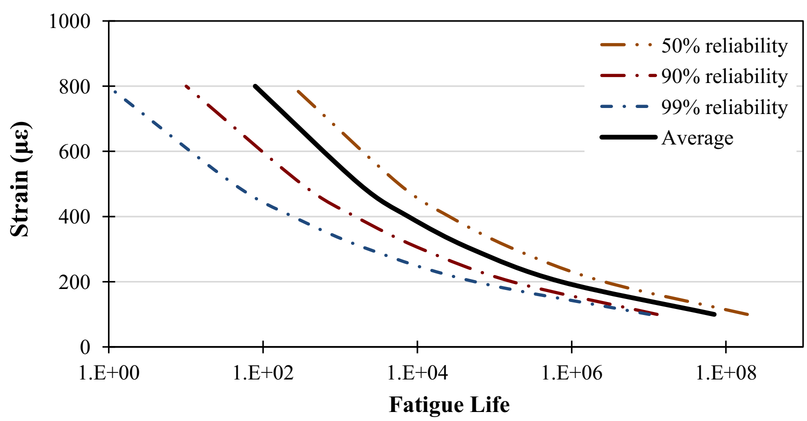
Strain (με)	Kolmogorov–Smirnov test		Coefficient of determination from linear fit to probability plot	
	Lognormal	Weibull	Lognormal	Weibull
200	0.37	0.51	0.83	0.87
300	0.55	0.71	0.87	0.92
400	0.47	0.61	0.88	0.93
500	0.49	0.62	0.90	0.94
800	0.84	0.88	0.92	0.96

and 3-parameter Weibull distribution are presented in Fig. 7a and b, respectively. Fatigue curves obtained using mixture-averaged values are also presented in same figures. It may be noted from Fig. 7 that fatigue curve obtained using mixture-averaged values is conservative when compared to fatigue curve obtained using Weibull distribution at 50% reliability.

Fig. 8 presents comparison of probabilistic fatigue curves developed using 2-parameter and 3-parameter Weibull distribution. The 3-parameter Weibull distribution provides a better fit when compared to 2-parameter Weibull distribution. Fig. 8 indicates until 95% reliability level, fatigue curves obtained using 2-parameter and 3-parameter Weibull distribution curves are almost overlapping.



(a) Using 2-parameter distribution



(b) Using 3-parameter distribution

Fig. 7. Probabilistic fatigue curves obtained using Weibull distribution

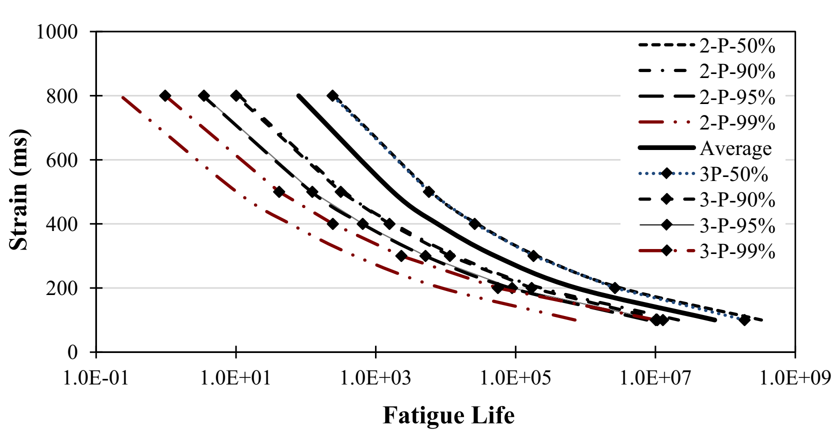


Fig. 8. Comparison of 2-parameter and 3-parameter probabilistic fatigue curves.

However at 99% reliability, fatigue curve obtained using 2-parameter Weibull distribution is more conservative when compared with fatigue curve obtained using 3-parameter Weibull distribution.

7. Summary and conclusions

Current viscoelastic continuum damage models assume fatigue damage as deterministic process. Any predicted material behavior using such an approach is very sensitive to input parameters. Further, any wrong input can lead to erroneous conclusions. Thus, careful attention is required during fatigue testing and analysis. This article presented a novel approach to develop probabilistic fatigue curve for asphalt concrete mixtures using VECD approach. Asphalt concrete specimens conforming to NHDOT specifications were tested for viscoelastic and damage properties. The data gathered were used to develop fatigue curve using VECD and probabilistic approach. Statistical analysis indicated 3-parameter Weibull distribution, 2-parameter Weibull distribution and lognormal distribution can describe predicted fatigue lives well (in decreasing order of ranking). Statistical analysis of computed Weibull distribution parameters indicates fatigue life dependency on applied strain amplitude. Also, fatigue life distribution was found to be skewed. Fatigue life predicted using deterministic values fails to capture underlying statistical distribution in fatigue lives. Thus, probabilistic fatigue curve development methodology presented in this article is efficient and improvement over current practice of VECD based fatigue life prediction. Use of probabilistic fatigue curve proposed in this research can provide safety margin in fatigue life estimation to certain extent. This methodology can be easily merged with currently used reliability based pavement design approaches.

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