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Improving bounds on γ in $B^\pm \to DK^\pm$ and $B^{\pm,0} \to DX_s^{\pm,0}$

Michael Gronau [1]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

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Abstract

In view of recent experimental progress in rate and CP asymmetry measurements in $B^\pm \to DK^\pm$, we reconsider information on the weak phase γ which can be obtained from these processes. Model-independent inequalities are proven for $\sin^2\gamma$ in terms of two ratios of partial rates for $B^{\pm,0} \to DX_s^{\pm,0}$, where X_s is any multiparticle charmless state carrying strangeness ± 1. Good prospects are shown to exist for using these inequalities and CP asymmetry measurements in two body and multibody decays in order to improve present bounds on γ.

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The observation of CP violation in decays of B mesons to J/ψ and neutral kaons [1] is in good agreement with the prediction of the Standard Model, in which CP violation originates in a single phase $\gamma \equiv \mathrm{Arg}\, V_{ub}^*$ of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Further measurements of CP asymmetries in other B decay processes are needed in order to establish the CKM hypothesis for CP violation on a firm ground, or to observe deviations from this simple picture. So far CP violation in B decays was observed only in processes involving B^0–$\overline{B}^0$ mixing, whereas the phase γ has not yet been put to a direct test in B decays. It is therefore of great importance to search for *direct* CP violation in processes unaffected by uncertainties due to penguin amplitudes [2], where CP asymmetries have clean theoretical interpretations in terms of the weak phase γ.

One of the very early proposals for a clean measurement of γ is based on decays of the type $B^\pm \to DX_s^\pm$ [3], where $X_s^\pm$ stands for a charged kaon or *any* few particle state with the same flavor quantum numbers as a charged kaon, e.g., $X_s = K, K^*, K\pi, K^*\pi$. The weak phase γ occurs as the relative phase between two B^- decay amplitudes into D^0 and $\overline{D}^0$ flavor states, from $b \to c\bar{u}s$ and $b \to u\bar{c}s$, both contributing in decays to CP eigenstates, $D^0_{\mathrm{CP}\pm} = (D^0 \pm \overline{D}^0)/\sqrt{2}$. In the original proposal all three B^- decay amplitudes and a corresponding B^+ decay amplitude for a CP-eigenstate had to be measured in order to determine γ. In the simplest case of two body decays, $X_s = K$, one of the four amplitudes, $A(B^- \to \overline{D}^0 K^-)$, is color-suppressed. Its measurement using hadronic $\overline{D}^0$ decays is prohibited [4] due to interference with a comparable contribution from $B^- \to D^0 K^-$ followed by

E-mail address: michael.gronau@cern.ch (M. Gronau).

[1] Permanent address: Physics Department, Technion—Israel Institute of Technology, 32000 Haifa, Israel.

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doubly-Cabibbo-suppressed (DCS) D^0 decays. Nevertheless, it was noted in Ref. [5] that useful constraints on γ can also be obtained without measuring this difficult mode. Several variants of this basic scheme were suggested, some of which rely on hitherto unmeasured and more difficult B and D decay modes [6], and others which require extra assumptions about negligible rescattering effects [7].

The magnitudes of all five amplitudes required for an implementation of this proposal, $A(B^- \to D^0 K^-)$ and the four amplitudes $A(B^\pm \to D^0_{\mathrm{CP}\pm} K^\pm)$, have already been measured. The decay $B^- \to D^0 K^-$ and its charge-conjugate were observed several years ago [8]. Recently branching ratios for the processes $D^0_{\mathrm{CP}} K^\pm$ were measured by the Belle Collaboration [9] both for CP-even and odd states, and by the BABAR Collaboration [10] for CP-even states. CP asymmetry measurements in decays involving D^0 CP-eigenstates [9,10] are approaching a level for setting interesting bounds on the asymmetries. In addition, there exists new experimental information [11] indicating that color-suppression of the ratio

$$r \equiv \left|A\left(B^- \to \overline{D}^0 K^-\right)/A\left(B^- \to D^0 K^-\right)\right| \tag{1}$$

is less effective than anticipated. This improves the feasibility of this method.

