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Vacuum expectation value of a Wegner–Wilson loop near the light-cone

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Abstract

Vacuum expectation values for one Wegner–Wilson loop representing a moving quark–antiquark pair are calculated in four-dimensional Euclidean and Minkowski space–time. The calculation uses gluon field strength correlators with perturbative gluon exchange and non-perturbative correlations from the stochastic vacuum model. The expectation value of a Wegner–Wilson loop forming a hyperbolic angle in Minkowski space–time is connected by an analytical continuation to the expectation value of the Wegner–Wilson loop in Euclidean four-space. The obtained result shows how confinement enters into the light-cone Hamiltonian for valence quarks independently of the chosen model.

1. Introduction

One of the challenges in quantum chromodynamics (QCD) is the relativistic bound state problem. In the light-cone Hamiltonian approach [1] light-cone wave functions can be constructed in a boost invariant way. It is necessary to have reliable light-cone wave functions if one wants to calculate high energy scattering, especially exclusive reactions. Many parametrizations assume separability of the dependence on the longitudinal momentum fraction and transverse momentum which is very unlikely since the two momenta are coupled in the kinetic energy operator. Various approaches have been tried to compute such wave functions. One can use the usual equal time Hamiltonian [2] and transform the resulting wave functions into light-cone form with the help of kinematical on-shell equations. The light-cone Hamiltonian in a string picture is formulated in Ref. [3]. More ambitious is the construction of an effective Hamiltonian including the gauge degrees of freedom explicitly and then solving the bound state problem. For mesons this approach [4,5] still needs many parameters to be fixed. Attempts have been made to solve the valence quark wave function for mesons in a simple Hamiltonian with a two-body potential [6].

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A necessary input is an adequate potential for the light-cone Hamiltonian. For the equal time Hamiltonian and heavy quarks the calculation of Wegner–Wilson loops gives the form of the non-perturbative potential for long distances. The correlator model [7] allows to calculate vacuum expectation values of gauge invariant Wegner–Wilson loops using perturbative and non-perturbative field strength correlation functions as input. One computes the loop expectation value $\langle W_r[C]\rangle$ in terms of a gauge invariant bilocal gluon field strength correlator integrated over minimal surfaces by using non-Abelian Stokes' theorem. Then the matrix cumulant expansion in the Gaussian approximation is applied.

The basic object of the correlator model is the gauge invariant bilocal gluon field strength correlator $F_{\mu\nu\rho\sigma}(X_1, X_2, O; C_{x_1o}, C_{x_2o})$. The strings C_{x_1o}, C_{x_2o} connect the coordinates X_1, X_2 in the correlation function of the two-field strengths to a common reference point O. We define

$$\frac{1}{4}\delta^{ab} F_{\mu\nu\rho\sigma}(X_1, X_2, O; C_{x_1o}, C_{x_2o}) := \left\langle \frac{g^2}{4\pi^2}\big[G^a_{\mu\nu}(O, X_1; C_{x_1o}) G^b_{\rho\sigma}(O, X_2; C_{x_2o})\big]\right\rangle_G. \tag{1}$$

The gluon field correlator has a perturbative (P) and a non-perturbative (NP) component. The stochastic vacuum model is used for the non-perturbative low frequency background field and the perturbative gluon exchange for the additional high frequency contributions. The most general form of the correlator respecting translational, Lorentz and parity invariance reads in Euclidean space [7]

$$\begin{aligned} F^{\mathrm{NP}}_{\mu\nu\rho\sigma}(Z) &= F^{\mathrm{NP}c}_{\mu\nu\rho\sigma}(Z) + F^{\mathrm{NP}nc}_{\mu\nu\rho\sigma}(Z) \\ &= \frac{1}{3(N_c^2-1)} G_2 \bigg\{ \kappa(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) D\big(Z^2\big) \\ &\quad + (1-\kappa)\frac{1}{2}\bigg[\frac{\partial}{\partial Z_\nu}(Z_\sigma\delta_{\mu\rho} - Z_\rho\delta_{\mu\sigma}) + \frac{\partial}{\partial Z_\mu}(Z_\rho\delta_{\nu\sigma} - Z_\sigma\delta_{\nu\rho})\bigg] D_1\big(Z^2\big)\bigg\} \end{aligned} \tag{2}$$

