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Estimation of the complex frequency of a harmonic signal based on a linear least squares method

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ABSTRACT

In this study, we propose a simple linear least squares estimation method (LLS) based on a Fourier transform to estimate the complex frequency of a harmonic signal. We first use a synthetically-generated noisy time series to validate the accuracy and effectiveness of LLS by comparing it with the commonly used linear autoregressive method (AR). For an input frequency of 0.5 mHz, the calculated deviations from the theoretical value were 0.004‰ and 0.008‰ for the LLS and AR methods respectively; and for an input 5×10^{-6} attenuation, the calculated deviations for the LLS and AR methods were 2.4% and 1.6%. Though the theory of the AR method is more complex than that of LLS, the results show LLS is a useful alternative method. Finally, we use LLS to estimate the complex frequencies of the five singlets of the ${}_0S_2$ mode of the Earth's free oscillation. Not only are the results consistent with previous studies, the method has high estimation precisions, which may prove helpful in determining constraints on the Earth's interior structures.

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1. Introduction

For the Fourier analysis of a signal, in most cases one must obtain the complex frequencies (frequency *f* and quality factor Q) and amplitudes of the signal, such as those recorded in studies about the tidal, normal modes, or polar motion of the Earth. It is therefore necessary to estimate those parameters with high precision; there are many methods currently in use for doing so.

Among numerous complex frequency estimation methods [1], early observations of α_k, the attenuation of the kth mode, most of which were obtained by a time lapse method. This method is relatively cost effective although does not easily lend itself to application of tapers, which are essential to the estimation of Q [2]. It has been proposed a very fast and reliable method, the autoregressive (AR) method [3], which can be used to estimate the four parameters of a signal and their accuracies, where A_k is the complex amplitude of the kth mode, ω_k ($= 2\pi f_k$) is the frequency of the kth mode, ϕ_k is

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the phase of the kth mode, and α_k is as previously defined. Additionally, a non-linear least squares fitting method termed as the least squares (LS) algorithm [2], which can be also used to estimate A_k, ω_k, α_k, and ϕ_k, and their accuracies. Moreover, the early measurement techniques, that the measurements of Q must be used the tapering process, and concluded that the most frequently used traditional methods, such as the time lapse method, should be replaced by the AR or LS method. It assumed that each resonance of the spectrum was produced by a damped harmonic oscillator, and then used a numerical method to obtain A_k, ω_k, and α_k [4]. An improved method to estimate the attenuation α_k based on the time lapse method and a non-linear damped harmonic analysis method to estimate the complex frequencies of a normal mode [5–7]. Those previous methods are non-linear algorithms, which in general are unstable and computationally time-consuming. Given that, we will introduce a linear algorithm as an alternative.

In this paper, we are concerned only with estimating ω_k, and α_k of a target signal, because the estimation of complex amplitudes are based on the estimation of the complex frequencies. If the latter can be accurately estimated, the former is determined by a simple linear least square process [3]. Note that the method we propose here is a linear method which can be seen as an alternative to the previous methods.

2. Methodology

A discrete time series consisting of M decaying functions can be expressed as

$$a(t) = \sum_{k=1}^{M} A_k \cos(\omega_k t + \phi_k) e^{-\alpha_k t} \tag{1}$$

where A_k is the complex amplitude of the kth mode, α_k is the attenuation, ω_k ($= 2\pi f_k$) is the frequency, and ϕ_k is the phase.

