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Soft semirings

Feng Feng [a,*], Young Bae Jun [b], Xianzhong Zhao [c]

[a] Department of Applied Mathematics and Applied Physics, Xi'an Institute of Posts and Telecommunications, Xi'an 710061, PR China

[b] Department of Mathematics Education (and RINS), Gyeongsang National University, Chinju 660-701, Republic of Korea

[c] Department of Mathematics, Northwest University, Xi'an 710069, PR China

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ABSTRACT

Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. In this paper, we initiate the study of soft semirings by using the soft set theory. The notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms are introduced, and several related properties are investigated.

1. Introduction

To solve complicated problems in economics, engineering, environmental science, medical science and social science, methods in classical mathematics may not be successfully used because of various uncertainties arising in these problems. Alternatively, mathematical theories such as probability theory, fuzzy set theory [1,2], rough set theory [3,4], vague set theory [5] and the interval mathematics [6] were established by researchers to modelling uncertainties appearing in the above fields. In 1992, Molodtsov [7] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields. At present, works on soft set theory are progressing rapidly. Maji et al. [8] discussed the application of soft set theory to a decision making problem. Chen et al. [9] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. In theoretical aspects, Maji et al. [10] defined and studied several operations on soft sets. Aktaş and Çağman [11] compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups, and derived some related properties. Furthermore, Jun [12] introduced and investigated the notion of soft BCK/BCI-algebras. Jun and Park [13] discussed the applications of soft sets in ideal theory of BCK/BCI-algebras.

On the other hand, semirings have been found useful for dealing with problems in different areas of applied mathematics and information sciences, as the semiring structure provides an algebraic framework for modelling and investigating the key factors in these problems. The applications of semirings to areas such as optimization theory, graph theory, theory of discrete event dynamical systems, generalized fuzzy computation, automata theory, formal language theory, coding theory and analysis of computer programs have been extensively studied in the literature (cf. [14,15]). It is also well-known that

* Corresponding author. Tel.: +86 29 85384065.
E-mail addresses: fengnix@hotmail.com (F. Feng), skywine@gmail.com (Y.B. Jun), xianzhongzhao@263.net (X. Zhao).

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ideals usually play a fundamental role in algebra, especially in the study of rings. Nevertheless, ideals in a semiring S do not in general coincide with the usual ring ideals if S is a ring, and so many results in ring theory have no analogues in semirings using only ideals. Consequently, some more restricted concepts of ideals such as k-ideals [16] and h-ideals [17] have been introduced in the study of the semiring theory. Moreover, the fuzzy set theory initiated by Zadeh has been successfully applied to generalize many basic concepts in algebra. In fact, several researchers have investigated a fuzzy theory in semirings (see [18–29]). They introduced the notions of fuzzy semirings, fuzzy (prime) ideals, fuzzy k-ideals, fuzzy h-ideals and L-fuzzy ideals in semirings, and obtained many related results.

In this paper, we deal with the algebraic structure of semirings by applying soft set theory. We define the notion of a soft semiring and focus on the algebraic properties of soft semirings. We introduce the notions of soft ideals and idealistic soft semirings, and give several illustrating examples. We investigate relations between soft semirings and idealistic soft semirings. We also establish the bi-intersection, union, "AND" operation, and "OR" operation of soft ideals and idealistic soft semirings. Moreover, the notion of soft semiring homomorphisms is introduced and illustrated by a corresponding example.

2. Preliminaries

A *semiring* S is a structure consisting of a nonempty set S together with two binary operations on S called *addition* and *multiplication* (denoted in the usual manner) such that

- S together with addition is a semigroup,
- S together with multiplication is a semigroup, and
- $a(b+c) = ab + ac$ and $(a+b)c = ac + bc$ for all $a, b, c \in S$.

A semiring S is said to be *additively commutative* if $a + b = b + a$ for all $a, b \in S$. A zero element of a semiring S is an element 0 such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. A nonempty subset I of a semiring S is called a *left* [*right*] *ideal* of S if I is closed under addition and $SI \subseteq I$ [$IS \subseteq I$, respectively]. We say that I is an *ideal* of S, denoted by $I \lhd S$, if it is both a left and a right ideal of the semiring S. Let R and S be semirings. A mapping $f : R \to S$ is called a *homomorphism* of semirings if it satisfies

$$f(a+b) = f(a) + f(b) \quad \text{and} \quad f(ab) = f(a)f(b)$$

for all $a, b \in R$. That is, the mapping f preserves the semiring operations. A semiring homomorphism $f : R \to S$ is called a *monomorphism* [resp. *epimorphism, isomorphism*] if it is an injective [resp. surjective, bijective] mapping.

Molodtsov [7] defined the notion of a soft set in the following way: Let U be an initial universe set and E be a set of parameters. The power set of U is denoted by $\mathscr{P}(U)$ and A is a subset of E.

Definition 2.1 ([7]). A pair (η, A) is called a *soft set* over U, where η is a mapping given by $\eta : A \to \mathscr{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\eta(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (η, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [7]. These examples were also discussed in [11,10].

Maji et al. [10] introduced and investigated several binary operations such as intersection, union, AND-operation, and OR-operation of soft sets.

Definition 2.2 ([10]). Let (η, A) and (γ, B) be two soft sets over a common universe U. The *intersection* of (η, A) and (γ, B) is defined to be the soft set (ϑ, C) satisfying the following conditions: (i) $C = A \cap B$; (ii) for all $e \in C$, $\vartheta(e) = \eta(e)$ or $\gamma(e)$ (as both are same set). In this case, we write $(\eta, A) \widetilde{\cap} (\gamma, B) = (\vartheta, C)$.

