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A class of semihypergroups connected to preordered weak Γ-semigroups

Morteza Jafarpour [a,*], Seyed S. Mousavi [b], V. Leoreanu-Fotea [c]

[a] Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran
[b] Shahid Bahonar University of Kerman, Kerman, Iran
[c] Faculty of Mathematics, Al.I. Cuza University, 6600 Iasi, Romania

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ABSTRACT

We introduce the concept of weak Γ-semigroups as a generalization of Γ-semigroups. Using preordered weak Γ-semigroups, we obtain a class of semihypergroups and we analyze them in this paper. A connection between morphisms of semihypergroups associated with preordered Γ-semigroups and morphisms of preordered structures is also investigated.

1. Introduction

Hyperstructure theory was born in 1934 at the 8th congress of Scandinavian Mathematicians, where Marty [1] introduced the hypergroup notion as a generalization of groups and after that he proved its utility in solving some problems of groups, algebraic functions and rational fractions. Surveys of the theory can be found in the books of Corsini [2], Vougiouklis [3], Corsini and Leoreanu [4]. In [5] Sen and Saha introduced the notion of a Γ-semigroup. Since then, many papers have been published on the topic of Γ-semigroups.

Let S and Γ be two non-empty sets. S is called a *Γ-semigroup* if there exists a mapping $S \times \Gamma \times S \to S$, written as $(a, \alpha, b) \to a\alpha b$, satisfying $(a\alpha b)\beta c = a\alpha(b\beta c)$, for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. In this paper we introduce and study weak Γ-semigroups which are generalizations of Γ-semigroups. We construct semihypergroups from preordered weak Γ-semigroups. Basic definitions and results concerning hypergroupoid theory can be found in [2,4]. To facilitate the understanding of the results of this paper, in the following, we provide a brief account of the topic. Let H be a non-empty set and $P^*(H)$ be the set of all non-empty subsets of H. Let $\star$ be a *hyperoperation* (or *join operation*) on H, that is $\star$ is a function from $H \times H$ into $P^*(H)$. If $(a, b) \in H \times H$, the image of a and b under $\star$ in $P^*(H)$ is denoted by $a \star b$ or ab. The join operation can be extended to subsets of H in a natural way, that is $A \star B = \bigcup\{ab \mid a \in A, b \in B\}$. The notation aA is used for $\{a\} \star A$ and Aa for $A \star \{a\}$. Generally, the singleton $\{a\}$ is identified with its member a. The structure $(H, \star)$ is called a *semihypergroup* if $a(bc) = (ab)c$ for all $a, b, c \in H$ and is called a *hypergroup* if it is a semihypergroup and $aH = Ha = H$ for all $a \in H$. A hypergroup is named *proper* if it is not a group. A hypergroup is called *commutative* if $ab = ba$ for every $a, b \in H$. A commutative hypergroup $(H, \star)$ is a *join space* if for all $(a, b, c, d) \in H^4$, the following implication holds:

$$a/b \cap c/d \neq \emptyset \Rightarrow ad \cap bc \neq \emptyset,$$

* Corresponding author. Fax: +98 3913202231.
E-mail addresses: m.j@mail.vru.ac.ir, rmo4909@yahoo.com (M. Jafarpour), smousavi@mail.uk.ac.ir (S.S. Mousavi), leoreanu2002@yahoo.com (V. Leoreanu-Fotea).

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where $a/b = \{x|a \in xb\}$. If $(H, \star)$ and $(H', \star')$ are two semihypergroups, then a function $f : H \to H'$ is called a *homomorphism* if $f(a \star b) \subseteq f(a) \star' f(b)$ for all a, b in H. We say that f is a *good homomorphism* if for all a, b in $H, f(a \star b) = f(a) \star' f(b)$.

