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Lévy–Ornstein–Uhlenbeck transition semigroup as second quantized operator

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Abstract

Let μ be an invariant measure for the transition semigroup (P_t) of the Markov family defined by the Ornstein–Uhlenbeck type equation

$$\mathrm{d}X = AX\,\mathrm{d}t + \mathrm{d}L$$

on a Hilbert space E, driven by a Lévy process L. It is shown that for any $t \geqslant 0$, P_t considered on $L^2(\mu)$ is a second quantized operator on a Poisson Fock space of e^{At}. From this representation it follows that the transition semigroup corresponding to the equation on $E = \mathbb{R}$, driven by an α-stable noise L, $\alpha \in (0, 2)$, is neither compact nor symmetric.

Keywords: Lévy–Ornstein–Uhlenbeck processes; Poisson chaos decomposition

1. Introduction

Let μ be an invariant measure for a Markov family $X = (X^x)$ on a measurable space $(E, \mathcal{B})$, with the transition semigroup

$$P_t\psi(x) = \mathbb{E}\psi\big(X^x(t)\big), \quad t \geqslant 0,\ x \in E.$$

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Then (P_t) is a semigroup of contractions on any $L^p(\mu) := L^p(E, \mathcal{B}(E), \mu)$-space, $p \in [1, +\infty]$. From a probabilistic and an analytic point of view, there is motivation to analyze (P_t) and μ as μ can be treated as a reference measure (especially in infinite-dimensional case) and the generator of (P_t) as a reference operator.

In the present paper X is defined by the Ornstein–Uhlenbeck type equation

$$\mathrm{d}X = AX\,\mathrm{d}t + \mathrm{d}L, \qquad X(0) = x \in E, \tag{1}$$

where $(A, D(A))$ generates a C_0-semigroup e^{At}, $t \geqslant 0$, on a Hilbert space $(E, \langle \cdot, \cdot \rangle_E)$. The main result covers the general case of the so-called cylindrical process; that is the case where L is a Lévy process taking values in a Hilbert space $\tilde{E} \hookleftarrow E$, for more details see Section 5.

Analytic properties of the transition semigroup (P_t) corresponding to the equation on a finite or infinite-dimensional state space, driven by a Wiener process have been studied for many years and now they are rather well understood. For example, there are if and only if conditions for its compactness, self-adjointness, analyticity, and hypercontractivity (see e.g. [7–11,14,13,23–26]). In particular, in the case of $E = \mathbb{R}$, (P_t) is symmetric, compact (even nuclear) and hypercontractiv. It will be shown in Theorem 7.3 of the present paper, that in the case of an α-stable noise, $\alpha \neq 2$, (P_t) in neither symmetric nor compact. The fact that the transition semigroup (P_t) is not symmetric has been shown using different methods, by Albeverio, Rüdiger, and Wu [1] for α-stable processes, and by Applebaum and Goldys [3] for general noise, whereas the fact that (P_t) is not hyperbounded; that is $\|P_t\|_{L(L^p(\mu), L^q(\mu))} = \infty$ for any $t > 0$ and $1 < p < q < \infty$, was shown by Röckner and Wang [30].

Ornstein–Uhlenbeck equations with Lévy noise have been studied for over twenty years (see e.g. [2,5,4,6,15,20,21,28–30]) and now they are a subject of intensive studies (see e.g. [5,4,28]). However, even in the case of $E = \mathbb{R}$, our knowledge on properties of their transition semigroups is rather limited. Namely, apart of the Albeverio, Rüdiger, and Wu, and the Applebaum and Goldys results on the lack of symmetry of (P_t), Lescot and Röckner [20,21] identified the generator of (P_t) as a pseudo-differential operator with an explicit symbol, and obtained an explicit formula for the square field operator of (P_t), next Röckner and Wang [30] established Poincaré and Harnack type inequalities and showed that generally (P_t) is not hyperbounded (for related results see [16]).