In contrast with the above definition of soft set intersections, we alternatively define and use the following binary operation, called *bi-intersection* of two soft sets.

Definition 2.3. The *bi-intersection* of two soft sets (α, A) and (β, B) over a common universe U is defined to be the soft set (γ, C), where $C = A \cap B$ and $\gamma : C \to \mathscr{P}(U)$ is a mapping given by $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$. This is denoted by $(\alpha, A) \widetilde{\sqcap} (\beta, B) = (\gamma, C)$.

Definition 2.4 ([10]). Let (η, A) and (γ, B) be two soft sets over a common universe U. The *union* of (η, A) and (γ, B) is defined to be the soft set (ϑ, C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $e \in C$,

$$\vartheta(e) = \begin{cases} \eta(e) & \text{if } e \in A \setminus B, \\ \gamma(e) & \text{if } e \in B \setminus A, \\ \eta(e) \cup \gamma(e) & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(\eta, A) \widetilde{\cup} (\gamma, B) = (\vartheta, C)$.

As a generalization of the union of two soft sets, we define the union of a nonempty family of soft sets in the following way.

Definition 2.5. Let $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The *union* of these soft sets is defined to be the soft set (β, B) such that $B = \cup_{i\in I} A_i$ and for all $x \in B$, $\beta(x) = \cup_{i\in I(x)} \alpha_i(x)$ where $I(x) = \{i \in I \mid x \in A_i\}$. In this case, we write $\widetilde{\cup}_{i\in I}(\alpha_i, A_i) = (\beta, B)$.

Definition 2.6 (*[10]*). If (η, A) and (γ, B) are two soft sets over a common universe U, then " (η, A) *AND* (γ, B)" denoted by $(\eta, A)\widetilde{\wedge}(\gamma, B)$ is defined by $(\eta, A)\widetilde{\wedge}(\gamma, B) = (\vartheta, A \times B)$, where $\vartheta(x, y) = \eta(x) \cap \gamma(y)$ for all $(x, y) \in A \times B$.

Definition 2.7 (*[10]*). If (η, A) and (γ, B) are two soft sets over a common universe U, then " (η, A) *OR* (γ, B)" denoted by $(\eta, A)\widetilde{\vee}(\gamma, B)$ is defined by $(\eta, A)\widetilde{\vee}(\gamma, B) = (\vartheta, A \times B)$, where $\vartheta(x, y) = \eta(x) \cup \gamma(y)$ for all $(x, y) \in A \times B$.

Definition 2.8 (*[10]*). For two soft sets (η, A) and (γ, B) over a common universe U, we say that (η, A) is a *soft subset* of (γ, B), denoted by $(\eta, A)\widetilde{\subset}(\gamma, B)$, if it satisfies: (i) $A \subset B$; (ii) For every $\varepsilon \in A$, $\eta(\varepsilon)$ and $\gamma(\varepsilon)$ are identical approximations.

3. Soft semirings and soft ideals

In the sequel, let S be a semiring and A be a nonempty set. ρ will refer to an arbitrary binary relation between an element of A and an element of S, that is, ρ is a subset of $A \times S$ without otherwise specified. A set-valued function $\eta : A \to \mathscr{P}(S)$ can be defined as $\eta(x) = \{y \in S \mid (x, y) \in \rho\}$ for all $x \in A$. The pair (η, A) is then a soft set over S, which is derived from the relation ρ. The concept of a support is defined for both fuzzy sets and formal power series in the literature. Here we define a similar notion for soft sets. For a soft set (η, A), the set $\mathrm{Supp}\,(\eta, A) = \{x \in A \mid \eta(x) \neq \emptyset\}$ is called the *support* of the soft set (η, A). Thus a *null soft set* in [10] is indeed a soft set with an empty support, and we say that a soft set (η, A) is *non-null* if $\mathrm{Supp}\,(\eta, A) \neq \emptyset$.

Definition 3.1. Let (η, A) be a non-null soft set over a semiring S. Then (η, A) is called a *soft semiring* over S if $\eta(x)$ is a subsemiring of S for all $x \in \mathrm{Supp}\,(\eta, A)$.

Theorem 3.2. *Let (α, A) and (β, B) be soft semirings over S. Then the soft set $(\alpha, A)\widetilde{\wedge}(\beta, B)$ is a soft semiring over S if it is non-null.*

Proof. By Definition 2.6, let $(\alpha, A)\widetilde{\wedge}(\beta, B) = (\gamma, C)$, where $C = A \times B$ and $\gamma(x, y) = \alpha(x) \cap \beta(y)$ for all $(x, y) \in C$. Then by the hypothesis, (γ, C) is a non-null soft set over S. If $(x, y) \in \mathrm{Supp}\,(\gamma, C)$, then $\gamma(x, y) = \alpha(x) \cap \beta(y) \neq \emptyset$. It follows that the nonempty sets $\alpha(x)$ and $\beta(y)$ are both subsemirings of S. Hence $\gamma(x, y)$ is a subsemiring of S for all $(x, y) \in \mathrm{Supp}\,(\gamma, C)$, and so $(\gamma, C) = (\alpha, A)\widetilde{\wedge}(\beta, B)$ is a soft semiring over S as required. □

Definition 3.3. Let (α, A) and (β, B) be soft semirings over S. Then the soft semiring (β, B) is called a *soft subsemiring* of (α, A) if it satisfies:

(1) $B \subset A$;
(2) $\beta(x)$ is a subsemiring of $\alpha(x)$ for all $x \in \mathrm{Supp}\,(\beta, B)$.