After a Fourier transform, consider that each of the spectral peaks corresponds to an independent ω_k, and therefore one can estimate the four parameters of a mode in the frequency domain. Note that a Hanning taper is required to multiply the given record prior to the fast Fourier transform (FFT) to weaken spectral leakage [3,8,9], there after the Fourier spectrum of (1) can be written (ignoring the form of the window function) as:

$$F_a(\omega) = \frac{1}{2}\left[\frac{A_k e^{i\phi_k}}{i(\omega - \omega_k) + \alpha_k} + \frac{A_k e^{-i\phi_k}}{i(\omega + \omega_k) + \alpha_k}\right] \tag{2}$$

For a normal mode, $\alpha_k >> 1$, we can see that:

$$|F_a(-\omega_k)| \approx |F_a(\omega_k)| \tag{3}$$

Hence, for a given spectrum, we only need to consider the positive frequencies, $\omega_k > 0$, in the frequency domain [10]. Therefore equation (2) can be replaced by

$$F_a(\omega) = \frac{1}{2}\frac{A_k e^{i\phi_k}}{i(\omega - \omega_k) + \alpha_k} = \frac{A_k e^{i\phi_k}}{2}\frac{\alpha_k - i(\omega - \omega_k)}{(\omega - \omega_k)^2 + \alpha_k^2} \tag{4}$$

Let

$$\begin{cases} C_k(\omega) = \dfrac{1}{2}\dfrac{\alpha_k - i(\omega - \omega_k)}{(\omega - \omega_k)^2 + \alpha_k^2} \\ \boldsymbol{a}_k = z_1 + iz_2 = A_k e^{i\phi_k} \end{cases} \tag{5}$$

Then one can get

$$F_a(\omega) = \boldsymbol{a}_k C(\omega) \tag{6}$$

Hence, the power spectrum can be written as

$$P_a(\omega) = |\boldsymbol{a}_k| \left|C_k(\omega)^2\right| / N = \frac{0.25|\boldsymbol{a}_k|/N}{(\omega - \omega_k)^2 + \alpha_k^2} \tag{7}$$

where N is the number of data points. Letting $B_k = 0.25|\boldsymbol{a}_k|/N$, (note that in a narrow target frequency band $P_a(\omega) \neq 0$), taking the reciprocal of both sides of equation (7) one can get that

$$\frac{1}{P_a(\omega)} = \frac{\omega^2}{B_k} - \frac{2\omega_k}{B_k}\omega + \frac{1}{B_k}(\omega_k^2 + \alpha_k^2) \tag{8}$$

Note that while B_k ensures that $B_k/\alpha_k^2 = P_a(\omega)|_{\max}$ in a given narrow frequency band, there is no information about complex amplitude in any power spectra [11], and all frequency points ω in (8) have to cross the target peak, which is located in the given narrow frequency band. Let $y(\omega) = 1/P_a(\omega)$, $a = 1/B_k$, $b = -2\omega_k/B_k$, and $c = (\omega_k^2 + \alpha_k^2)/B_k$, then

$$y(\omega) = a\omega^2 + b\omega + c \tag{9}$$

It is apparent that equation (9) is linear for a, b, and c; and according to Least Square Estimation (LSE), we only need three values of y to obtain an estimate for the three parameters. Since

$$B_k = \frac{1}{a}, \quad \omega_k = \frac{-b}{2a}, \quad \alpha_k = \sqrt{\frac{4ac - b^2}{4a^2}} \tag{10a}$$

where ω_k and α_k can also be estimated. It should be noted that B_k contains two unknown parameters A_k and ϕ_k, and a non-fixed value N, hence we only use the above mentioned process to estimate ω_k and α_k. Clearly, by simply taking the reciprocal of a given spectrum, decouples the estimations of complex amplitude ($A_k e^{i\phi_k}$) and complex frequency $\omega_k + i\alpha_k$.

To achieve a more accurate complex frequency estimation of the modes, we adopt a repeating estimation process. Furthermore, given the limited number of frequency points across the single target spectral peak, numerous frequency points can be increased by zero-padding or linear interpolation to achieve more accurate estimations; another process is just using discrete Fourier transform (DFT) to obtain the Fourier spectrum of any frequency point below the Nyquist frequency. In fact, for a given spectral peak, there are often more than three points, which cross the target peak. How to realize a multiple least squares estimation will be detailed below.