A semihypergroup $(H, \star)$ is called *complete* if for all $(x_1, x_2, \ldots, x_n) \in H^n$ and $(y_1, y_2, \ldots, y_m) \in H^m$, where $n, m \geqslant 2$ the following implication holds:

$$\prod_{i=1}^{n} x_i \bigcap \prod_{j=1}^{m} y_j \neq \emptyset \Rightarrow \prod_{i=1}^{n} x_i = \prod_{j=1}^{m} y_j.$$

The notion of a complete semihypergroup was defined by Corsini [6], while complete hypergroups were introduced by Koskas [7]. In [8] Corsini and Romeo proved that the class of complete hypergroups and the class of associativity hypergroups coincide. On the other hand, not all complete semihypergroups are associativity semihypergroups; see for instance [9–14]. Associativity semihypergroups and associativity hypergroups were introduced by Koskas and then they were analyzed by Corsini, Kepka, Jezek, Drbohlav, Nemec, and Niemenmaa. Complete hypergroups have been analyzed and generalized also by Migliorato; see [15].

2. Preordered weak Γ-semigroups

In this section we introduce the notion of weak Γ-semigroups and then we introduce a class of semihypergroups associated with weak Γ-semigroups. Moreover, necessary and sufficient conditions are given for such semihypergroups, such that they become hypergroups.

Let S be a non-empty set and $\lesssim$ be a preorder on S, that is $\lesssim$ is reflexive and transitive.

A set equipped with a preorder is called a *preordered set*.

Let $(S, \lesssim)$ be a preordered set and for $x \in S$, we define

$$L(x) = \{h \in S | h \lesssim x\} \quad \text{and for } \emptyset \neq X \subseteq S, \quad L(X) = \bigcup_{x \in X} L(x).$$

We now introduce the notion of preordered weak Γ-semigroup.

Definition 2.1. Let S and Γ be two non-empty sets. S is called a weak Γ*-semigroup* if there exists a mapping $S \times \Gamma \times S \to S$, written as $(a, \alpha, b) \to a\alpha b$, satisfying the following condition:

$$(a\alpha b)\beta c \in \{a\alpha(b\beta c), a\beta(b\alpha c)\},$$

for all $(\alpha, \beta) \in \Gamma^2$ and $(a, b, c) \in S^3$.

Definition 2.2. A weak Γ-semigroup S is called *commutative* if $a\alpha b = b\alpha a$ for all $(a, b) \in S^2$ and $\alpha \in \Gamma$.

Definition 2.3. Let S be a weak Γ-semigroup and $\lesssim$ be a preorder on S. We say that $(S, \Gamma, \lesssim)$ is a *preordered weak* Γ*-semigroup* if the following axioms are fulfilled:

(i) $x \lesssim y$ implies that $x\alpha z \lesssim y\alpha z$, for every $(x, y, z) \in S^3$ and $\alpha \in \Gamma$;
(ii) $x \lesssim y$ implies that $z\alpha x \lesssim z\alpha y$, for every $(x, y, z) \in S^3$ and $\alpha \in \Gamma$.

If Γ is a singleton in a preordered weak Γ-semigroup $(S, \Gamma, \lesssim)$, then it becomes a preordered semigroup and we denote it by $(S, \cdot, \lesssim)$.

Example 2.4. Let S be the set of all 3×2 matrices with positive entries and Γ be a non-empty subset of all 2×3 matrices with positive entries. For $A, B \in S$, the usual matrix multiplication AB cannot be defined, i.e., S is not a semigroup under the usual matrix multiplication.

But for all $A, B, C \in S$ and $P, Q \in \Gamma$ we have $APB \in S$ and since the matrix multiplication is associative, we obtain $(APB)QC = AP(BQC)$. Hence S is a Γ-semigroup. Moreover, the triple $(S, \Gamma, \lesssim)$ is a preordered Γ-semigroup, where

$$A = (a_{ij}) \lesssim (b_{ij}) = B \Leftrightarrow a_{ij} \leq b_{ij},$$

for all $1 \leq i \leq 3$ and $1 \leq j \leq 2$.

If S is a weak Γ-semigroup, α is a fixed element of Γ and we define $a \cdot b = a\alpha b$ for all $a, b \in S$, then $(S, \cdot)$ is a semigroup that we denote by S_α.

A (weak) Γ-semigroup S is called a (weak) Γ*-group* if S_α is a group for every $\alpha \in \Gamma$.