From the above definition, one easily deduces that if (β, B) is a soft subsemiring of (α, A), then $\mathrm{Supp}\,(\beta, B) \subset \mathrm{Supp}\,(\alpha, A)$.

Proposition 3.4. *Let (α, A) and (β, A) be soft semirings over S. Then we have the following:*

(1) *The bi-intersection $(\alpha, A)\widetilde{\sqcap}(\beta, A)$ is a soft semiring over S if it is non-null.*
(2) *If $\beta(x) \subset \alpha(x)$ for all $x \in A$, then (β, A) is a soft subsemiring of (α, A).*
(3) *$(\alpha, A)\widetilde{\sqcap}(\beta, A)$ is a soft subsemiring of both (α, A) and (β, A) if it is non-null.*

Proof. (1) By Definition 2.3, we can write $(\alpha, A)\widetilde{\sqcap}(\beta, A) = (\gamma, A)$, where $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in A$. Suppose that (γ, A) is a non-null soft set over S. If $x \in \mathrm{Supp}\,(\gamma, A)$, then $\gamma(x) = \alpha(x) \cap \beta(x) \neq \emptyset$. Thus the nonempty sets $\alpha(x)$ and $\beta(x)$ are both subsemirings of S. Hence $\gamma(x)$ is a subsemiring of S for all $x \in \mathrm{Supp}\,(\gamma, A)$, and so $(\gamma, A) = (\alpha, A)\widetilde{\sqcap}(\beta, A)$ is a soft semiring over S as required.

(2) Straightforward.

(3) It follows immediately from (1) and (2). □

Generalizing the corresponding definitions in Section 2, we formulate the following definitions.

Definition 3.5. Let $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The *bi-intersection* of these soft sets is defined to be the soft set (β, B) such that $B = \cap_{i\in I} A_i$, and $\beta(x) = \cap_{i\in I} \alpha_i(x)$ for all $x \in B$. In this case, we write $\widetilde{\sqcap}_{i\in I}(\alpha_i, A_i) = (\beta, B)$.

Definition 3.6. Let $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The *AND-soft set* $\widetilde{\wedge}_{i\in I}(\alpha_i, A_i)$ of these soft sets is defined to be the soft set (β, B) such that $B = \prod_{i\in I} A_i$, and $\beta(x) = \cap_{i\in I} \alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.

Similarly, the *OR-soft set* $\widetilde{\vee}_{i\in I}(\alpha_i, A_i)$ of these soft sets is defined to be the soft set (γ, B) such that $B = \prod_{i\in I} A_i$, and $\gamma(x) = \cup_{i\in I} \alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.

Note that if $A_i = A$ and $\alpha_i = \alpha$ for all $i \in I$, then $\widetilde{\wedge}_{i\in I}(\alpha_i, A_i)$ and $\widetilde{\vee}_{i\in I}(\alpha_i, A_i)$ are denoted by $\widetilde{\wedge}_{i\in I}(\alpha, A)$ and $\widetilde{\vee}_{i\in I}(\alpha, A)$ respectively. In this case, $\prod_{i\in I} A_i = \prod_{i\in I} A$ means the direct power A^I.

Theorem 3.7. *Let $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft semirings over a semiring S. Then we have the following:*

(1) *$\widetilde{\wedge}_{i\in I}(\alpha_i, A_i)$ is a soft semiring over S if it is non-null.*
(2) *If $\{A_i \mid i \in I\}$ are pairwise disjoint, i.e., $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\widetilde{\bigcup}_{i\in I}(\alpha_i, A_i)$ is a soft semiring over S.*
(3) *$\widetilde{\sqcap}_{i\in I}(\alpha_i, A_i)$ is a soft semiring over S if it is non-null.*

Proof. (1) By Definition 3.6, let $\widetilde{\wedge}_{i\in I}(\alpha_i, A_i) = (\beta, B)$, where $B = \prod_{i\in I} A_i$, and $\beta(x) = \bigcap_{i\in I} \alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$. Suppose that the soft set (β, B) is non-null. If $x = (x_i)_{i\in I} \in \mathrm{Supp}\,(\beta, B)$, then $\beta(x) = \bigcap_{i\in I} \alpha_i(x_i) \neq \emptyset$. Thus the nonempty set $\alpha_i(x_i)$ is a subsemiring of S since (α_i, A_i) is a soft semiring over S for all $i \in I$. Hence $\beta(x)$ is a subsemiring of S for all $x \in \mathrm{Supp}\,(\beta, B)$, and so $\widetilde{\wedge}_{i\in I}(\alpha_i, A_i) = (\beta, B)$ is a soft semiring over S.