In view of these important developments, we wish to reconsider in this Letter the implications which further improvements in these measurements will have on constraining γ. In particular, we make use of two inequalities [5]

$$\sin^2\gamma \leqslant R_{\mathrm{CP}\pm}, \tag{2}$$

where we define for each of the two CP-eigenstates a ratio of charge-averaged rates

$$R_{\mathrm{CP}\pm} \equiv \frac{2[\Gamma(B^- \to D^0_{\mathrm{CP}\pm} K^-) + \Gamma(B^+ \to D^0_{\mathrm{CP}\pm} K^+)]}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \overline{D}^0 K^+)}. \tag{3}$$

We will find that, although these two constraints do not depend explicitly on r, and do not require a knowledge of r, in general they become stronger with increasing values of this parameter. For a reasonable estimate, $r \sim 0.2$, one may encounter one of two possible situations: if the relevant final state interaction phase δ is large, then one should soon measure for the first time direct CP violation in B decays. On the other hand, if $\delta \leqslant 30^\circ$, which can be verified by improving bounds on CP asymmetries, then the above constraints improve present bounds on γ.

In the second part of the Letter we proceed to a general discussion of decays of the form $B^\pm \to D X_s^\pm$ and $B^0(\overline{B}^0) \to D X_s^0(\overline{X}_s^0)$, where $X_s^\pm$ and $X_s^0(\overline{X}_s^0)$ are arbitrary charmless multiparticle states with strangeness ± 1. We will prove a generalization of Eq. (2) to multibody decays of this type. In the absence of color-suppression in most multibody decays, which implies larger values of corresponding r parameters in these processes, these bounds are likely to provide stronger constraints on γ than in the case of two body decays. Our considerations apply to any multibody decay of this kind, for instance, $B^\pm \to D K^\pm \pi^0$ and the self-tagged $B^0(\overline{B}^0) \to D K^\pm \pi^\mp$, and are model-independent [12].

Using notations for amplitudes as in [3,5] and disregarding a common strong phase,

$$A\left(B^- \to D^0 K^-\right) = |A|, \qquad A\left(B^- \to \overline{D}^0 K^-\right) = |\bar{A}| e^{i\delta} e^{-i\gamma}, \tag{4}$$

we define in addition to the two ratios of charge-averaged rates (3) two pseudo asymmetries

$$\mathcal{A}_{\mathrm{CP}\pm} \equiv \frac{2[\Gamma(B^- \to D^0_{\mathrm{CP}\pm} K^-) - \Gamma(B^+ \to D^0_{\mathrm{CP}\pm} K^+)]}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \overline{D}^0 K^+)}, \tag{5}$$

from which ordinary CP asymmetries are obtained, $A_{\mathrm{CP}\pm} = \mathcal{A}_{\mathrm{CP}\pm}/R_{\mathrm{CP}\pm}$. Expressions of these measurables in terms of $r = |\bar{A}/A|, \delta$ and γ are readily obtained, neglecting tiny D^0–$\overline{D}^0$ mixing [13] and using $D^0_{\mathrm{CP}\pm} = (D^0 \pm \overline{D}^0)/\sqrt{2}$,

$$R_{\mathrm{CP}\pm} = 1 + r^2 \pm 2r\cos\delta\cos\gamma, \tag{6}$$

$$\mathcal{A}_{\mathrm{CP}\pm} = \pm 2r\sin\delta\sin\gamma, \tag{7}$$

where (6) implies

$$\frac{1}{2}(R_{\mathrm{CP}+} + R_{\mathrm{CP}-}) = 1 + r^2, \tag{8}$$

and

$$R_{\mathrm{CP}+} - R_{\mathrm{CP}-} = 4r\cos\delta\cos\gamma. \tag{9}$$

The quantities $R_{\mathrm{CP}\pm}$ and $\mathcal{A}_{\mathrm{CP}\pm}$ hold information from which r, δ and γ can in principle be determined. The parameter r is given by (8), and γ is obtained up to a discrete ambiguity from $R_{\mathrm{CP}\pm}$ and $\mathcal{A}_{\mathrm{CP}\pm}$,

$$R_{\mathrm{CP}\pm} = 1 + r^2 \pm \sqrt{4r^2\cos^2\gamma - \mathcal{A}^2_{\mathrm{CP}\pm}\cot^2\gamma}, \tag{10}$$

where the $\pm$ signs on the right-hand side correspond to even and odd CP states for $\cos\delta\cos\gamma > 0$.