The following result holds:

Proposition 2.5. *Let S be a Γ-semigroup. If S_α is a group for some $\alpha \in \Gamma$, then S_α is a group for all $\alpha \in \Gamma$.*

Proof. Let $(\alpha, \beta) \in \Gamma^2$ and S_α be a group with identity e. There exists $f \in S$ such that $(e\beta e)\alpha f = e = f\alpha(e\beta e)$ and hence $e\beta f = e\beta(e\alpha f) = e = (f\alpha e)\beta e = f\beta e$. Now let $a \in S$. From $a\beta f = (a\alpha e)\beta f = a\alpha(e\beta f) = a\alpha e = a$ and $f\beta a = f\beta(e\alpha a) = (f\beta e)\alpha a = e\alpha a = a$ we conclude that f is an identity of S_β. Moreover, we have the equations

$a\beta(f\alpha a^{-1}\alpha f) = (a\beta f)\alpha(a^{-1}f) = a\alpha(a^{-1}\alpha f) = e\alpha f = f$ and similarly $(f\alpha a^{-1}\alpha f)\beta a = f$, where a^{-1} is the inverse of a in S_α. We have shown that $f\alpha a^{-1}\alpha f$ is the inverse of a in S_β and hence S_β is a group. □

The following example shows that Proposition 2.5 is not true for the class of weak Γ-semigroups.

Example 2.6. Let $S = \{e, a, b\}$ and $\Gamma = \{\alpha, \beta\}$ be the non-empty set of binary operations defined below:

α	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

β	e	a	b
e	e	a	b
a	a	b	a
b	b	a	b

We mention that S is a weak Γ-semigroup and S_α is a group but S_β is not a group.

Example 2.7. Let $G = \{e, a, b, c\}$ and $\Gamma = \{\alpha, \beta\}$ be the non-empty set of binary operations defined below:

α	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

β	e	a	b	c
e	a	b	c	e
a	b	c	e	a
b	c	e	a	b
c	e	a	b	c

It is not problematic to see that G is a Γ-group.

Example 2.8. Let $G = \{e, a, b, c, d, f\}$ and $\Gamma = \{\alpha, \beta\}$, where

α	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	c	d	f	e
b	b	c	d	f	e	a
c	c	d	f	e	a	b
d	d	f	e	a	b	c
f	f	e	a	b	c	d

β	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	d	f	e	a	b
d	d	f	c	a	b	e
f	f	c	d	b	e	a

By the above tables, we have

$$(a\alpha b)\beta d = c\beta d = a = a\beta(b\alpha d) = a\beta e \neq a\alpha(b\beta d) = a\alpha c = d.$$

We notice that G is a weak Γ-group.

In what follows, starting with a preordered weak Γ-semigroup, we construct a semihypergroup and we analyze it when it is a hypergroup.

Let $(S, \Gamma, \lesssim)$ be a preordered weak Γ-semigroup. We define a hyperoperation $*$ on S, as follows:

$$\forall (a, b) \in S^2, \quad a * b = \cup_{\alpha\in\Gamma} L(a\alpha b).$$

Proposition 2.9. *$(S, *)$ is a semihypergroup, called an $(S, \Gamma, \lesssim)$-associated semihypergroup.*

Proof. It is clear that $*$ is well defined, so we must check that $*$ is an associative hyperoperation. We assert that $(a*b)*c \subseteq a*(b*c)$. Set $x \in (a*b)*c$. So $x \lesssim t_0\alpha c$, for some $t_0 \lesssim a\beta b$ and $\alpha, \beta \in \Gamma$. Thus by Definition 2.1, we have $x \lesssim (a\beta b)\alpha c \in \{a\beta(b\alpha c), a\alpha(b\beta c)\}$. If $(a\beta b)\alpha c = a\beta(b\alpha c)$, set $v = b\alpha c$, so $x \in a * v$, where $v \in b * c$ and hence $x \in a * (b * c)$. Similarly we analyze the other case and also the opposite assertion. □

Remark 1. If S is a weak Γ-semigroup and $\lesssim$ is the discrete relation on S (i.e. $\lesssim = \{(s, s)|s \in S\}$), then the associated hyperoperation $(*)$ is

$$a * b = a\Gamma b = \{a\alpha b : \alpha \in \Gamma\},$$

for every $(a, b) \in S^2$ and $(S, *)$ is called the *natural associated semihypergroup* of S.