As mentioned above, for the determination of ω_k and α_k, the values of three power spectral points must be known, but to make more accurate estimations, additional observations are required; therefore we will repeat the estimation processes multiple times, interpolating the power spectrum sequence by zero-padding. Assume that there are M ($M \geq 5$) frequency points, which cross the target peak before the interpolation, the corresponding frequency sequence and power spectral sequence are $\omega[k]$ and $P(\omega[k])$ ($k = 1,2,\ldots,M$; $\omega=\{\omega[k]\}$ is called "sequence $\omega[k]$" hereafter when referring to the entire finite-duration sequence; otherwise, $\omega[k]$ denotes the kth number in the finite-duration sequence), and the frequency point $\omega[m_0]$ which corresponds to the maximum of $P(\omega[k])$ is located near the middle of the sequence $\omega[k]$ (i.e., $m_0 = (M+1)/2$, if M is odd; otherwise, $m_0 = M/2$). If the frequency spacing of sequence $\omega[k]$ is denoted as $\Delta\omega$, the frequency interpolation interval can be equal to $\Delta\omega/n$ (where $n \geq 2$ is the length of the zero-padding), then we can get a new frequency sequence $\omega_I[j]$ (where the subscript I indicates the interpolated sequence) and a new power spectral sequence $P(\omega_I[j])$ ($j = 1,2,\ldots,N; N = (M-1)n-1$). Obviously, the maximum value of the power spectral sequence and its corresponding frequency will be the same, but $\omega[m_0]$ will be noted by Ref. $\omega_I[m]$ in the new sequence $\omega_I[j]$ ($m = (M-1)n/2+1$, if M is odd; otherwise, $m = (M-2)n/2+1$). To perform multiple estimations, five observations are chosen. Three pairs of observations will be used in each estimation, they are $\{\omega_I[m],P(\omega_I[m])\}$, $\{\omega_I[1],P(\omega_I[1])\}$ and $\{\omega_I[2m-1],P(\omega_I[2m-1])\}$, the two other observations ($\{\omega_I[m-i], P(\omega_I[m-i])\}$, $\{\omega_I[m+i], P(\omega_I[m+i])\}$) are selected symmetrically from reference $\omega_I[m]$. To ensure the uniqueness of these additional observation equations, $i \neq m-1$, namely $i = 1,2,\ldots,m-2$; $m-2$ is the number of the multiple estimations. Because we have assumed that $M \geq 5$, so $m-2 \geq 5$, and multiple estimations can be implemented.

Thus, for the ith estimation

$$\begin{cases} P(\omega_I(m)) = a_i(\omega_I(m))^2 + b_i\omega_I(m) + c_i \\ P(\omega_I(1)) = a_i(\omega_I(1))^2 + b_i\omega_I(1) + c_i \\ P(\omega_I(2m-1)) = a_i(\omega_I(2m-1))^2 + b_i\omega_I(2m-1) + c_i, \\ \qquad i = 1,2,\ldots,m-2 \\ P(\omega_I(m-i)) = a_i(\omega_I(m-i))^2 + b_i\omega_I(m-i) + c_i \\ P(\omega_I(m+i)) = a_i(\omega_I(m+i))^2 + b_i\omega_I(m+i) + c_i \end{cases} \tag{10b}$$

Power spectral sequence P and coefficient matrix C can be written as

$$P = \begin{bmatrix} P(\omega_I(m)) \\ P(\omega_I(1)) \\ P(\omega_I(2m-1)) \\ P(\omega_I(m-i)) \\ P(\omega_I(m+i)) \end{bmatrix}, \quad C = \begin{bmatrix} (\omega_I(m))^2 & \omega_I(m) & 1 \\ (\omega_I(1))^2 & \omega_I(1) & 1 \\ (\omega_I(2m-1))^2 & \omega_I(m-i) & 1 \\ (\omega_I(m-i))^2 & \omega_I(m-i) & 1 \\ (\omega_I(m+i))^2 & \omega_I(m+i) & 1 \end{bmatrix} \tag{11}$$