(2) By Definition 2.5, we can write $\widetilde{\bigcup}_{i\in I}(\alpha_i, A_i) = (\beta, B)$. Then $B = \bigcup_{i\in I} A_i$ and for all $x \in B$, $\beta(x) = \bigcup_{i\in I(x)} \alpha_i(x)$ where $I(x) = \{i \in I \mid x \in A_i\}$. Note first that (β, B) is non-null since $\mathrm{Supp}\,(\beta, B) = \bigcup_{i\in I} \mathrm{Supp}\,(\alpha_i, A_i) \neq \emptyset$. Let $x \in \mathrm{Supp}\,(\beta, B)$. Then $\beta(x) = \bigcup_{i\in I(x)} \alpha_i(x) \neq \emptyset$, and so we have $\alpha_{i_0}(x) \neq \emptyset$ for some $i_0 \in I(x)$. But by the hypothesis, we know that $\{A_i \mid i \in I\}$ are pairwise disjoint. Hence the above i_0 is indeed unique, and so $\beta(x)$ coincides with $\alpha_{i_0}(x)$. Moreover, since (α_{i_0}, A_{i_0}) is a soft semiring over S, we deduce that the nonempty set $\alpha_{i_0}(x)$ is a subsemiring of S. It follows that $\beta(x) = \alpha_{i_0}(x)$ is a subsemiring of S for all $x \in \mathrm{Supp}\,(\beta, B)$. Therefore $\widetilde{\bigcup}_{i\in I}(\alpha_i, A_i) = (\beta, B)$ is a soft semiring over S.

(3) By Definition 3.5, let $\widetilde{\sqcap}_{i\in I}(\alpha_i, A_i) = (\beta, B)$, where $B = \bigcap_{i\in I} A_i$, and $\beta(x) = \bigcap_{i\in I} \alpha_i(x)$ for all $x \in B$. Suppose that the soft set (β, B) is non-null. If $x \in \mathrm{Supp}\,(\beta, B)$, then $\beta(x) = \bigcap_{i\in I} \alpha_i(x) \neq \emptyset$. It follows that for all $i \in I$, the nonempty set $\alpha_i(x)$ is a subsemiring of S since (α_i, A_i) is a soft semiring over S. Hence $\beta(x)$ is a subsemiring of S for all $x \in \mathrm{Supp}\,(\beta, B)$, and so $\widetilde{\sqcap}_{i\in I}(\alpha_i, A_i) = (\beta, B)$ is a soft semiring over S. This completes the proof. □

Definition 3.8. Let (η, A) be a soft semiring over a semiring S. A non-null soft set (γ, I) over S is called a *soft ideal* of (η, A), denoted by $(\gamma, I)\widetilde{\triangleleft}(\eta, A)$, if it satisfies:

(i) $I \subset A$;
(ii) $\gamma(x)$ is an ideal of $\eta(x)$ for all $x \in \mathrm{Supp}\,(\gamma, I)$.

It is clear that every soft ideal of a soft semiring (η, A) over S is a soft subsemiring of (η, A), but not every soft subsemiring of (η, A) is a soft ideal.

Example 3.9. Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ be the semiring of integers module 6. Let (η, A) be a soft set over Z_6, where $A = Z_6$ and $\eta : A \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\eta(x) = \{y \in Z_6 \mid x\,\rho\,y \Leftrightarrow xy \in \{0, 2, 4\}\}$$

for all $x \in A$. Then $\eta(0) = Z_6$, $\eta(1) = \{0, 2, 4\}$, $\eta(2) = Z_6$, $\eta(3) = \{0, 2, 4\}$, $\eta(4) = Z_6$ and $\eta(5) = \{0, 2, 4\}$ are subsemirings of Z_6. Hence (η, A) is a soft semiring over Z_6. Let (γ, I) be a soft set over Z_6, where $I = \{0, 1, 2\}$ and $\gamma : I \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\gamma(x) = \{y \in Z_6 \mid x\,\rho\,y \Leftrightarrow xy = 0\}$$

for all $x \in I$. Then $\gamma(0) = Z_6 \triangleleft Z_6 = \eta(0)$, $\gamma(1) = \{0\} \triangleleft \{0, 2, 4\} = \eta(1)$, and $\gamma(2) = \{0, 3\} \triangleleft Z_6 = \eta(2)$. Hence (γ, I) is a soft ideal of (η, Z_6).

Theorem 3.10. *Let (γ_1, I_1) and (γ_2, I_2) be soft ideals of a soft semiring (η, A) over a semiring S. Then the soft set $(\gamma_1, I_1)\,\widetilde{\sqcap}\,(\gamma_2, I_2)$ is a soft ideal of (η, A) if it is non-null.*

Proof. Assume that $(\gamma_1, I_1)\widetilde{\triangleleft}\,(\eta, A)$ and $(\gamma_2, I_2)\widetilde{\triangleleft}\,(\eta, A)$. By Definition 2.3, we can write $(\gamma_1, I_1)\,\widetilde{\sqcap}\,(\gamma_2, I_2) = (\gamma, I)$, where $I = I_1 \cap I_2$ and $\gamma(x) = \gamma_1(x) \cap \gamma_2(x)$ for all $x \in I$. Obviously, we have $I \subset A$. Suppose that the soft set (γ, I) is non-null. If $x \in \mathrm{Supp}\,(\gamma, I)$, then $\gamma(x) = \gamma_1(x) \cap \gamma_2(x) \neq \emptyset$. Since $(\gamma_1, I_1)\widetilde{\triangleleft}\,(\eta, A)$ and $(\gamma_2, I_2)\widetilde{\triangleleft}\,(\eta, A)$, we deduce that the nonempty sets $\gamma_1(x)$ and $\gamma_2(x)$ are both ideals of $\eta(x)$. It follows that $\gamma(x)$ is an ideal of $\eta(x)$ for all $x \in \mathrm{Supp}\,(\gamma, I)$. Therefore $(\gamma_1, I_1)\,\widetilde{\cap}\,(\gamma_2, I_2) = (\gamma, I)$ is a soft ideal of (η, A) as required. □