Proposition 2.10. *If S is a commutative Γ-group then the natural associated semihypergroup $(S, *)$ is a join space.*

Proof. It is clear that $(S, *)$ is a commutative hypergroup, so it is sufficient to prove that the following implication is satisfied:

$$a/b \cap c/d \neq \emptyset \Rightarrow a * d \cap b * c \neq \emptyset.$$

Let us suppose that $x \in a/b \cap c/d$, so there exists $(\alpha, \beta) \in \Gamma^2$ such that $a = x\alpha b$ and $c = x\beta d$. Therefore $a\beta d = (x\alpha b)\beta d = (b\alpha x)\beta d = b\alpha(x\beta d) = b\alpha c$ and hence $a\beta d = b\alpha c \in a * d \cap b * c \neq \emptyset$. □

Example 2.11. The next table describes the associated join space obtained from the Γ-group introduced in Example 2.6.

$*$	e	a	b	c
e	e, a	a, b	b, c	e, c
a	a, b	b, c	e, c	e, a
b	b, c	e, c	e, a	a, b
c	e, c	e, a	a, b	b, c

Theorem 2.12. *Let $(S, \Gamma, \lesssim)$ be a preordered weak Γ-semigroup and $(S, *)$ be the associated semihypergroup. Then the following conditions are equivalent:*

(i) *for all $(a, b) \in S^2$ there exist $(c, c') \in S^2$ and $(\alpha, \beta) \in \Gamma^2$ such that $a \lesssim b\alpha c$ and $a \lesssim c'\beta b$;*
(ii) *$(S, *)$ is a hypergroup.*

Proof. (i) $\Rightarrow$ (ii): Set $t \in S$. So the inclusions $t * S \subseteq S$ and $S * t \subseteq S$, are obviously fulfilled. We must prove the opposite inclusion. For any $s, t \in S$ there exist $c, c' \in S$ and a pair $(\alpha, \beta) \in \Gamma^2$ such that $s \lesssim t\alpha c$ and $s \lesssim c'\beta t$, whence $s \in L(t\alpha c) \subseteq t * c \subseteq t * S$ and $s \in L(c'\beta t) \subseteq c' * t \subseteq S * t$, and consequently $S \subseteq t * S$ and $S \subseteq S * t$.

(ii) $\Rightarrow$ (i): Let $a, b \in S$ be arbitrary elements. So $b * S = S * b = S$ and it follows that $a \in b * S = \bigcup_{t \in S} b * t$ which means that $a \in b * c$ for some $c \in S$, i.e., $a \lesssim b\alpha c$ for some $\alpha \in \Gamma$. Similarly $a \in S * b$, which implies that $a \lesssim c'\beta b$, for some $c' \in S$ and $\beta \in \Gamma$.

Example 2.13. Let $S = \{e, a, b, c\}$ and $\Gamma = \{\alpha, \beta\}$ be the non-empty set of the binary operations defined below:

α	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

β	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

We can see that S is a weak Γ-semigroup which is not a Γ-semigroup and the natural associated semihypergroup is the following one:

$*$	e	a	b	c
e	e	a	b	c
a	a	e, b	c	e, b
b	b	c	e	a
c	c	e, b	a	e, b

Theorem 2.14. *Every complete semihypergroup is derived from a preordered semigroup.*

Proof. Let $(H, *)$ be a complete semihypergroup. According to Theorem 45 of [4], $H = \bigcup_{s \in S} A_s$, where S is a semigroup and $\{A_s\}_{s \in S}$ is a non-empty family of sets which are mutually disjoint.

Let C be a choice function over the set $\{A_s | s \in S\}$, i.e.,

$$C : \{A_s | s \in S\} \to \bigcup_{s \in S} A_s = H \ ,$$

where $C(A_s) \in A_s$.