In the case of a Wiener noise; Chojnowska-Michalik and Goldys [8] following Simon [32] and Feyel and de La Pradelle [12] showed that for each $t \geqslant 0$, P_t is equal to the second quantized operator $\Gamma(S_0^*(t))$ of the adjoint semigroup $S_0^*(t)$, $t \geqslant 0$, where $S_0(t)$ is the original semigroup e^{At}, $t \geqslant 0$, "regarded" on the Reproducing Hilbert Kernel Space of μ. This representation is very useful for study properties of the transition semigroup (see Section 2 where the Chojnowska-Michalik and Goldys results will be sketched). The goal of the present paper is to formulate an analogous result for the Markov family defined by equation with Lévy noise (see Section 5).

We will use the fact that μ is the distribution of

$$Y_\infty = \int_0^\infty \mathrm{e}^{At}\,\mathrm{d}L(t), \tag{2}$$

and that the transition semigroup (P_t) is given by the *generalized Mehler formula* (see e.g. [2,15])

$$P_t f(x) = \int_E f\left(e^{At}x + y\right)\mu_t(dy), \quad x \in E,\ t \geqslant 0, \tag{3}$$

where μ_t is the distribution of

$$Y_t := \int_0^t e^{A(t-s)}\, dL(s) \tag{4}$$

or equivalently of $\int_0^t e^{As}\, dL(s)$.

2. Gaussian case

In this section we are dealing with (1) driven by a Wiener process. Namely, we assume that $L(t) = BW(t)$, where W is a cylindrical Wiener process on E, see e.g. [8,11,28], B is a bounded linear operator on E, and e^{At}, $t \geqslant 0$, is an exponentially stable semigroup on E satisfying

$$\int_0^\infty \left\|e^{At}B\right\|^2_{L_{(HS)}(E,E)}\, dt < \infty.$$

Note that this estimate is in fact if and only if condition for the existence of an invariant measure μ. Moreover, due to the stability of the semigroup e^{At}, $t \geqslant 0$, μ is unique. Finally μ is the distribution of Y_∞ given by (2), and μ is mean-zero, Gaussian with the covariance operator

$$Q_\infty := \int_0^\infty e^{As}BB^*e^{A^*s}\, ds.$$

To simplify the exposition we assume that $\operatorname{Ker} Q_\infty = \{0\}$.

Let us recall that the *Reproducing Hilbert Kernel Space* of μ is the space $E_0 := \operatorname{Range} Q_\infty^{1/2}$, equipped with the scalar product

$$\left\langle Q_\infty^{1/2}u, Q_\infty^{1/2}v\right\rangle_{E_0} = \langle u, v\rangle_E, \quad u, v \in E.$$

2.1. Second quantization

Given $h \in E_0$ define a linear functional $\psi_h(x) := \langle Q_\infty^{-1/2}h, x\rangle_E$, $x \in E$. Then

$$\int_E \psi_h(x)\psi_u(x)\,\mu(dx) = \left\langle Q_\infty Q_\infty^{-1/2}h, Q_\infty^{-1/2}u\right\rangle_E = \langle h, u\rangle_E, \quad h, u \in E_0. \tag{5}$$

Since E_0 is dense in E, for any $h \in E$ there is a sequence $(h_n) \subset E_0$ converging in E to h. By (5),

(ψ_{h_n}) converges in $L^2(\mu)$. The limit will be denoted by ψ_h. Clearly,

$$\int_E \psi_h(x)\psi_u(x)\,\mu(\mathrm{d}x) = \langle h, u\rangle_E, \quad \forall h, u \in E.$$

Let $\mathcal{P}_n$ be the closed subspace of $L^2(\mu)$ spanned by $p(\psi_{h_1}, \dots, \psi_{h_k})$, $k \in \mathbb{N}$, $h_1, \dots, h_k \in E$, and p is a polynomial of order $\leqslant n$. Let $\mathcal{H}_0$ be the space of all constant functions, and let $\mathcal{H}_n$, $n \in \mathbb{N}$, be the orthogonal complement of $\mathcal{H}_{n-1}$ in $\mathcal{P}_n$. The Itô–Wiener chaos decomposition says that

$$L^2(\mu) = \bigoplus_{n=0}^{\infty} \mathcal{H}_n.$$

Let Pr_n be the orthogonal projection of $L^2(\mu)$ into $\mathcal{H}_n$. Given an $R \in L(E, E)$ define $\Gamma_n(R) : \mathcal{H}_n \mapsto \mathcal{H}_n$, $n = 0, 1 \dots$, by

$$\Gamma_n(R) Pr_n(\psi_{h_1} \dots \psi_{h_n}) := Pr_n(\psi_{Rh_1} \dots \psi_{Rh_n}), \quad h_1, \dots, h_n \in E.$$