Given that the frequency points come from the same sequence $\omega_I[i]$, the weight matrix can simply be taken as an $n \times n$ identity matrix. Based on the LSE, we have

$$[a_i \quad b_i \quad c_i]^T = (\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T\mathbf{P} \tag{12}$$

and

$$[e(a_i) \quad e(b_i) \quad e(c_i)]^T = \sqrt{\frac{\mathbf{V}^T\mathbf{V}(\mathbf{C}^T\mathbf{C})^{-1}}{5-3}} \tag{13}$$

where $e(x)$ is the accuracy of x, and $\mathbf{V} = \mathbf{C}[a_i \quad b_i \quad c_i]^T - \mathbf{P}$. According to equation (10),

$$\omega_k(i) = \frac{b_i}{-2a_i}, \quad \alpha_k(i) = \sqrt{\frac{4a_ic_i - b_i^2}{4a_i^2}}$$

Then we can get that [12]:

$$\begin{cases} e(\omega_k(i)) = \sqrt{\left[\frac{e(b_i)}{2a_i}\right]^2 + \left[\frac{b_ie(a_i)}{2a_i^2}\right]^2} \\ e(\alpha_k(i)) = \frac{1}{\alpha_k(i)}\sqrt{\left(\frac{\left(b_i^2 - 2a_ic_i\right)e(a_i)}{2a_i^3}\right)^2 + \left(\frac{b_ie(b_i)}{2a_i^2}\right)^2 + \left(\frac{e(c_i)}{a_i}\right)^2} \end{cases} \tag{14}$$

Finally, we have

$$\begin{cases} \omega_k = \frac{1}{m-2}\sum_{i=1}^{m-2}\omega_k(i), \quad e(\omega_k) = \frac{1}{m-2}\sqrt{\sum_{i=1}^{m-2}e(\omega_k(i))^2} \\ \alpha_k = \frac{1}{m-2}\sum_{i=1}^{m-2}\alpha_k(i), \quad e(\alpha_k) = \frac{1}{m-2}\sqrt{\sum_{i=1}^{m-2}e(\alpha_k(i))^2} \end{cases} \tag{15}$$

By the above procedure (equations (10)–(15)), the frequency and attenuation of a given spectral peak can be accurately estimated. The procedure for the estimation of complex frequency is referred to as the linear least squares estimation method (LLS) in this paper. On the basis of LLS, the knowledge of the spectral sequence is enough to estimate the complex frequency of a given spectral peak, therefore, it is suitable for estimating the complex frequencies in a product spectrum, which is obtained by multiplying the spectrums of different records.

3. Verification of the LLS

3.1. *Synthetic series*

First we will use a simple figure to explain the process given in section 2. As shown in Fig.1, the solid curve is the direct Fourier spectrum without interpolation, while the dashed curve is the interpolated spectrum (five times zero-padding). The target peak is located between the two vertical dotted lines, and while only three original frequency points (represented by circles) can be used in the estimation, there are 23 points (represented by stars) can be used to estimate after interpolation, so the latter can carry out the LLS.

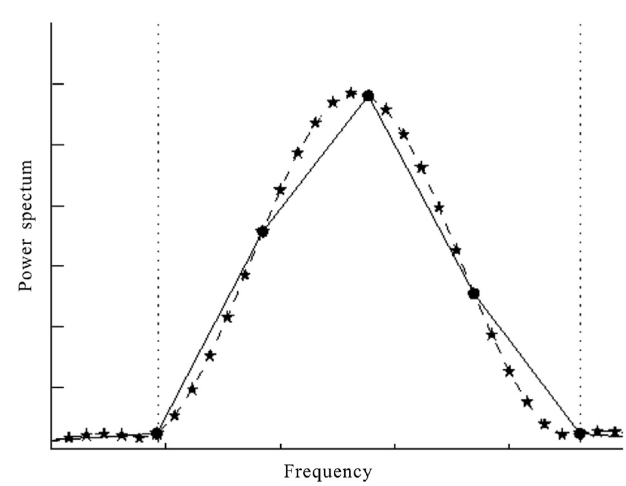

Fig. 1 – A simple example to explain the interpolation used to carry out the LLS estimation.