Plots of $R_{\mathrm{CP}\pm}$ as function of γ, for a few values of r around 0.2 and asymmetries in the range of 0–30%, may be borrowed from [14] plotting analogous quantities for the processes $B^0 \to K^+\pi^-$ and $B^+ \to K^0\pi^+$, which involve a similar algebra relating γ to $B \to K\pi$ decay rates. Here one defines a ratio of charge-averaged decay rates, $R \equiv \Gamma(B \to K^\pm\pi^\mp)/\Gamma(B^\pm \to K\pi^\pm)$, which is given in terms of a pseudo asymmetry (A_0) in $B^0 \to K^+\pi^-$ and a ratio (r) of tree and penguin amplitudes. In contrast to these decays, which involve a single ratio R, in $B \to DK$ one measures two ratios for even and odd CP states. This resolves an ambiguity in the plots between $R > 1 + r^2$ and $R < 1 + r^2$ and allows for another measurable (9). The plots of $R_{\mathrm{CP}\pm}$ as function of γ can be used to study the precision in r, $R_{\mathrm{CP}\pm}$ and $\mathcal{A}_{\mathrm{CP}\pm}$ required to measure γ to a given accuracy. In our case the accuracy is seen to improve with increasing values of r due to a larger interference between $A(B^- \to D^0K^-)$ and $A(B^- \to \overline{D}^0K^-)$.

A crucial point is the actual value of r. New experimental information exists which relates to this value. Previously arguments based on naive factorization [15] seemed to imply that the amplitude $A(B^- \to \overline{D}^0K^-)$ involves a suppression factor, $|a_2/a_1| = 0.25$ [16], for the fact that the quark and antiquark making the kaon in $B^- \to \overline{D}^0K^-$ do not originate in the same weak current of the effective Hamiltonian describing $b \to u\bar{c}s$. This has led to a commonly accepted estimate $r \approx (|V_{ub}V^*_{cs}|/|V_{cb}V^*_{us}|)(|a_2/a_1|) \approx 0.1$. Recent measurements [11] of the color-suppressed process $\overline{B}^0 \to D^0\pi^0$ show, however, that color-suppression is less effective in this process than anticipated [17], implying $a_2/a_1 \simeq 0.44$. Therefore, a more reasonable estimate is

$$r \sim 0.2. \tag{11}$$

As noted in the past [5], it is difficult to associate a theoretical uncertainty with this value. While the amplitude for $\overline{B}^0 \to D^0\pi^0$ involves a $b \to c$ transition with a heavy quark in the final state, $B^- \to \overline{D}^0K^-$ follows from a $b \to u$ transition with a light quark in the final state. The different flavor structure of the two operators and the different kinematics with which the heavy and light quarks emerge from the weak interaction imply different hadronic final state interaction effects in the two cases. This is expected to result in different color-suppression factors. Thus, while we will be using the value (11) as a guide for testing the sensitivity of this method, one should not exclude different values of r.

An important task of future studies is to determine r experimentally without measuring $B^- \to \overline{D}^0K^-$ [18]. A useful lower bound on r is obtained from Eq. (9),

$$r \geqslant \frac{1}{4}|R_{\mathrm{CP}+} - R_{\mathrm{CP}-}|. \tag{12}$$

If r is as small as (11) it will be very difficult to determine its precise value from Eq. (8), since the right-hand side is quadratic in r and is expected to be only a few percent larger than one. Setting upper bounds on r would also be useful.