One can show that $\Gamma_n(R)$ is well defined and $\|\Gamma_n(R)\|_{L(\mathcal{H}_n,\mathcal{H}_n)} = \|R\|^n_{L(E,E)}$. Hence for any linear contraction R on E,

$$\Gamma(R) := \sum_{n=0}^{\infty} \Gamma_n(R) Pr_n$$

defines a contraction on $L^2(\mu)$. We call $\Gamma(R)$ the *second quantized operator* of R, and Γ the *second quantization operator*. In the Gaussian case the action of the second quantization operator is well understood. In fact, the following lemma gathers some basic properties of Γ. For its proof we refer the reader to Lemma 2 and Proposition 2 from [8], and to Chapter 1 of [32].

Theorem 2.1. *Assume that R, R_1, R_2 are contractions on E. Then*:

(a) $\Gamma(I_E) = I_{L^2(\mu)}$, *where I_E and $I_{L^2(\mu)}$ are the identity operators.*
(b) $\Gamma(R_1R_2) = \Gamma(R_1)\Gamma(R_2)$, $\Gamma(R^*) = \Gamma(R)^*$.
(c) $\Gamma(R)1 = 1$, *and $\Gamma(R)$ is positivity preserving*; *that is if $f \geqslant 0$, μ-a.s., then $\Gamma(R)f \geqslant 0$, μ-a.s.*
(d) *The operator $\Gamma(R)$ has an extension* (*restriction*) *to a positive contraction on every $L^p(\mu)$ for $p \geqslant 1$.*
(e) *For any $p \geqslant 1$ and*

$$q_0 = 1 + \frac{p-1}{\|R\|^2_{L(E,E)}},$$

we have $\|\Gamma(R)\|_{L(L^p(\mu),L^{q_0}(\mu))} = 1$ and if $q > q_0$, then $\|\Gamma(R)\|_{L(L^p(\mu),L^q(\mu))} = \infty$.

(f) *If R is self-adjoint with a complete set of eigenvectors (v_k), then $\Gamma(R)$ is also self-adjoint with the complete orthogonal set of eigenvectors*

$$\prod_{j=1}^{\infty} Pr_{a_j}\big(\psi_{v_j}^{a_j}\big) : (a_j) \subset \mathbb{N} \cup \{0\} \quad \textit{and} \quad \sum_j a_j < \infty.$$

(g) *Let $p, q \geqslant 1$ and $R \neq 0$. Then $\Gamma(R) : L^p(\mu) \mapsto L^q(\mu)$ is compact if and only if R is a compact strict contraction and $q < q_0$, q_0 is given in* (e).

(h) *The operator $\Gamma(R)$ is Hilbert–Schmidt on $L^2(\mu)$ if and only if R is a strict Hilbert–Schmidt contraction. Moreover,*

$$\big\|\Gamma(R)\big\|_{L_{(HS)}(L^2(\mu), L^2(\mu))} = \frac{1}{\sqrt{\det(I - R^*R)}}.$$

2.2. *Second quantization of Mehler semigroup*

The following lemma and theorem were formulated and proven in [8], see Lemma 4, and Theorems 1, 2, 3. Let μ_t be the distribution of the random variable Y_t given by (4). Clearly, μ_t is mean-zero Gaussian with the covariance

$$Q_t := \int_0^t \mathrm{e}^{As} BB^* \mathrm{e}^{A^*s}\, \mathrm{d}s.$$

It is convenient to formulate the following condition

$$\text{Range}\, Q_t^{1/2} = \text{Range}\, Q_\infty^{1/2} = E_0. \tag{6}$$

Lemma 2.2. *For any $t \geqslant 0$, $\mathrm{e}^{At} E_0 \subset E_0$, and $S_0(t) = Q_\infty^{-1/2} \mathrm{e}^{At} Q_\infty^{1/2}$, $t \geqslant 0$, is a C_0-semigroup of contractions on E. Moreover, $\|S_0(t)\|_{L(E,E)} < 1$ if and only if* (6) *holds.*