The synthetically-generated series consists of a cosine signal with 5 MHz frequency and 1 unit amplitude, and applied white noise. The waveform and the power spectrum are shown in Fig.2.

A comparison between the estimated values by both the LLS and the AR methods [3] are shown in Table 1. Clearly, the estimated frequency from using LLS is more accurate than that found by using the AR method, while the latter obtains a better estimate for the attenuation. However, since the attenuation is harder to precisely estimate, we consider the result from our LLS estimation to be valid. This finding indicates that the LLS is at least an acceptable alternative for the AR method. In addition, according to multiple tests, we find the length of the zero-padding $n = 4$ is enough to obtain a stable estimation.

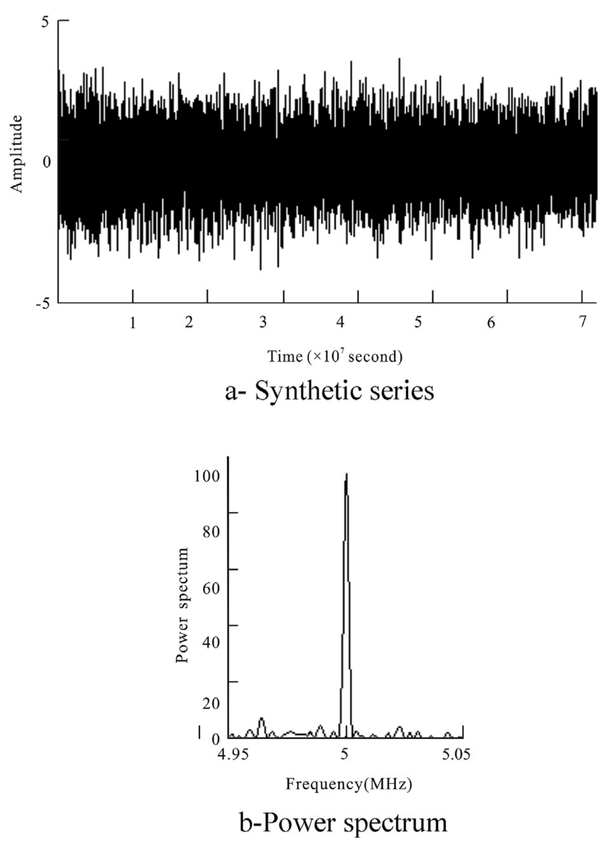

Fig. 2 – Waveform and the power spectrum.

Table 1 – The estimated values for the synthetic series by the LLS and the AR methods.		
	Frequency (MHz)	Attenuation
Theoretical value	5.00000	5.00×10^{-6}
LLS	$4.99998 \pm 2.1 \times 10^{-6}$	$5.12 \times 10^{-6} \pm 3.4 \times 10^{-8}$
AR	$5.00004 \pm 4.7 \times 10^{-6}$	$5.08 \times 10^{-6} \pm 2.9 \times 10^{-8}$

3.2. *The estimate for the normal mode ${}_0S_2$*

Recently, author in reference [13] claimed that they have obtained the most precise estimate for the frequencies for the multiplet ${}_0S_2$, and the estimated errors presented there are also of the same order of precision as this study. Hence, we also choose ${}_0S_2$ to test our method, using a simple example. Namely, we will use the superconducting gravity record from the Strasbourg, France station after the 2004 Sumatra earthquake as an example. Although the optimum record length is about 1.0 Q-cycle of the modes (i.e., about 450 h for ${}_0S_2$) [3,8], we find that the signal-to-noise ratios for ${}_0S_2$ is higher when the length is 300 h, and in recent results [14–17], the record length is also chosen to be 300 h. Hence, in this paper, the record started 5 h after the event, with a length of 300 h, with an interval of 1 min.