The information on δ and γ obtained from $\mathcal{A}_{\mathrm{CP}\pm}$ and $R_{\mathrm{CP}\pm}$ is complementary to each other. While the asymmetries become larger for large values of $\sin\delta\sin\gamma$, both the deviation of $R_{\mathrm{CP}\pm}$ from one and the difference $R_{\mathrm{CP}+} - R_{\mathrm{CP}-}$ increase with $\cos\delta\cos\gamma$. The two asymmetries (7), which are equal in magnitude and opposite in

Table 1
Upper bounds on γ (in degrees) obtained using Eqs. (6) and (14). Numbers in parentheses are corresponding maximal values of $R_{CP\pm}$ for one of the two CP-eigenstates

Input value of γ	Upper bound on γ assuming $\|\delta\| \leqslant 30°$			$\delta = 0$	$\delta = 60$
	$r = 0.1$	$r = 0.2$	$r = 0.4$	$r = 0.4$	$r = 0.4$
50	71 (0.90)	65 (0.82)	58 (0.71)	53.5 (0.65)	72 (0.90)
60	74 (0.92)	69 (0.87)	64 (0.81)	60.7 (0.76)	78 (0.96)
70	77 (0.95)	74 (0.92)	74 (0.92)	70.3 (0.89)	– (1.02)
80	82 (0.98)	82 (0.98)	– (1.04)	– (1.02)	– (1.09)

sign, may be combined to yield an overall asymmetry, $\mathcal{A}_{CP+} - \mathcal{A}_{CP-} = 4r \sin\delta \sin\gamma$. Sizable CP asymmetries $\mathcal{A}_{CP\pm}$ at a level of 20%, which could soon be observed [9,10], require a large value of $|\sin\delta|$ corresponding to $|\delta| \geqslant 30°$ (mod π). A nonzero asymmetry would be an important observation by itself, demonstrating for the first time direct CP violation in B decays. Observing nonzero CP asymmetries may, however, be difficult if δ is small. Currently there exists no information about δ. A corresponding strong phase difference between isospin amplitudes in $B \to \overline{D}\pi$ decays was measured recently [19] in the range 16°–38°. QCD considerations suggest that this final state interaction phase occurring in $b \to c$ transitions is either perturbative or power suppressed in $1/m_b$ [20]. On the other hand, the phase δ is due to $b \to u$ transitions and may be different as mentioned above.

Anticipating that bounds on CP asymmetries will soon be improved to the level of 20%, thereby setting an upper bound on $|\delta|$, we will show that new constraints on γ follow from $R_{CP\pm}$ if $|\delta|$ is assumed to be smaller than about 30°. This will be contrasted with weaker constraints in case that nonzero CP asymmetries are measured indicating larger values of $|\delta|$.

Rewriting

$$R_{CP\pm} = \sin^2\gamma + (r \pm \cos\delta\cos\gamma)^2 + \sin^2\delta\cos^2\gamma, \tag{13}$$

one obtains the two simultaneous inequalities

$$\sin^2\gamma \leqslant R_{CP\pm}. \tag{14}$$

These inequalities become useful when $R_{CP\pm} < 1$ holds for either even or odd CP states. This condition is fulfilled in a major part of the r, δ, γ parameter space because of the two opposite signs of the last term in (6). The condition is equivalent to a rather weak requirement, $|\cos\delta\cos\gamma| > r/2$. Namely, Eq. (14) imply nontrivial constraints on γ when neither γ nor δ lies too close to $\pi/2$. Since we are assuming this to be true for the strong phase δ, Eq. (14) provides useful bounds on γ for values different from $\pi/2$. We note that, while the bounds (14) themselves are not too useful when γ is near $\pi/2$, such values of γ can be tested and excluded by measuring $R_{CP+} - R_{CP-} = 4r\cos\delta\cos\gamma$.