Theorem 3.11. *Let (γ, I) and (ϑ, J) be soft ideals of a soft semiring (η, A) over a semiring S. If I and J are disjoint, then $(\gamma, I)\,\widetilde{\bigcup}\,(\vartheta, J)$ is a soft ideal of (η, A).*

Proof. Assume that $(\gamma, I)\widetilde{\triangleleft}\,(\eta, A)$ and $(\vartheta, J)\widetilde{\triangleleft}\,(\eta, A)$. According to Definition 2.4, we can write $(\gamma, I)\,\widetilde{\bigcup}\,(\vartheta, J) = (\kappa, U)$, where $U = I \cup J$ and for every $x \in U$,

$$\kappa(x) = \begin{cases} \gamma(x) & \text{if } x \in I \setminus J, \\ \vartheta(x) & \text{if } x \in J \setminus I, \\ \gamma(x) \cup \vartheta(x) & \text{if } x \in I \cap J. \end{cases}$$

Clearly, we have $U \subset A$. Suppose that I and J are disjoint, i.e., $I \cap J = \emptyset$. Then for every $x \in \mathrm{Supp}(\kappa, U)$, we know that either $x \in I \setminus J$ or $x \in J \setminus I$. If $x \in I \setminus J$, then $\kappa(x) = \gamma(x) \neq \emptyset$ is an ideal of $\eta(x)$ since $(\gamma, I)\tilde{\lhd}(\eta, A)$. Similarly, if $x \in J \setminus I$, then $\kappa(x) = \vartheta(x) \neq \emptyset$ is an ideal of $\eta(x)$ since $(\vartheta, J)\tilde{\lhd}(\eta, A)$. Thus we conclude that $\kappa(x) \lhd \eta(x)$ for all $x \in \mathrm{Supp}(\kappa, U)$, and so $(\gamma, I)\,\tilde{\cup}\,(\vartheta, J) = (\kappa, U)$ is a soft ideal of (η, A). □

If I and J are not disjoint in Theorem 3.11, then Theorem 3.11 is not true in general as seen in the following example.

Example 3.12. Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ be the semiring of integers module 6. Let (η, A) be a soft set over Z_6, where $A = Z_6$ and $\eta : A \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\eta(x) = \{y \in Z_6 \mid x\,\rho\,y \Leftrightarrow xy \in \{0, 2, 4\}\}$$

for all $x \in A$. Then as shown in Example 3.9, (η, A) is a soft semiring over Z_6.

Let (α, I) be a soft set over Z_6, where $I = \{2, 3, 4, 5\}$ and $\alpha : I \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\alpha(x) = \{y \in Z_6 \mid x\,\rho\,y \Leftrightarrow xy = 0\}$$

for all $x \in I$. Then $\alpha(2) = \{0, 3\} \lhd Z_6 = \eta(2)$, $\alpha(3) = \{0, 2, 4\} \lhd \{0, 2, 4\} = \eta(3)$, $\alpha(4) = \{0, 3\} \lhd Z_6 = \eta(4)$, and $\alpha(5) = \{0\} \lhd \{0, 2, 4\} = \eta(5)$, and so (α, I) is a soft ideal of (η, A). Let (β, J) be a soft set over Z_6, where $J = \{4\}$ and $\beta : J \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\beta(x) = \{0\} \cup \{y \in Z_6 \mid x + y \in \{0, 2\}\}$$

for all $x \in J$. Then $\beta(4) = \{0, 2, 4\} \lhd Z_6 = \eta(4)$, and so (β, J) is a soft ideal of (η, A). Nevertheless, $(\xi, U) = (\alpha, I)\,\tilde{\cup}\,(\beta, J)$ is not a soft ideal of (η, A) since $\xi(4) = \alpha(4) \cup \beta(4) = \{0, 2, 3, 4\}$ is not an ideal of $\eta(4)$ for $3 + 4 = 1 \notin \xi(4)$.

Theorem 3.13. *Let (η, A) be a soft semiring over S and $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft ideals of (η, A). Then we have the following:*

(1) *If both $\tilde{\wedge}_{i\in I}(\eta, A)$ and $\tilde{\wedge}_{i\in I}(\alpha_i, A_i)$ are non-null, then $\tilde{\wedge}_{i\in I}(\alpha_i, A_i)$ is a soft ideal of the soft semiring $\tilde{\wedge}_{i\in I}(\eta, A)$.*
(2) *If $\{A_i \mid i \in I\}$ are pairwise disjoint, i.e., $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\tilde{\bigcup}_{i\in I}(\alpha_i, A_i)$ is a soft ideal of (η, A).*
(3) *$\tilde{\sqcap}_{i\in I}(\alpha_i, A_i)$ is a soft ideal of (η, A) if it is non-null.*