with $G_2 = \langle \frac{g^2}{4\pi^2} G^a_{\mu\nu}(O) G^a_{\mu\nu}(O)\rangle$ as the gluon condensate. The term proportional to κ is the non-Abelian confining part $F^{\mathrm{NP}c}$ of the correlator, in contrast, the tensor structure $F^{\mathrm{NP}nc}_{\mu\nu\rho\sigma}$ is characteristic for Abelian gauge theories and does not lead to confinement. The correlation functions are a simple exponential of range a:

$$D\big(Z^2\big) = D_1\big(Z^2\big) = e^{-|Z|/a}. \tag{3}$$

The calculation of a Wegner–Wilson loop along the imaginary time directions gives the heavy quark–antiquark potential with color-Coulomb behavior for small and confining linear rise for large source separations [7]. Since the computation of the VEV for one Wegner–Wilson loop can be done completely analytically, also other orientations of the loop can be chosen, e.g., a loop where the quark–antiquark pair moves along the z-direction. By transforming to Minkowski space–time the dependence of the interaction potential on longitudinal and transverse separation of the pair can be obtained this way. In Section 2 we describe the calculation in Euclidean space–time, in Section 3 in Minkowski space–time and in Section 4 we derive the potential in a light-cone Hamiltonian for valence quarks.

2. Vacuum expectation value for a tilted Wegner–Wilson loop in Euclidean space–time

The vacuum expectation value (VEV) of a tilted Wegner–Wilson loop represents a moving quark–antiquark pair

$$W[C] = \mathrm{Tr}\,\mathcal{P}\exp\bigg(-ig\oint_C dZ_\mu\, G^a_\mu(Z) t^a\bigg). \tag{4}$$

The group generators t^a are in the fundamental representation of $SU(3)$, g is the strong coupling constant and $\mathcal{P}$ symbolizes path ordering of the closed path C in space–time. The loop C with spatial extension R_0 and temporal

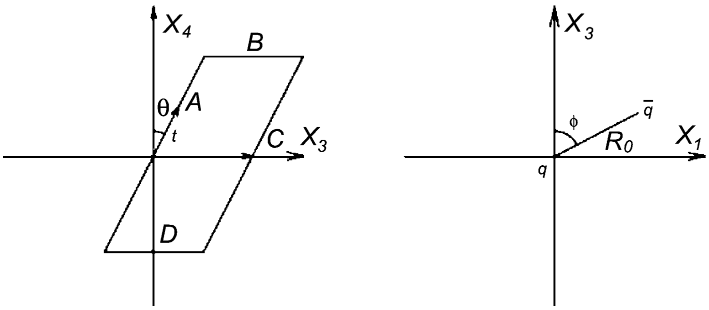

Fig. 1. Configuration of the Wegner–Wilson loop in Euclidean space–time.

extension T has the following parametrization in four-dimensional Euclidean space–time (Fig. 1)

$$C = C_A \cup C_B \cup C_C \cup C_D,$$

where

$$C_A = \{ut_\mu,\ u \in [-T/2, T/2]\}, \qquad C_B = \{T/2 t_\mu + vR_0 r_\mu,\ v \in [0,1]\},$$
$$C_C = \{-ut_\mu + R_0 r_\mu,\ u \in [-T/2, T/2]\}, \qquad C_D = \{-T/2 t_\mu + (1-v)R_0 r_\mu,\ v \in [0,1]\}, \tag{5}$$

and the parametrization of the surface

$$X_\mu = ut_\mu + vR_0 r_\mu, \quad u \in [-T/2, T/2],\ v \in [0,1], \tag{6}$$

where

$$r = \begin{pmatrix} \sin\phi \\ 0 \\ \cos\phi \\ 0 \end{pmatrix}, \qquad t = \begin{pmatrix} 0 \\ 0 \\ \sin\theta \\ \cos\theta \end{pmatrix}. \tag{7}$$

The expression for the Wegner–Wilson loop simplifies with the help of the Casimir operator in the fundamental representation $C_2(3) = t^2 = 4/3$

$$\langle W[C] \rangle_G = \exp\left[-\frac{C_2(3)}{2}\chi_{ss}\right], \tag{8}$$

where χ_{ss} is the double area integral of the correlation function over the surface

$$\chi_{ss} := \frac{\pi^2}{4}\int_S d\sigma_{\mu\nu}(X_1)\int_S d\sigma_{\rho\sigma}(X_2)\, F_{\mu\nu\rho\sigma}(X_1, X_2, O; C_{x_1o}, C_{x_2o}). \tag{9}$$