The bounds on γ depend on the value of r. Since we are assuming the CKM framework, we disregard a discrete ambiguity in γ, taking its value to be smaller than $\pi/2$ [21]. For illustration, we calculate in Table 1 upper bounds on γ obtained from Eqs. (6) and (14) for three values of r, $r = 0.1,\ 0.2,\ 0.4$. The value $r = 0.4$, which may be an overestimate for the case of two body decays, is a realistic value for multibody decays which we discuss below. We include it in the present discussion for a later reference. Values of $R_{CP\pm}$ which depend on γ and resulting bounds on this phase are computed for input values in the range $50° \leqslant \gamma \leqslant 80°$ permitted by CKM fits [21]. In computing the bounds for $r = 0.1,\ 0.2,\ 0.4$ in the second, third and fourth columns we assume $|\delta| \leqslant 30°$ (mod π) which can be verified by CP asymmetry measurements. The bounds in the fifth and sixth columns correspond to $\delta = 0$ and $\delta = 60°$, respectively. Also given in parentheses in the second, third and fourth columns are maximal values of $R_{CP\pm}$ for one of the two CP-eigenstates, which are obtained for $\delta_{max} = 30°$ (mod π). Smaller values of $R_{CP\pm}$, and corresponding stronger upper bounds on γ, are obtained for $\delta = 0$ as shown in the fifth column. On the other hand, the bounds become weaker when δ is large, as demonstrated for $\delta = 60°$ in the last column. In this case one should soon observe a CP asymmetry in $B^\pm \to D^0_{CP\pm}K^\pm$.

We note that as r increases the bounds on γ become stronger. For $r = 0.2$, as estimated in (11), and assuming $|\delta| \leqslant 30^\circ \pmod{\pi}$, the upper limits on γ already provide useful information on the weak phase beyond CKM fits. For instance, for $\gamma = 50^\circ$ the deviation of the lower $R_{\mathrm{CP}\pm}$ value from one is substantial, implying $\gamma \leqslant 65^\circ$. The corresponding difference $|R_{\mathrm{CP}+} - R_{\mathrm{CP}-}| = 0.45$ is quite large. For $r = 0.4$ the limits are rather close to the input values of γ, in particular, for $\delta = 0 \pmod{\pi}$. If the actual value of γ is near 50° then the bound fixes the weak phase to a very narrow range of several degrees. In this case the values of $|R_{\mathrm{CP}+} - R_{\mathrm{CP}-}|$ are 0.89 and 1.03, corresponding to $\delta = 30^\circ \pmod{\pi}$ and $\delta = 0 \pmod{\pi}$, respectively. One notes that, while bounds on CP asymmetries do not distinguish between the two possibilities that δ is near zero or near π, the former case seems to be favored by theory [20]. Therefore, one expects $R_{\mathrm{CP}+} > R_{\mathrm{CP}-}$. We conclude that, although r may not be determined accurately experimentally, there exist good prospects, in terms of reasonable values of r and δ which is assumed not to be too large, for improving present bounds on γ using measured values of $R_{\mathrm{CP}\pm}$.

The small value of $r \equiv |A(B^- \to \overline{D}{}^0 K^-)/A(B^- \to D^0 K^-)|$ in (11) follows from color-suppression in the two body decay $B^- \to \overline{D}{}^0 K^-$ on top of a modest CKM-suppression. Color suppression arguments are unknown to hold in practice in multibody decays, and furthermore do not apply formally to most multibody decays of the type $B^-(\overline{B}{}^0) \to \overline{D}{}^0 X_s^{-,0}$.[2] For instance, the processes $B^- \to \overline{D}{}^0 K^- \pi^0$, $B^- \to \overline{D}{}^0 K^- \pi^+ \pi^-$ and the self-tagged $\overline{B}{}^0 \to \overline{D}{}^0 K^- \pi^+$ involve the same color factors as the corresponding processes with D^0 in the final state. Thus, disregarding a possible suppression due to form factors or other dynamical factors, one expects the corresponding ratio of $\overline{D}{}^0$ and D^0 amplitudes in most multibody $B \to D X_s$ decays to be larger than in two body decays. As we saw now, the two inequalities (14) become stronger as r increases. An interesting question is therefore whether inequalities similar to (14) hold also in multibody decays. If this were the case, then one would be able to apply such inequalities to these decays in order to obtain stronger constraints on γ. In the remaining part of this Letter we will prove that such inequalities hold in general and we will study their consequences.