The corresponding power spectrum of the chosen record is shown in Fig. 3. The five singlets of ${}_0S_2$ can be completely identified. The corresponding estimates of those singlets are tabulated in Table 2. Clearly, our estimate values are very close to the previous studies, especially in that the Q-values quite close to some previous studies [15]. It is well known that the Q-value of a signal is difficult to precisely determine [3,6,15].

This example again validates the usefulness of LLS, and importantly, this method is quite easy to carry out. We expect that it might be useful in the estimations of the normal mode frequency and other geophysical signals, such as the polar motion or the tides of the Earth.

4. Conclusion

Benefiting from the zero padding of FFT (or using the DFT), we introduce a simple LLS method to estimate the complex frequency of a harmonic signal. Generally, a linear process is less complex and time-consuming than a non-linear process; hence, we compare LLS with a previous linear method—the AR method [3]. The complex frequency estimated from both

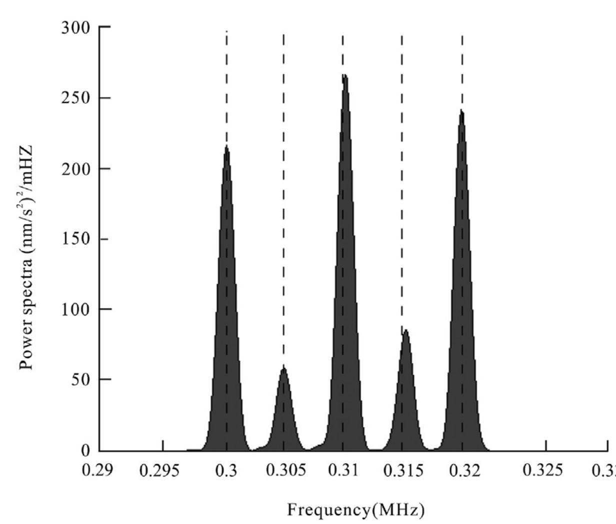

Fig. 3 – The Fourier power spectrum of the Strasbourg record after the 2004 Sumatra earthquake.

Table 2 – The observed weighted average values of the frequencies and Qs of $_0S_2$, compared with the previous estimates and the PREM predictions.