The lengthy calculation of χ_{ss} is standard and follows the lines of Ref. [7], we will give here only these parts which are relevant to understand the calculation of the tilted loops. One gets for the non-perturbative confinement component

$$\chi_{ss}^{\mathrm{NP}\,c} = \frac{\pi^2 G_2 \kappa}{3(N_c^2-1)}\int_0^1 dv_1 \int_0^1 dv_2 \int_{-T/2}^{T/2} du_1 \int_{-T/2}^{T/2} du_2 \left[t^2 \cdot r^2 - (t\cdot r)^2\right] D(Z^2) \tag{10}$$

$$= \frac{\pi^2 G_2 \kappa}{3(N_c^2-1)} R_0^2\left(1 - \cos^2\phi \sin^2\theta\right)\int_0^1 dv_1 \int_0^1 dv_2 \int_{-T/2}^{T/2} du_1 \int_{-T/2}^{T/2} du_2\, D(Z^2). \tag{11}$$

Correlated points on the surface have the distance $Z = (u_1 - u_2)t + (v_1 - v_2)R_0 r$. The geometry of the loop orientation enters via the factor α

$$\alpha^2 = 1 - \cos^2\phi \sin^2\theta. \tag{12}$$

The confining χ_{ss} has the following final form:

$$\chi_{ss}^{\mathrm{NP}\,c} = \lim_{T\to\infty} \frac{2\pi^2 G_2 \kappa T}{3(N_c^2 - 1)} \int_0^{R_0\alpha} d\rho\, (R_0\alpha - \rho) \cdot 2\rho K_1\left(\frac{\rho}{a}\right). \tag{13}$$

At large distances $R_0\alpha \gg 2a$ one recognizes that the confining interaction leads to a VEV of the tilted Wilson loop which is consistent with the area law R_0

$$\langle W[C] \rangle = e^{-\sigma R_0 \alpha T}, \tag{14}$$

$$\sigma = \frac{\pi^3 G_2 a^2 \kappa}{18}, \tag{15}$$

where σ is the string tension [7] and the area is obtained from

$$\mathrm{Area} = T R_0 \int_{-1/2}^{1/2} du \int_0^1 dv \sqrt{\left(\frac{dX_\mu}{du}\right)^2 \left(\frac{dX_\mu}{dv}\right)^2 - \left(\frac{dX_\mu}{du}\frac{dX_\mu}{dv}\right)^2} \tag{16}$$

$$= T R_0 \alpha. \tag{17}$$

The non-confining χ_{ss} functions give the short range attractive quark–antiquark interaction from massive correlator and gluon exchange

$$\chi_{ss}^{\mathrm{NP}nc} = -\lim_{T\to\infty} \frac{2\pi^2 G_2 (1-\kappa) a T}{3(N_c^2 - 1)} \cdot R_0^2 \alpha^2 \cdot K_2\left(\frac{R_0\alpha}{a}\right), \tag{18}$$

$$\chi_{ss}^{\mathrm{P}} = -\lim_{T\to\infty} \frac{2g^2 T \exp(-m_G R_0 \alpha)}{4\pi R_0 \alpha}. \tag{19}$$

In the limit of straight loops ($\theta = 0$) all results agree with previous calculations [7].

3. Vacuum expectation value for one Wegner–Wilson loop in Minkowskian space–time near the light-cone

In this section the vacuum expectation value (VEV) of one Wegner–Wilson loop near the light-cone is computed in Minkowskian space–time. As before we use the correlator model for the non-perturbative low frequency background field and perturbative gluon exchange for the additional high frequency contribution. The path of the color dipole in four-dimensional Euclidean space–time is represented by a light-like QCD Wegner–Wilson loop: to accomplish the transition from Euclidean to Minkowski space–time we have to make the following replacements:

$$X_4 \to i x^0, \qquad X_i \to x^i. \tag{20}$$

Here $X = (\vec{X}, X_4)$ is the Euclidean space–time point and $x = (x^0, \vec{x})$ the Minkowskian vector. Of course, also the Euclidean correlation functions of the field strengths have to be changed to the Minkowskian correlation functions [8].