In a multibody decay of the type $B^- \to D X_s^-$, and in a similar neutral B decay $\overline{B}{}^0 \to D X_s^0$, one may generalize Eq. (4) to hold at any point p in the multibody phase space,

$$A\big(B^- \to \big(D^0 X_s^-\big)_p\big) = A_p, \qquad A\big(B^- \to \big(\overline{D}{}^0 X_s^-\big)_p\big) = \bar{A}_p e^{-i\gamma}, \tag{15}$$

and consequently

$$A\big(B^- \to \big(D^0_{\mathrm{CP}\pm} X_s^-\big)_p\big) = \frac{1}{\sqrt{2}}\big(A_p \pm \bar{A}_p e^{-i\gamma}\big), \tag{16}$$

where A_p and $\bar{A}_p$ are complex amplitudes involving final state interaction phases which depend on the point p in phase space. For instance, in $B^- \to D K^- \pi^0$ p is a point in a Dalitz plot and the magnitudes and complex phases of A_p and $\bar{A}_p$, which depend on resonance structures in the two channels, vary from one point to another. This seems to pose a serious problem in applying the method [3] or any of its variants to multibody decays in order to determine γ. Any such attempt would be strongly model-dependent, since it requires modeling the amplitudes A_p and $\bar{A}_p$ as functions of p in terms of assumed resonance structures in the two channels. For a very recent attempt, see Ref. [12]. Our following considerations are, however, model-independent.

Let us consider partial rates for the four processes in Eqs. (15) and (16),

$$\Gamma\big(B^- \to D^0 X_s^-\big) = \int dp\, |A_p|^2, \qquad \Gamma\big(B^- \to \overline{D}{}^0 X_s^-\big) = \int dp\, |\bar{A}_p|^2, \tag{17}$$

$$\Gamma\big(B^- \to D^0_{\mathrm{CP}\pm} X_s^-\big) = \frac{1}{2}\left(\int dp\, |A_p|^2 + \int dp\, |\bar{A}_p|^2\right) \pm \int dp\, \mathrm{Re}\big(A_p \bar{A}_p^* e^{i\gamma}\big), \tag{18}$$

[2] Color factors in multibody decays are inferred formally from quark diagrams in which a light $q\bar{q}$ pair implies an extra factor of N_c. In the special case of $B^- \to \overline{D}{}^0 K_S \pi^-$ the amplitude obtains only a color-suppressed contribution.

where integration over phase space may be either complete or partial. The corresponding B^+ decay rates for decays to CP-eigenstates are obtained by changing the sign of γ in Eq. (18). Defining ratios of partial rates and pseudo-asymmetries,

$$r_s^2 \equiv \frac{\Gamma(B^- \to \overline{D}^0 X_s^-)}{\Gamma(B^- \to D^0 X_s^-)}, \tag{19}$$

$$R_{\mathrm{CP}\pm}(X_s) \equiv \frac{2[\Gamma(B^- \to D^0_{\mathrm{CP}\pm} X_s^-) + \Gamma(B^+ \to D^0_{\mathrm{CP}\pm} X_s^+)]}{\Gamma(B^- \to D^0 X_s^-) + \Gamma(B^+ \to \overline{D}^0 X_s^+)}, \tag{20}$$

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) \equiv \frac{2[\Gamma(B^- \to D^0_{\mathrm{CP}\pm} X_s^-) - \Gamma(B^+ \to D^0_{\mathrm{CP}\pm} X_s^+)]}{\Gamma(B^- \to D^0 X_s^-) + \Gamma(B^+ \to \overline{D}^0 X_s^+)}, \tag{21}$$

we find

$$R_{\mathrm{CP}\pm}(X_s) = 1 + r_s^2 \pm 2\cos\gamma \frac{\mathrm{Re}(\int dp\, A_p \bar{A}_p^*)}{\int dp\, |A_p|^2}, \tag{22}$$

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) = \pm 2\sin\gamma \frac{\mathrm{Im}(\int dp\, A_p \bar{A}_p^*)}{\int dp\, |A_p|^2}. \tag{23}$$

Denoting

$$\kappa e^{i\delta_s} \equiv \frac{\int dp\, A_p \bar{A}_p^*}{\sqrt{\int dp\, |A_p|^2 \int dp\, |\bar{A}_p|^2}}, \tag{24}$$

where the Schwarz inequality for the two complex vectors A_p and $\bar{A}_p$ implies $0 \leqslant \kappa \leqslant 1$, one obtains

$$R_{\mathrm{CP}\pm}(X_s) = 1 + r_s^2 \pm 2\kappa r_s \cos\delta_s \cos\gamma, \tag{25}$$