$_0S_2$		$m = -2$	$m = -1$	$m = 0$	$m = +1$	$m = +2$
PREM	f	0.30000117	0.30449303	0.30906353	0.31371556	0.31845238
	Q	494.6	501.8	509.3	517.0	525.0
Buland et al. [18]	f	0.30001	0.30480	0.30949	0.31400	0.31850
Rosat et al. [7]	f	$0.29997 \pm 6.3 \times 10^{-6}$	$0.30458 \pm 4.7 \times 10^{-6}$	$0.30924 \pm 6.0 \times 10^{-6}$	$0.31381 \pm 1.1 \times 10^{-5}$	$0.31843 \pm 4.6 \times 10^{-6}$
Roult et al. [5]	f	$0.299779 \pm 7.4 \times 10^{-5}$	$0.304624 \pm 2.9 \times 10^{-5}$	$0.309397 \pm 1.57 \times 10^{-4}$	$0.313892 \pm 6.9 \times 10^{-5}$	$0.318465 \pm 8.9 \times 10^{-5}$
	Q	490.6 ± 14.0	562.9 ± 4.0	395.6 ± 11.3	495.3 ± 4.0	480.2 ± 14.9
Rosat et al. [7]	f	$0.299951 \pm 1.5 \times 10^{-6}$	$0.304599 \pm 1.6 \times 10^{-6}$	$0.3092607 \pm 2.5 \times 10^{-7}$	$0.3138446 \pm 2.6 \times 10^{-7}$	$0.3184385 \pm 2.8 \times 10^{-7}$
	Q	449.3 ± 0.1	481.5 ± 0.1	506.7 ± 0.4	457.7 ± 0.3	518.7 ± 0.4
Abd El-Gelil et al. [19]	f	$0.300001 \pm 1.2 \times 10^{-6}$	$0.304533 \pm 1.1 \times 10^{-6}$	$0.309296 \pm 1.1 \times 10^{-6}$	$0.313882 \pm 0.5 \times 10^{-6}$	$0.318402 \pm 1.0 \times 10^{-6}$
	Q	509.9 ± 3.9	677.9 ± 11.5	512.3 ± 3.9	592.7 ± 8.1	520.3 ± 3.1
Roult et al. [20]	f	$0.29998 \pm 3.313 \times 10^{-4}$	$0.30447 \pm 4.985 \times 10^{-4}$	$0.30922 \pm 3.560 \times 10^{-4}$	$0.31374 \pm 4.480 \times 10^{-4}$	$0.31835 \pm 3.548 \times 10^{-4}$
Deuss et al. [21]	f	0.29993	0.30463	0.30928	0.31386	0.31840
Hafner and Widmer [13]	f	$0.299948 \pm 9.0 \times 10^{-6}$	$0.304612 \pm 6.0 \times 10^{-6}$	$0.309269 \pm 1.6 \times 10^{-5}$	$0.313840 \pm 5.0 \times 10^{-6}$	$0.318429 \pm 9.0 \times 10^{-6}$
Ding and Shen [14]		$0.299965 \pm 2.0 \times 10^{-5}$	$0.304536 \pm 6.2 \times 10^{-5}$	$0.309193 \pm 3.7 \times 10^{-5}$	$0.313843 \pm 5.5 \times 10^{-5}$	$0.318433 \pm 1.8 \times 10^{-5}$
Ding and Shen [15]	f	$0.299958 \pm 8.1 \times 10^{-6}$	$0.304588 \pm 4.6 \times 10^{-6}$	$0.309263 \pm 1.1 \times 10^{-5}$	$0.313835 \pm 1.4 \times 10^{-6}$	$0.318422 \pm 7.4 \times 10^{-6}$
	Q	509.4 ± 12.1	484.7 ± 9.3	394.4 ± 14.3	520.2 ± 8.1	532.7 ± 10.1
Shen and Ding [17]	f	$0.299994 \pm 1.1 \times 10^{-5}$	$0.304618 \pm 7.8 \times 10^{-6}$	$0.309278 \pm 9.1 \times 10^{-6}$	$0.313865 \pm 7.7 \times 10^{-6}$	$0.318424 \pm 9.6 \times 10^{-6}$
Ding and Chao [16]	f	$0.299967 \pm 1.4 \times 10^{-5}$	$0.304587 \pm 7.8 \times 10^{-6}$	$0.309372 \pm 5.0 \times 1010^{-5}$e-5	$0.313850 \pm 6.9 \times 10^{-6}$	$0.318396 \pm 1.3 \times 10^{-5}$
This paper	f	$0.299983 \pm 6.9 \times 10^{-6}$	$0.3045867 \pm 7.2 \times 10^{-6}$	$0.309255 \pm 6.0 \times 10^{-6}$	$0.313842 \pm 8.0 \times 10^{-6}$	$0.318456 \pm 6.6 \times 10^{-6}$
	Q	505.3 ± 17.4	488.5 ± 23.5	399.1 ± 16.7	527.6 ± 25.0	525.8 ± 19.5

the LLS and AR methods based on a signal obtained from noisy synthetic records are all almost same as the input complex frequency, namely, LLS can accurately estimate the complex frequency of a target signal just as well as the AR method, which has been validated by lots of previous studies [3,11,16]. Given that the theory of the LLS method is easier than the AR method, the corresponding results show LLS is a useful alternative method. Because the $_0S_2$ mode of the free oscillation of the Earth has been well studied, we estimated the complex frequencies of its five singlets by using the LLS would be a very favorable verification of the LLS method. On the basis of the SG records after the 2004 Sumatra earthquake, our results from the LLS method are consistent with previous studies, and the method has high estimation precisions.

Because the proposed LLS method is a useful alternative method for calculating the complex frequency of a signal, our estimations for the complex frequencies of $_0S_2$ may also help to determine constraints on the Earth's interior structures.

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