Proof. (1) By Definition 3.6, we can write $\tilde{\wedge}_{i\in I}(\eta, A) = (\beta, B)$, where $B = \prod_{i\in I} A$ and $\beta(x) = \bigcap_{i\in I} \eta(x_i)$ for all $x = (x_i)_{i\in I} \in B$. Similarly, let $\tilde{\wedge}_{i\in I}(\alpha_i, A_i) = (\gamma, C)$, where $C = \prod_{i\in I} A_i$, and $\gamma(x) = \bigcap_{i\in I} \alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in C$. Suppose that the soft sets (β, B) and (γ, C) are non-null. Then by Theorem 3.7 (1), we have that (β, B) is a soft semiring over S. It remains to show that the non-null soft set (γ, C) is a soft ideal of the soft semiring (β, B). In fact, note first that $C \subset B$ since $A_i \subset A$ for all $i \in I$. Moreover, if $x = (x_i)_{i\in I} \in \mathrm{Supp}(\gamma, C)$, then $\gamma(x) = \bigcap_{i\in I} \alpha_i(x_i) \neq \emptyset$. Since (α_i, A_i) is a soft ideal of (η, A) for all $i \in I$, we deduce that the nonempty set $\alpha_i(x_i)$ is an ideal of $\eta(x_i)$ for all $i \in I$. It follows that $\gamma(x) = \bigcap_{i\in I} \alpha_i(x_i)$ is an ideal of $\beta(x) = \bigcap_{i\in I} \eta(x_i)$ for all $x = (x_i)_{i\in I} \in \mathrm{Supp}(\gamma, C)$. Hence we conclude that $\tilde{\wedge}_{i\in I}(\alpha_i, A_i) = (\gamma, C)$ is a soft ideal of the soft semiring $\tilde{\wedge}_{i\in I}(\eta, A) = (\beta, B)$.

The proofs of (2) and (3) are similar to those of the corresponding parts of Theorem 3.7. □

4. Idealistic soft semirings

Definition 4.1. Let (η, A) be a non-null soft set over S. Then (η, A) is called an *idealistic soft semiring* over S if $\eta(x)$ is an ideal of S for all $x \in \mathrm{Supp}(\eta, A)$.

Example 4.2. Consider the product semiring $S := R \times \mathbb{Z}$, where R is a semiring with zero and $\mathbb{Z}$ is the semiring of integers. Let $\eta : S \to \mathscr{P}(S)$ be a set-valued function defined as follows:

$$\eta(y, n) := \begin{cases} R \times n\mathbb{Z} & \text{if } n \in \mathbb{N}, \\ \{(0, 0)\} & \text{otherwise}, \end{cases} \tag{1}$$

for all $(y, n) \in S$, where $\mathbb{N}$ is the set of all nonnegative integers. Then (η, S) is an idealistic soft semiring over S.

Since every ideal of a semiring S is a subsemiring of S, we know that every idealistic soft semiring over S is a soft semiring over S. Nevertheless, the following example shows that the converse is not true in general.

Example 4.3. Let $S = \{0, a, b, c\}$ be a semiring with the operation tables given in Table 1. Let $A = S$ and let $\eta : A \to \mathscr{P}(S)$ be a set-valued function defined as follows:

$$\eta(x) = \{y \in S \mid x\,\rho\,y \Leftrightarrow y = x^n,\ \text{for some } n \in \mathbb{N}\}$$

for all $x \in A$. Here $x^n = xx\cdots x$ means the n-fold product of x, and $x^0 = 0$. Then $\eta(0) = \{0\}$, $\eta(a) = \{0, a\}$, $\eta(b) = \{0, b\}$ and $\eta(c) = \{0, c\}$, which are all subsemirings of S. Hence (η, A) is a soft semiring over S. But $\eta(c) = \{0, c\}$ is not an ideal of S since $cb = b \notin \eta(c)$. Therefore (η, A) is not an idealistic soft semiring over S.

Table 1
The operation tables of the semiring S

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Proposition 4.4. *Let (η, A) be a soft set over a semiring S and let $B \subset A$. If (η, A) is an idealistic soft semiring over S, then so is (η, B) whenever it is non-null.*

Proof. Straightforward. $\square$

The converse of Proposition 4.4 is not true in general as seen in the following example.

Example 4.5. Let (η, A) be the soft set given in Example 4.3. Note that (η, A) is not an idealistic soft semiring over S. But if we take $B = \{0, a, b\} \subset A$, then $(\eta\,|_B, B)$ is an idealistic soft semiring over S.

Theorem 4.6. *Let (α, A) and (β, B) be idealistic soft semirings over S. Then $(\alpha, A)\,\widetilde{\sqcap}\,(\beta, B)$ is an idealistic soft semiring over S if it is non-null.*

Proof. By Definition 2.3, we can write $(\alpha, A)\,\widetilde{\sqcap}\,(\beta, B) = (\gamma, C)$, where $C = A \cap B$ and $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$. Suppose that (γ, C) is a non-null soft set over S. If $x \in \mathrm{Supp}\,(\gamma, C)$, then $\gamma(x) = \alpha(x) \cap \beta(x) \neq \emptyset$. Thus the nonempty sets $\alpha(x)$ and $\beta(x)$ are both ideals of S. It follows that $\gamma(x)$ is an ideal of S for all $x \in \mathrm{Supp}\,(\gamma, C)$. Hence $(\gamma, C) = (\alpha, A)\,\widetilde{\sqcap}\,(\beta, B)$ is an idealistic soft semiring over S. $\square$

Theorem 4.7. *Let (η, A) and (γ, B) be idealistic soft semirings over S. If A and B are disjoint, then the union $(\eta, A)\,\widetilde{\cup}\,(\gamma, B)$ is an idealistic soft semiring over S.*