The loop C with spatial extent R_0 and temporal extension T is now placed in four-dimensional Minkowski space–time. The quark–antiquark pair is moving with velocity β

$$\beta = \frac{\sinh(\psi)}{\cosh(\psi)} \tag{21}$$

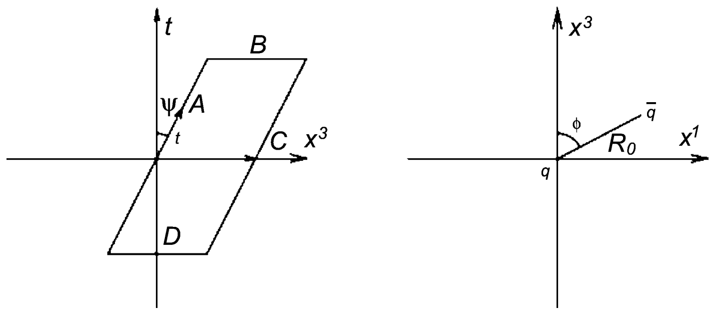

Fig. 2. Configuration of the Wegner–Wilson loop in Minkowski space–time.

and the hyperbolic angle ψ defines the boost (Fig. 2).

The loop has the same parametrization as before but with Minkowskian vectors r^μ and t^μ

$$r^\mu = \begin{pmatrix} 0 \\ \sin\phi \\ 0 \\ \cos\phi \end{pmatrix}, \qquad t^\mu = \begin{pmatrix} \cosh\psi \\ 0 \\ 0 \\ \sinh\psi \end{pmatrix}. \tag{22}$$

For the Wegner–Wilson loop in Minkowski space–time we define χ_{ss} in the same way as in Ref. [8]

$$\langle W[C]\rangle_G = \exp\left[-i\frac{C_2(3)}{2}\chi_{ss}\right]. \tag{23}$$

The phase factor χ_{ss} is given as the double area integral over the surface

$$\chi_{ss} := -i\frac{\pi^2}{4}\int_S d\sigma^{\mu\nu}(x_1)\int_S d\sigma^{\rho\sigma}(x_2)\,F_{\mu\nu\rho\sigma}(x_1, x_2, O; C_{x_1 O}, C_{x_2 O}). \tag{24}$$

The expression for the Wegner–Wilson loop is the same as before Eq. (8) and Eq. (9). We use the above convention in order to be consistent with the notation of Ref. [8]. In the course of the calculation we find

$$\chi_{ss}^{\mathrm{NP}c} = -\frac{\pi^2 G_2\kappa}{12(N_c^2-1)}\int_S d\sigma^{\mu\nu}(x_1)\int_S d\sigma^{\rho\sigma}(x_2)\,i\,D\big(z^2/a^2\big)\{g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\}, \tag{25}$$

where the changed metric tensor in Minkowski space comes from replacing the Euclidean metric tensor by the Minkowski tensor in the correlation function. Using the Minkowskian vectors r, t with the following properties

$$t^2 = \cosh^2\psi - \sinh^2\psi = 1, \qquad r^2 = r_\mu r^\mu = -1,$$
$$(t\cdot r)^2 = \big(t_\mu r^\mu\big)\cdot\big(t^\nu r_\nu\big) = \cos^2\phi\sinh^2\psi, \qquad t^2\cdot r^2 - (t\cdot r)^2 = -\big(1+\cos^2\phi\sinh^2\psi\big)$$

one obtains

$$\chi_{ss}^{\mathrm{NP}c} = \lim_{T\to\infty}\frac{2\pi^2 G_2\kappa T}{3(N_c^2-1)}\int_0^{R_0\alpha_M} d\rho\,(R_0\alpha_M - \rho)\,i\,D^{(3)}\big(\rho^2\big).$$

The three-dimensional correlation function $D^{(3)}(\rho^2)$ in Minkowski space is obtained by analytical continuation of the Euclidean function. Since the argument of the function is given by the magnitude of a three-vector the analytic continuation leads to the same modified Bessel function which was obtained in Euclidean space, cf. also

Appendix B of Ref. [8]. The resulting χ_{ss} is real. The geometry enters via the factor

$$\alpha_M^2 = 1 + \cos^2\phi \sinh^2\psi, \tag{26}$$

which is consistent with the analytical continuation of the Euclidean expression $\alpha = 1 - \cos^2\phi \sin^2\theta$ into Minkowski space by transforming the angle $\theta \to i\psi$. This analytical continuation is similar to the analytical continuation used in high energy scattering [9–11] where the angle between two Wilson loops transforms in the same way.

