

Sixth International Conference on Sensitivity Analysis of Model Output

Sensitivity Analysis in Stochastic Second Order Cone Programming for Mobile Ad Hoc Networks

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Abstract

In this paper sensitivity analysis is adopted in order to understand the randomness of a stochastic second order cone program for mobile ad hoc networks [3]. The algorithm looks for a destination node and sets up a route by means of the *expected zone*, the region where the sender node expects to find the destination node and the *requested zone* defined by the sender node for spreading the route request to the destination node. Sensitivity analysis is performed by considering different costs of flooding and latency penalty. Evaluation of EVPI and VSS [2]-[4] allows us to find the range of values in which it is safe to save time by using a deterministic approach instead of a stochastic one.

Keywords: sensitivity analysis, stochastic programming, mobile ad hoc network, second order cone programming, evaluation of deterministic solution.

1. Main text

Sensitivity analysis is adopted by researchers in order to study the robustness of solutions of deterministic problems, that is, to see how it might change if the data were different. A change in the solution or its structure would indicate the need for further investigation. In the case neither changes, the proposed solution can be considered as an appropriate guide for making decision. However, when the decision has to be made under *uncertainty*, the solution given by sensitivity analysis could no longer be appropriate: it reflects deterministic properties of the model and cannot capture the possibility of a response to information at times at which decisions are made (see [5] and [6], the latter with a worked-out example).

Nevertheless, there could be stochastic programs where uncertainty is crucial for the problem, but actually not revealed by the solutions, which are the same for all possible values of the random parameters. Even so, the objective function value may be very dependent by the scenarios considered. In such a case, sensitivity analysis techniques are useful to understand the stochasticity of the problem, that is, to find the range for parameters within

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which the solutions are scenarios dependent. This is the case considered in this paper where sensitivity analysis is applied to a stochastic second order cone program for mobile ad hoc networks [3].

The algorithm considered, known as Stochastic Location-Aided Routing (SLAR) [1], uses a procedure to search for a destination node D and set up a route. The main concepts behind the algorithm are the *expected zone* and *requested zone* inside a network of wireless mobile nodes. The former is the region in the shape of a circle, where the sender node S expects to find the destination node D after an elapsed time, while the latter is the region defined by the sender node for spreading the route request to the destination node. The movements of D are represented by ellipsoid scenarios, randomly generated by uniform and normal distribution in a neighborhood of the starting position of the destination node. A stochastic second order cone model allows us to solve problems with a much larger number of scenarios (20250) than what is possible with the semidefinite model [1].

Sensitivity analysis is then performed by considering different costs of flooding and making corrections on the radius of the request zone. A larger expected zone will be picked up for a smaller flooding cost coefficient and a smaller one in the opposite case in order to minimize the number of nodes involved in the flooding. On the other hand, a higher cost on the radius of the requested zone puts more penalty on the route discovery latency indicating that it is preferable to find node D in the first stage. As a consequence the expected zone corresponds to the largest area already containing all the ellipsoid scenarios and the associated *Expected Value of Perfect Information* EVPI [2] stabilizes to a plateau region because of the rejection of the uploading at the second stage.

The convex behaviour of the *Value of the Stochastic Solution* VSS [2] against the latency penalty shows the range of values in which it is safe to save time by solving the deterministic *mean value problem* (where stochastic parameters are replaced by their mean values) instead of considering a complex stochastic structure [3].

The results obtained show that also in a stochastic context sensitivity analysis represents a useful tool for a deeper understanding of the problem considered and to reveal the randomness of the model hidden in the wrong choice of parameters.

2. References

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