The following assertions can be easily checked:

(i) $(H, \circ)$ is a semigroup, where for all $(a, b) \in A_s \times A_t$ and $s, t \in S$,

$$a \circ b \overset{\text{def}}{\coloneqq} C(A_{st});$$

(ii) the relation $\lesssim$ defined over H by:
 (1) for all $a \in H,\ a \lesssim a$;
 (2) for all $s, t \in S$ and $(a, b) \in A_s \times A_t$ and for all $x \in A_{st},\ a \circ b \lesssim x$,

is a preorder relation on H and the associated semihypergroup of the preordered semigroup $(H, \lesssim)$ is just $(H, *)$. □

The notion of general mutually associative hypergroupoids was introduced in [16], as follows: two (partial) hypergroupoids $(H, *_1)$ and $(H, *_2)$ are called *general mutually associative*, or for short g.m.a., if for any $(x, y, z) \in H^3$, we have

$$(x *_1 y) *_2 z \cup (x *_2 y) *_1 z = x *_1 (y *_2 z) \cup x *_2 (y *_1 z).$$

We obtain:

Proposition 2.15. *If S is a weak Γ-semigroup, then for all $(\alpha, \beta) \in \Gamma^2$ the two semigroups S_α and S_β are g.m.a.*

Proposition 2.16. *Let $(S, \cdot, \lesssim)$ be a preordered semigroup and $(S, *)$ be the associated semihypergroup. The following assertions hold:*
(i) *the pair $((S, \cdot), (S, *))$ is g.m.a.;*
(ii) *for all $(x, y, z, t) \in S^4$ if $x \cdot y \in z * t$, then $x * y \subseteq z * t$.*

Proof. (i) It is obvious.
(ii) If $u \in x * y$, then $u \lesssim x \cdot y$, and since $x \cdot y \in z * t$, we obtain that $x \cdot y \lesssim z \cdot t$. Thus $u \lesssim x \cdot y \lesssim z \cdot t$; by transitivity it follows that $u \lesssim z \cdot t$. Therefore $u \in z * t$. □

Theorem 2.17. *Let $(S, *)$ be a hypergroup associated with a preordered semigroup $(S, \lesssim)$. Then $(S, *)$ is a group if and only if it contains a left scalar identity (i.e. $\exists e \in S$ such that $e * x = x$, for all $x \in S$).*

Proof. Let $(x, y) \in S^2$ and e be a left scalar identity of $(S, *)$. Now we have $x \cdot y = e * (x \cdot y)$, and using the previous proposition we conclude that $x * y \subseteq e * (x \cdot y)$, so $|x * y| = 1$ thus $(S, *)$ is a group. The converse is obvious. □

Remark 2. In [12], Hort introduced a class of hypergroups associated with preordered semigroups. The previous theorem shows that the associated hypergroup of Example 2.13 cannot be produced from a preordered semigroup, so the class of hypergroups associated with weak Γ-semigroups is bigger than the class of hypergroups associated with preordered semigroups.

Corollary 2.18. *The class of hypergroups which contain a left scalar identity and the class of proper hypergroups associated with preordered semigroups are disjoint.*

3. On morphisms of associated semihypergroups

In this section we give a connection between morphisms of the semihypergroups associated with preordered Γ-semigroups and morphisms of preordered structures.

Proposition 3.1. *Let $(S_1, \lesssim_1)$ and $(S_2, \lesssim_2)$ be preordered sets and $f : S_1 \to S_2$ be a mapping. The following conditions are equivalent:*
(i) *f is isotone;*
(ii) *$f(L(x)) \subseteq L(f(x))$ for all $x \in S_1$;*
(iii) *$L(f^{-1}(f(x))) \subseteq f^{-1}(L(f(x)))$ for all $x \in S_1$.*

Proof. (i) $\Rightarrow$ (ii): Let $x \in S_1$ be an arbitrary element and set $y \in f(L(x))$. Then there exists $z \in L(x)$, i.e., $z \lesssim_1 x$ such that $y = f(z)$. Since f is isotone, $f(z) \lesssim_2 f(x)$ which implies that $y \in L(f(x))$. Thus $f(L(x)) \subseteq L(f(x))$.

(ii) $\Rightarrow$ (iii): Let $x \in S_1$ be an arbitrary element and set $y \in L(f^{-1}(f(x)))$. Then there exists $z \in f^{-1}(f(x))$, i.e., $f(z) = f(x)$ such that $y \lesssim_1 z$ which means that $y \in L(z)$. Therefore $f(y) \in f(L(z)) \subseteq L(f(z)) = L(f(x))$ and hence $y \in f^{-1}(f(y)) \subseteq f^{-1}(L(f(x)))$. Consequently $L(f^{-1}(f(x))) \subseteq f^{-1}(L(f(x)))$.