The different contributions to χ_{ss} read:

$$\chi_{ss}^{\mathrm{NP}c} = \lim_{T\to\infty} \frac{2\pi^2 G_2 \kappa T}{3(N_c^2-1)} \int\limits_0^{R_0\alpha_M} d\rho\,(R_0\alpha_M - \rho)\cdot 2\rho K_1\left(\frac{\rho}{a}\right),$$

$$\chi_{ss}^{\mathrm{NP}nc} = -\lim_{T\to\infty} \frac{2\pi^2 G_2(1-\kappa)aT}{3(N_c^2-1)} \cdot R_0^2\alpha_M^2 \cdot K_2\left(\frac{R_0\alpha_M}{a}\right), \tag{27}$$

$$\chi_{ss}^{\mathrm{P}} = -\lim_{T\to\infty} \frac{2g^2 T \exp(-m_G R_0\alpha_M)}{4\pi R_0\alpha_M}. \tag{28}$$

We remark that the same calculation with a time-like vector $t\cdot t = 0$ and a quark separation r^μ oriented orthogonally to t, i.e., $r\cdot t = 0$ gives the well-known result that the expectation value of the loop equals unity as has been shown in the original references on high energy scattering. In the formulation of high energy scattering [8] with the correlation function summed to all orders in the S-matrix the respective phase factors from single loops totally cancel. So the here obtained expectation value does not change the results of the previous calculations even if one goes away from the light-cone.

4. Using the VEV of the light-like Wilson loop to derive the quark–antiquark potential in the light-cone Hamiltonian

The exponent giving the expectation value of the Wilson loop acquires a new meaning for a tilted loop. In order to interprete the result of the preceding section one must define the four-velocity of the particles described by the tilted loop

$$u_\mu = (\gamma, 0_\perp, \gamma\beta). \tag{29}$$

With the help of the four-velocity we can rewrite the loop as:

$$e^{-ig\int d\tau\, A^\mu u_\mu} = e^{-ig\int d\tau\,(\gamma A^0 - \gamma\beta A^3)}. \tag{30}$$

The line integral of the gauge potential acts as a phase factor on a Dirac wave function ψ which splits up into a leading dynamical component ψ_+ and a dependent component ψ_-. For very fast quarks the mass term and transverse momenta are negligible compared with the energy and longitudinal momentum. In this eikonal approximation the Dirac equation of the leading component decouples from the small component:

$$i\partial_-\psi_+ = P^-_{\mathrm{pot}}(A^-)\psi_+ \tag{31}$$

$$= gA^-\psi_+. \tag{32}$$

With $\beta \approx 1$ the phase factor in the tilted Wilson loop integrates A^- and leads to a VEV for the loop containing $P^-_{\mathrm{pot}} = \frac{1}{\sqrt{2}}(P^0 - P^3)|_{\mathrm{pot}}$

$$\langle W_r[C]\rangle = e^{-i\gamma(P^0-P^3)|_{\mathrm{pot}}T}. \tag{33}$$

The light-cone potential energy arising from the confining part of the correlation function has a term of order $O(1/P^+) = O(1/\gamma)$, where $P^+ = \frac{1}{\sqrt{2}}(P^0 + P^3)$ is the light-cone momentum

$$P^-_{\text{pot}} = \frac{1}{\sqrt{2}}\left(\sigma R_0\sqrt{\cos(\phi)^2 + \sin(\phi)^2/\gamma^2}\right). \tag{34}$$

Terms involving transverse momenta and masses of the same order $O(1/P^+)$ are not included in the loop as it has been calculated. Two of these terms give the standard kinetic energy term of free particles, which contributes to the total light-cone energy. Terms with spin cannot be obtained from the straight Wilson loop and are not discussed here. We introduce the relative + momentum k^+ and transverse momentum $k_\perp$ for the quarks with mass μ. By adding the above "potential" term to the kinetic term of relative motion of the two particles we complete our approximate derivation of the light-cone energy P^-

$$P^- = \frac{(\mu^2 + k_\perp^2)P^+}{2(1/4P^{+2} - k^{+2})} + \frac{1}{\sqrt{2}}\sigma\sqrt{x_3^2 + x_\perp^2/\gamma^2}. \tag{35}$$

To derive the light-cone Hamiltonian we multiply P^- with the plus component of the light-cone momentum $P^+ = \frac{1}{\sqrt{2}}(P^0 + P^3)$ and use that $P^+/M = \sqrt{2}\gamma$ to eliminate the boost variable from the Hamiltonian. Further we follow the notation of Ref. [12] and introduce the fraction $\xi = k^+/P^+$ with $|\xi| < 1/2$ and its conjugate the scaled longitudinal space coordinate $\sqrt{2}\rho = P^+x_3$ as dynamical variables. For our configuration the relative time of the quark and antiquark is zero

$$M_c^2 = 2P^+P^- = \frac{(\mu^2 + k_\perp^2)}{1/4 - \xi^2} + 2\sigma\sqrt{\rho^2 + M_c^2 x_\perp^2}. \tag{36}$$