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) = \pm 2\kappa r_s \sin\delta_s \sin\gamma. \tag{26}$$

Eq. (25) leads immediately to

$$\sin^2\gamma \leqslant R_{\mathrm{CP}\pm}(X_s). \tag{27}$$

The expressions of $R_{\mathrm{CP}\pm}(X_s)$ and $\mathcal{A}_{\mathrm{CP}\pm}(X_s)$ become identical to those of two body decays for $\kappa = 1$, namely, when A_p and $\bar{A}_p$ are parallel (i.e., proportional) to each other. This is the case in which rate and asymmetry measurements are most sensitive to the weak phase γ. An extreme and unfavorable case, $\kappa = 0$, occurs when A_p and $\bar{A}_p$ are orthogonal to each other. An upper bound on κ, which is saturated when relative phases between A_p and $\bar{A}_p$ vanish and which becomes weak when these phases are large, can be expressed in terms of measurable differential rates,

$$\kappa \leqslant \frac{\int dp\, |A_p||\bar{A}_p|}{\sqrt{\int dp\, |A_p|^2 \int dp\, |\bar{A}_p|^2}}. \tag{28}$$

Eqs. (25), (26) and the bound (27) are quite powerful. They apply to any multibody decay of the type under discussion and to an arbitrary choice of phase space over which one integrates. Their advantage over the corresponding relations in two body decays is threefold:

1. Multibody decays are expected to have larger branching ratios than two body decays.
2. Since most multibody decays of the type $B^- \to \overline{D}^0 X_s^-$ and $\overline{B}^0 \to \overline{D}^0 X_s^0$ are not color-suppressed, their measurements using hadronic $\overline{D}^0$ decays are less affected by interference with doubly Cabibbo-suppressed D^0 decays in $B^- \to D^0 X_s^-$ and $\overline{B}^0 \to D^0 X_s^0$, respectively. This allows a reasonably accurate direct measurement of r_s in these processes.

3. In general the parameter r_s is expected to be larger than r which is color-suppressed. A typical estimate, based only on CKM factors, is $r_s \approx |V_{ub}V^*_{cs}|/|V_{cb}V^*_{us}| \approx 0.4$. One expects that in certain decays and in specific regions of phase space the value of r_s may be larger than 0.4 due to a dynamical enhancement of $\bar{A}_p$ relative to A_p. The sensitivity to γ grows with κr_s, where there exists no direct measurement for κ except the upper bound (28). One may choose judiciously, or by scanning over different regions of phase space, regions which minimize the lower of the two $R_{\mathrm{CP}\pm}(X_s)$ values. This would correspond to maximizing the value of κr_s while keeping the phase δ_s as small as possible. On the other hand, large CP asymmetries correspond to large values of δ_s. For instance, in $B \to DK\pi$ the phase δ_s is expected to become small as one moves away from K and $D(D_s)$ resonance states, and to increase as one approaches the resonance bands [12,22]. Upper bounds on γ were calculated in the fourth and fifth columns of Table 1 for $\kappa = 1, r_s = 0.4, \delta_s \leqslant 30^\circ$ and $\delta_s = 0$, and were shown to be considerably stronger than for $r = 0.2$. They may correspond approximately to realistic situations in multibody decays. The strongest bounds on γ are obtained when applying $R_{\mathrm{CP}\pm}(X_s)$ and $A_{\mathrm{CP}\pm}(X_s)$ to regions of phase space in which A_p and $\bar{A}_p$ are proportional to each other, corresponding to $\kappa = 1$. In this case an algebraic solution for γ can be obtained as shown in Eq. (10).