(iii) $\Rightarrow$ (i): Let $x, y \in S_1$ be such that $x \lesssim_1 y$. Since $y \in f^{-1}(f(y))$, it follows that $x \in L(y) \subseteq L(f^{-1}(f(y))) \subseteq f^{-1}(L(f(y)))$. Thus $f(x) \in L(f(y))$ and hence $f(x) \lesssim_2 f(y)$. □

Definition 3.2. Let (S_1, Γ_1) and (S_2, Γ_2) be weak Γ-semigroups. A pair of mappings $f : S_1 \to S_2$ and $g : \Gamma_1 \to \Gamma_2$ is said to be a *homomorphism* if $f(a\alpha b) = f(a)g(\alpha)f(b)$, for all $(a, b) \in S_1^2$ and all $\alpha \in \Gamma_1$.

Theorem 3.3. *Let $(S_1, \Gamma_1, \lesssim_1)$ and $(S_2, \Gamma_2, \lesssim_2)$ be preordered weak Γ-semigroups and let (f, g) be a homomorphism, where $f : S_1 \to S_2$ is isotone and $g : \Gamma_1 \to \Gamma_2$ is a surjection map. Then f is a homomorphism between the associated semihypergroups $(S_1, *_1)$ and $(S_2, *_2)$.*

Proof. For any $x, y \in S_1$, we have

$$\begin{aligned} f(x *_1 y) &= f(\cup_{\alpha \in \Gamma_1} L(x\alpha y)) \\ &= \cup_{\alpha \in \Gamma_1} f(L(x\alpha y)) \\ &\subseteq \cup_{\alpha \in \Gamma_1} L(f(x\alpha y)) \\ &= \cup_{\alpha \in \Gamma_1} L(f(x)g(\alpha)f(y)) \\ &= f(x) *_2 f(y). \quad \square \end{aligned}$$

Proposition 3.4. *Let $(S_1, \lesssim_1)$ and $(S_2, \lesssim_2)$ be two preordered sets and $f : S_1 \to S_2$ be a mapping. The following conditions are equivalent:*
(i) *f is a strongly isotone mapping;*
(ii) *$f(L(x)) = L(f(x))$ for all x in S.*

Proof. (i) $\Rightarrow$ (ii): For a strongly isotone mapping f is enough to prove the set inclusion $L(f(x)) \subseteq f(L(x))$ according to Proposition 3.1. Let $y \in L(f(x))$ be an arbitrary element, that is $y \lesssim_2 f(x)$. Since the mapping f is strongly isotone, there exists $x' \in S_1$ such that $x' \lesssim_1 x$ and $f(x') = y$. Therefore $x' \in L(x)$ and hence $y = f(x') \in f(L(x))$ which means that $L(f(x)) \subseteq f(L(x))$.

(ii) $\Rightarrow$ (i): Let $x_1 \in S_1$ and $x_2 \in S_2$ be such that $x_2 \lesssim_2 f(x_1)$. Since $x_2 \in L(f(x_1)) = f(L(x_1))$, there exists $x_1' \in L(x_1)$, i.e., $x_1' \lesssim_1 x_1$ such that $f(x_1') = x_2$; consequently f is a strongly isotone mapping. □

Theorem 3.5. *Let $(S_1, \Gamma_1, \lesssim_1)$ and $(S_2, \Gamma_2, \lesssim_2)$ be preordered weak Γ-semigroups and let (f, g) be a homomorphism, where $f : S_1 \to S_2$ is strongly isotone and $g: \Gamma_1 \to \Gamma_2$ is a surjection map. Then f is a good homomorphism between the associated semihypergroups $(S_1, *_1)$ and $(S_2, *_2)$.*

Proof. The proof is similar to the proof of Proposition 3.4. □

4. Conclusion

In this paper, we introduce and analyze the notion of weak Γ-semigroups which is a generalization of the notion of Γ-semigroups. Several properties are investigated, such as connections with (semi)hypergroups. This research can be continued, for instance in the study of some particular classes of semihypergroups. Moreover, we intend to continue this study in order to obtain fuzzy weak Γ-semigroups, interval-valued fuzzy weak Γ-semigroups and (λ, μ)-fuzzy weak Γ-semigroups; see [11,10,17,18].

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