Proof. Using Definition 2.4, we can write $(\eta, A)\,\widetilde{\cup}\,(\gamma, B) = (\vartheta, C)$, where $C = A \cup B$ and for every $x \in C$,

$$\vartheta(e) = \begin{cases} \eta(x) & \text{if } x \in A \setminus B, \\ \gamma(x) & \text{if } x \in B \setminus A, \\ \eta(x) \cup \gamma(x) & \text{if } x \in A \cap B. \end{cases}$$

Suppose that $A \cap B = \emptyset$. Then for every $x \in \mathrm{Supp}\,(\vartheta, C)$, we know that either $x \in A \setminus B$ or $x \in B \setminus A$. If $x \in A \setminus B$, then $\vartheta(x) = \eta(x)$ is an ideal of S since (η, A) is an idealistic soft semiring over S. If $x \in B \setminus A$, then $\vartheta(x) = \gamma(x)$ is an ideal of S since (γ, B) is an idealistic soft semiring over S. Thus we conclude that $\vartheta(x)$ is an ideal of S for all $x \in \mathrm{Supp}\,(\vartheta, C)$. Hence $(\vartheta, C) = (\eta, A)\,\widetilde{\cup}\,(\gamma, B)$ is an idealistic soft semiring over S. $\square$

If A and B are not disjoint in Theorem 4.7, then Theorem 4.7 is not true in general as seen in the following example.

Example 4.8. Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ be the semiring of integers module 6. Let (η, A) be a soft set over Z_6, where $A = Z_6$ and $\eta : A \to \mathscr{P}(Z_6)$ is a set-valued function defined by

$$\eta(x) = \{y \in Z_6 \mid x\,\rho\,y \Leftrightarrow xy = 0\}$$

for all $x \in A$. Then $\eta(0) = Z_6$, $\eta(1) = \{0\}$, $\eta(2) = \{0, 3\}$, $\eta(3) = \{0, 2, 4\}$, $\eta(4) = \{0, 3\}$ and $\eta(5) = \{0\}$ are ideals of Z_6. Hence (η, A) is an idealistic soft semiring over Z_6. Let (γ, B) be a soft set over Z_6, where $B = \{4\}$ and $\gamma : B \to \mathscr{P}(X)$ is a set-valued function defined by

$$\gamma(x) = \{0\} \cup \{y \in Z_6 \mid x + y \in \{0, 2\}\}$$

for all $x \in B$. Then $\gamma(4) = \{0, 2, 4\}$ is an ideal of Z_6 and so (γ, B) is an idealistic soft semiring over Z_6. However, $(\xi, U) = (\eta, A)\,\widetilde{\cup}\,(\gamma, B)$ is not an idealistic soft semiring over Z_6 since $\xi(4) = \eta(4) \cup \gamma(4) = \{0, 2, 3, 4\}$ is not an ideal of Z_6 for $3 + 4 = 1 \notin \xi(4)$.

Theorem 4.9. *Let (η, A) and (γ, B) be idealistic soft semirings over S. Then $(\eta, A)\,\widetilde{\wedge}\,(\gamma, B)$ is an idealistic soft semiring over S if it is non-null.*

Table 2
The operation tables of the semiring S

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	0	0	b
b	0	0	0	0
c	0	b	0	a

Proof. By Definition 2.6, we can write $(\alpha, A)\,\widetilde{\wedge}\,(\beta, B) = (\gamma, C)$, where $C = A \times B$ and $\gamma(x, y) = \alpha(x) \cap \beta(y)$ for all $(x, y) \in C$. Suppose that (γ, C) is a non-null soft set over S. If $(x, y) \in \mathrm{Supp}\,(\gamma, C)$, then $\gamma(x, y) = \alpha(x) \cap \beta(y) \neq \emptyset$. Since (η, A) and (γ, B) are idealistic soft semirings over S, we deduce that the nonempty sets $\alpha(x)$ and $\beta(y)$ are both ideals of S. Hence $\gamma(x, y)$ is an ideal of S for all $(x, y) \in \mathrm{Supp}\,(\gamma, C)$, and so we conclude that $(\gamma, C) = (\alpha, A)\,\widetilde{\wedge}\,(\beta, B)$ is an idealistic soft semiring over S. □

Definition 4.10. An idealistic soft semiring (η, A) over a semiring S with zero is said to be *trivial* if $\eta(x) = \{0\}$ for all $x \in A$. An idealistic soft semiring (η, A) over S is said to be *whole* if $\eta(x) = S$ for all $x \in A$.

Example 4.11. Let $S = \{0, a, b, c\}$ be a semiring with the operation tables given in Table 2. For $A = S$, let $\eta : A \to \mathscr{P}(S)$ be a set-valued function defined by

$$\eta(x) = \{y \in S \mid x\,\rho\,y \Leftrightarrow xy \in \{0, a, b\}\}$$

for all $x \in A$. Then $\eta(x) = S$ for all $x \in A$, and so (η, A) is a whole idealistic soft semiring over S.

Let (η, A) be a soft set over a semiring R and let $f : R \to S$ be a mapping of semirings. Then we can define a soft set $(f(\eta), A)$ over S, where $f(\eta) : A \to \mathscr{P}(S)$ is given by $f(\eta)(x) = f(\eta(x))$ for all $x \in A$. By definition, it is easy to see that $\mathrm{Supp}\,(f(\eta), A) = \mathrm{Supp}\,(\eta, A)$.