Before concluding, let us comment on the measurement of $R_{\mathrm{CP}\pm}$ upon which the proposed bounds on γ depend. The ratios $R_{\mathrm{CP}\pm}$ involve B decay rates into CP and flavor states of neutral D mesons. They are measured by observing D^0 decay modes involving even CP (K^+K^-, $\pi^+\pi^-$), odd CP ($K_S\pi^0$, $K_S\phi$, $K_S\omega$, $K_S\rho$, $K_S\eta$, $K_S\eta'$), and flavor states ($K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^+\pi^-\pi^0$). It may seem that accurate measurements of $R_{\mathrm{CP}\pm}$ require precise knowledge of D^0 decay branching ratios into these states, which is the current situation in some decay modes but not in all. There exists, however, a way in which $R_{\mathrm{CP}\pm}$ may be measured independent of D^0 decay branching ratios. Let us define two ratios which do not depend on D^0 decay branching ratios,

$$R(K/\pi) \equiv \frac{\mathcal{B}(B^- \to D^0K^-)}{\mathcal{B}(B^- \to D^0\pi^-)}, \tag{29}$$

$$R(K/\pi)_{\mathrm{CP}\pm} \equiv \frac{\mathcal{B}(B^- \to D^0_{\mathrm{CP}\pm}K^-) + \mathcal{B}(B^+ \to D^0_{\mathrm{CP}\pm}K^+)}{\mathcal{B}(B^- \to D^0_{\mathrm{CP}\pm}\pi^-) + \mathcal{B}(B^+ \to D^0_{\mathrm{CP}\pm}\pi^+)}. \tag{30}$$

Using

$$A\big(B^+ \to D^0_{\mathrm{CP}\pm}\pi^+\big) \approx A\big(B^- \to D^0_{\mathrm{CP}\pm}\pi^-\big) \approx \frac{1}{\sqrt{2}}A\big(B^- \to D^0\pi^-\big), \tag{31}$$

where one neglects a term $(|V_{ub}V^*_{cd}/V_{cb}V^*_{ud}|)(|a_2/a_1|) = r|V_{us}V_{cd}/V_{ud}V_{cs}| \approx 0.01$, one finds

$$R_{\mathrm{CP}\pm} = \frac{R(K/\pi)_{\mathrm{CP}\pm}}{R(K/\pi)}. \tag{32}$$

The ratios (29) and (30) were measured in [8–10]. While the current average value, $R(K/\pi) = 0.0819 \pm 0.0037$, involves only a small error, errors are still large in the two measurements, $R(K/\pi)_{\mathrm{CP}+} = 0.125 \pm 0.036 \pm 0.010$ [9] and $0.074 \pm 0.017 \pm 0.006$ [10], as well as in the single measurement, $R(K/\pi)_{\mathrm{CP}-} = 0.119 \pm 0.028 \pm 0.006$ [9]. The implied averages, $R_{\mathrm{CP}+} = 1.15 \pm 0.22$ and $R_{\mathrm{CP}-} = 1.45 \pm 0.36$, are still consistent with $R_{\mathrm{CP}+} - R_{\mathrm{CP}-} = 4r\cos\delta\cos\gamma = 0$. This situation should change when errors are reduced. As we have argued, it is unlikely that both $R_{\mathrm{CP}+}$ and $R_{\mathrm{CP}-}$ are larger than one. Since $\cos\delta > 0$ seems to be favored theoretically, one would expect $R_{\mathrm{CP}-}$ to be smaller than one and to provide new bounds on γ. This requires some reduction of the errors in $R(K/\pi)_{\mathrm{CP}\pm}$.

In conclusion, we have shown that several two body decays $B^\pm \to DK^\pm$, for which data exist, have the potential of improving present bounds on the weak phase γ, in particular, if CP asymmetries are not soon observed in decays to D^0 CP-eigenstates. We argued that multibody decays of this class, for both charged and neutral B mesons, are expected to even do better. Measuring $R_{\mathrm{CP}\pm}(X_s) = 0.60 \pm 0.05$ for one of the two CP-eigenstates in any of these decays would determine γ to within several degrees, $\gamma = 51^\circ \pm 3^\circ$, approaching the present level of precision in β [21]. On the other hand, a measurement $R_{\mathrm{CP}\pm}(X_s) < 0.5$, corresponding to $\gamma < 45^\circ$, would be a signature for physics beyond the Standard Model.

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