We have obtained the light-cone Hamiltonian M_c^2 from the confining interaction in a Lorentz invariant manner, because the variables ξ, ρ, $k_\perp$ and $x_\perp$ are invariant under boosts. The valence quark light-cone Hamiltonian has a simple confining potential. The magnitude of the confining potential is set by the string tension σ. The effective "distance" of the quarks is given by scale-free light-cone longitudinal distance and the transverse distance multiplied by the bound state mass. The above equation agrees in the limit of one-dimensional motion with the equation for the yo–yo string derived in Ref. [12]. If there is only transverse motion ($\xi = 0$), the confinement has the usual form which is seen by using $M \approx 2\mu$. To solve the M_c^2 operator one can go over to M_s^2. Minimizing M_s^2 with respect to s one can replace the square root operator. Final self consistency must be reached with a guessed mass eigenvalue M_0

$$M_s^2 = \frac{(\mu^2 + k_\perp^2)}{1/4 - \xi^2} + \frac{1}{2}\left(4\sigma^2\frac{\rho^2 + M_0^2 x_\perp^2}{s} + s\right). \tag{37}$$

The other non-confining potentials from the Abelian correlator and the perturbative gluon exchange can be worked out similarly and one gets for the complete valence Hamiltonian

$$M^2 = \frac{(\mu^2 + k_\perp^2)}{1/4 - \xi^2} + 2\sigma r - \frac{4}{3}\left(\frac{2g^2M^2e^{-\frac{m_G r}{M}}}{4\pi r}\right) - 2\frac{\sigma(1-\kappa)r^2}{\kappa Ma\pi}K_2\left(\frac{r}{Ma}\right) \tag{38}$$

with the dimensionless variable

$$r = \sqrt{\rho^2 + M^2x_\perp^2}. \tag{39}$$

The relative weight of non-perturbative non-Abelian and Abelian contribution is fixed by $\kappa = 0.7$ in the parametrization of the correlation function [8]. We have used σ to parametrize also the Abelian non-confining potential, instead of giving the full expression with the gluon condensate. Of course, the Abelian part of the potential does not confine. The best way to find the two-body wave function is to keep ξ in the momentum representation and the transverse direction in configuration space $x_\perp$. It is to be expected that in this approximation the pion mass is not described correctly. Firstly, the spin structure of the meson is not reflected in the spin independent expression above. Secondly, one expects quark self energy corrections [13]. The typical mass scale of a meson estimated from a trial solution with a wave function $\psi(\xi, x_\perp) = A\cos(\xi\pi)e^{-x_\perp^2/x_0^2}$ comes out to be 1 GeV. The problem of the pion has to be addressed separately, since on the light-cone the mechanism of chiral symmetry breaking needs special care and is of particular interest. In the approach above confinement plays an important role in contrast to Nambu–Jona-Lasinio effective models which give an adequate description of spontaneous chiral symmetry breaking but do not include confinement.

The confining interaction in the light-cone Hamiltonian was derived in the specific model of the stochastic vacuum. But it also can be inferred from the simple Lorentz transformation properties of the phase in the Wilson loop and a lattice determination of the tilted Wilson expectation values. In this respect the final Hamiltonian is model independent.

The inclusion of confining forces in the initial and final state wave functions can put all scattering cross sections calculated with the stochastic vacuum model on a much safer base since wave functions and cross section are derived consistently. For low Q^2 the long distance part of the photon wave function matters strongly and confinement is important cf. [14]. Especially the diffractive cross section has a sizeable contribution from large dipole sizes and a correct behaviour can only be expected when the problem of the large dipole wave function is treated adequately. A very useful extension of the above calculation is the coupling of the initial $q\bar{q}$ state to higher Fock states $q\bar{q}g$ with gluons or fragmentation of the original $q\bar{q}$ state into $q\bar{q}q\bar{q}$ states which can also be estimated from Wilson loops near the light-cone in Minkowski space. On the problem of fragmentation the Lund model [15] has been very successful and it is interesting to see how the above model calculation fares in comparison.

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