Lemma 4.12. *Let $f : R \to S$ be an epimorphism of semirings. If (η, A) is an idealistic soft semiring over R, then $(f(\eta), A)$ is an idealistic soft semiring over S.*

Proof. Note first that $(f(\eta), A)$ is a non-null soft set over S since (η, A) is an idealistic soft semiring over R, which is a non-null soft set by definition. For every $x \in \mathrm{Supp}\,(f(\eta), A)$, we have $f(\eta)(x) = f(\eta(x)) \neq \emptyset$. Then the nonempty set $\eta(x)$ is an ideal of R, and so we deduce that its onto homomorphic image $f(\eta(x))$ is an ideal of S. Hence $f(\eta(x))$ is an ideal of S for every $x \in \mathrm{Supp}\,(f(\eta), A)$. That is, $(f(\eta), A)$ is an idealistic soft semiring over S as required. □

Theorem 4.13. *Let (η, A) be an idealistic soft semiring over a semiring R and let $f : R \to S$ be an epimorphism of semirings.*

(1) *If S is a semiring with zero and $\eta(x) = \ker(f)$ for all $x \in A$, then $(f(\eta), A)$ is the trivial idealistic soft semiring over S.*
(2) *If (η, A) is whole, then $(f(\eta), A)$ is the whole idealistic soft semiring over S.*

Proof. (1) Assume that $\eta(x) = \ker(f)$ for all $x \in A$. Then $f(\eta)(x) = f(\eta(x)) = \{0_S\}$ for all $x \in A$. Hence $(f(\eta), A)$ is the trivial idealistic soft semiring over S by Lemma 4.12 and Definition 4.10.

(2) Suppose that (η, A) is whole. Then $\eta(x) = R$ for all $x \in A$, and so $f(\eta)(x) = f(\eta(x)) = f(R) = S$ for all $x \in A$. It follows from Lemma 4.12 and Definition 4.10 that $(f(\eta), A)$ is the whole idealistic soft semiring over S. □

Definition 4.14. Let (η, A) and (γ, B) be soft semirings over two semirings R and S, respectively. Let $f : R \to S$ and $g : A \to B$ be two mappings. Then the pair (f, g) is called a *soft semiring homomorphism* if it satisfies the following conditions:

(i) f is an epimorphism of semirings.
(ii) g is a surjective mapping.
(iii) $f(\eta(x)) = \gamma(g(x))$ for all $x \in A$.

If there exists a soft semiring homomorphism between (η, A) and (γ, B), we say that (η, A) is *soft homomorphic* to (γ, B), which is denoted by $(\eta, A) \sim (\gamma, B)$. Moreover, if f is an isomorphism of semirings and g is a bijective mapping, then (f, g) is called a *soft semiring isomorphism*. In this case, we say that (η, A) is *soft isomorphic* to (γ, B), which is denoted by $(\eta, A) \simeq (\gamma, B)$.

Example 4.15. Denote by $\mathbb{Z}$ and Z_n the semiring of integers and the semiring of integers module (a positive integer) n, respectively. Let $f : \mathbb{Z} \to Z_n$ be the natural mapping defined by $f(x) = [x]$ for all $x \in \mathbb{Z}$. Evidently, f is an epimorphism of semirings. Let $\mathbb{Z}^+$ be the set of positive integers and define a mapping $g : \mathbb{Z}^+ \to Z_n$ by $g(x) = [x]$ for all $x \in \mathbb{Z}^+$. Then it is easy to see that the mapping g is surjective. Let $(\alpha, \mathbb{Z}^+)$ be a soft set over $\mathbb{Z}$, where $\alpha : \mathbb{Z}^+ \to \mathscr{P}(\mathbb{Z})$ is a set-valued function defined by $\alpha(x) = \{3xk \mid k \in \mathbb{Z}\}$ for all $x \in \mathbb{Z}^+$. One easily verifies that $\alpha(x) = 3x\mathbb{Z}$ is a subsemirings of $\mathbb{Z}$ for all $x \in \mathbb{Z}^+$. Thus $(\alpha, \mathbb{Z}^+)$ is a soft semiring over $\mathbb{Z}$. Let (β, Z_n) be a soft set over Z_n, where $\beta : Z_n \to \mathscr{P}(Z_n)$ is a set-valued function given by $\beta([x]) = \{[3xk] \mid k \in \mathbb{Z}\}$ for all $[x] \in Z_n$. Then one can also prove that (β, Z_n) is a soft semiring over Z_n. Moreover, since $f(\alpha(x)) = f(3x\mathbb{Z}) = \{[3xk] \mid k \in \mathbb{Z}\}$ and $\beta(g(x)) = \beta([x]) = \{[3xk] \mid k \in \mathbb{Z}\}$ for all $x \in \mathbb{Z}^+$, we deduce that $f(\alpha(x)) = \beta(g(x))$ for all $x \in \mathbb{Z}^+$. Hence (f, g) is a soft semiring homomorphism and $(\alpha, \mathbb{Z}^+) \sim (\beta, Z_n)$.

5. Conclusion

Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. We consider in the present paper the algebraic structure of semirings in a setting of soft set theory. We define the notion of a soft semiring and focus on the algebraic properties of soft semirings. We introduce some related notions such as soft ideals, idealistic soft semirings and soft semiring homomorphisms, with illustrating examples. We also investigate some basic properties of these concepts by using soft set theory. Based on these results, we could apply soft sets to other types of ideals in semirings, and do some further work on the properties of soft semirings, which may be useful to characterize the classical semirings.

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