Theorem 2.3. *For any $t \geqslant 0$, $P_t = \Gamma(S_0^*(t))$ and $P_t^* = \Gamma(S_0(t))$. Moreover, the following statements hold*:

(a) *Let $t \geqslant 0$. If* (6) *holds, then for any $p, q \geqslant 1$, $\|P_t\|_{L(L^p(\mu), L^q(\mu))} = 1$ if and only if*

$$q \leqslant 1 + \frac{p-1}{\|S_0(t)\|^2_{L(E,E)}}.$$

Otherwise, $\|P_t\|_{L(L^p(\mu), L^q(\mu))} = \infty$.

(b) *For $p, q \geqslant 1$ and $t \geqslant 0$, the operator P_t is compact from $L^p(\mu)$ into $L^q(\mu)$ if and only if* (6) *holds, $S_0(t)$ is compact and*

$$q < 1 + \frac{p-1}{\|S_0(t)\|^2_{L(E,E)}}.$$

(c) *The operator P_t is Hilbert–Schmidt on $L^2(\mu)$ if and only if $S_0(t)$ is Hilbert–Schmidt on E and* (6) *holds. In this case*

$$\|P_t\|_{L_{(HS)}(L^2(\mu),L^2(\mu))} = \frac{1}{\sqrt{\det(I - S_0(t)S_0^*(t))}}.$$

3. Chaos decomposition in Poisson case

For the convenience of the reader we recall here some basic facts on the chaos decomposition in a Poisson case. This section is based on Last and Penrose [19], see also [18,22,27] and references therein. Namely, let $(E, \mathcal{B})$ be a measurable space, and let Π be a Poisson random measure on E with intensity measure λ. We assume that Π is defined on a probability space $(\Omega, \mathfrak{F}, \mathbb{P})$. Let $\mathbb{Z}_+(E)$ be the space of integer-valued σ-finite measures on $(E, \mathcal{B})$ with the σ-field $\mathcal{G}$ generated by the family of functions

$$\mathbb{Z}_+(E) \ni \xi \mapsto \xi(A) \in \{0, 1, 2, \dots, +\infty\}, \quad A \in \mathcal{B}.$$

Denote by $\mathbb{P}_\Pi$ the law of Π in $(\mathbb{Z}_+(E), \mathcal{G})$. Let $L^2(\mathbb{P}_\Pi)$ be the space of all measurable $F : \mathbb{Z}_+(E) \mapsto \mathbb{R}$ such that $|F|^2_{L^2(\mathbb{P}_\Pi)} := \mathbb{E}F^2(\Pi) < \infty$.

Given $F : \mathbb{Z}_+(E) \mapsto \mathbb{R}$, and $y \in E$ write

$$D_y F(\xi) := F(\xi + \delta_y) - F(\xi), \quad \xi \in \mathbb{Z}_+(E).$$

Differences $D^n_{y_1,\dots,y_n} F$, $n \in \mathbb{N}$, $y_1, \dots, y_n$, are defined by induction. Note that

$$D^n_{y_1,\dots,y_n} F(\xi) = \sum_{I \subset \{1,\dots,n\}} (-1)^{n-|I|} F\left(\xi + \sum_{i \in I} \delta_{y_i}\right), \quad \xi \in \mathbb{Z}_+(E). \tag{7}$$

Set $T^0(F) := \mathbb{E}F(\Pi)$, and for $n \in \mathbb{N}$,

$$T^n F(y_1, \dots, y_n) := \mathbb{E}D^n_{y_1,\dots,y_n} F(\Pi) = \int_{\mathbb{Z}_+(E)} D^n_{y_1,\dots,y_n} F(\xi)\, \mathbb{P}_\Pi(\mathrm{d}\xi),$$

provided that the function $D^n_{y_1,\dots,y_n} F$ appearing on the right-hand side is integrable with respect to $\mathbb{P}_\Pi$. We denote by $L^2_{(s)}(E^n, \lambda^n)$ the (closed) subspace of symmetric functions from $L^2(E^n, \lambda^n)$, with the scalar product inherited from $L^2(E^n, \lambda^n)$. We set $L^2_{(s)}(E^0, \lambda^0) := \mathbb{R}$.

Theorem 3.1. *For any $F \in L^2(\mathbb{P}_\Pi)$ and for λ^n-almost all $y_1, \dots, y_n \in E$, $T^n F(y_1, \dots, y_n)$ is well defined and $T^n F \in L^2_{(s)}(E^n, \lambda^n)$. Moreover, for any $F, G \in L^2(\mathbb{P}_\Pi)$,*

$$\mathbb{E}F(\Pi)G(\Pi) = \mathbb{E}F(\Pi)\mathbb{E}G(\Pi) + \sum_{n=1}^{\infty} \frac{1}{n!} \langle T^n F, T^n G \rangle_{L^2(E^n,\lambda^n)}.$$

Corollary 3.2. *For any n, T^n is a bounded acting operator from $L^2(\mathbb{P}_\Pi)$ into $L^2_{(s)}(E^n,\lambda^n)$ and its operator norm is bounded by $\sqrt{n!}$.*

In what follows $T := T^1$. For $f \in L^2(E^n,\lambda^n)$ we denote by $I_n(f)$ the multiple Itô integral with respect to the compensated measure $\tilde{\Pi} := \Pi - \lambda$. We set $I_0(f) := f$.

Theorem 3.3. *Let $F \in L^2(\mathbb{P}_\Pi)$. Then*

$$F(\Pi) = \sum_{n=0}^{\infty} \frac{1}{n!} I_n\big(T^n F\big).$$

Let $\mathcal{H}_0 = \mathbb{R}$, and let

$$\mathcal{H}_n := \big\{I_n(f)\colon\ f \in L^2_{(s)}\big(E^n,\lambda^n\big)\big\},\quad n \in \mathbb{N}.$$

By Theorems 3.1 and 3.3, $\mathcal{H}_n$, $n \in \mathbb{N}\cup\{0\}$, are orthogonal closed subspaces of $L^2(\Omega,\mathcal{F},\mathbb{P})$, and the operators

$$\frac{1}{\sqrt{n!}} I_n : L^2_{(s)}\big(E^n,\lambda^n\big) \ni f \mapsto \frac{1}{\sqrt{n!}} I_n(f) \in L^2(\Omega,\mathcal{F},\mathbb{P}),\quad n \in \mathbb{N}, \tag{8}$$

are linear and isometric. Let Pr_n be the orthogonal projection of $L^2(\Omega,\sigma(\Pi),\mathbb{P})$ into $\mathcal{H}_n$. Combining Theorems 3.1 and 3.3 we obtain:

Corollary 3.4. *One has*

$$L^2\big(\Omega,\sigma(\Pi),\mathbb{P}\big) = \bigoplus_{n=0}^{\infty} \mathcal{H}_n$$

and for any $F \in L^2(\mathbb{P}_\Pi)$, $Pr_0 F(\Pi) = \mathbb{E}F(\Pi)$, and $Pr_n F(\Pi) = \frac{1}{n!} I_n(T^n F)$, $n \in \mathbb{N}$.

4. Second quantization in Poisson case

Given an $R \in L(E,E)$, and a real-valued function f on E^n write

$$\rho_R^n f(y_1,\ldots,y_n) := f(Ry_1,\ldots,Ry_n),\quad y_1,\ldots,y_n \in E.$$

The proof of the following lemma is elementary.

Lemma 4.1. *If $\rho_R := \rho_R^1$ is a contraction on $L^2(E,\lambda)$, then for any $n \in \mathbb{N}$, ρ_R^n is a contraction on $L^2_{(s)}(E^n,\lambda^n)$. Moreover, $\|\rho_R^n\| \leqslant \|\rho_R\|^n$, where $\|\cdot\|$ stands for the operator norm on $L^2_{(s)}(E^n,\lambda^n)$ and on $L^2(E,\lambda)$.*

Taking into account Theorem 3.1 and Lemma 4.1 for any $R \in L(E,E)$ such that ρ_R is a contraction on $L^2(E,\lambda)$ we can define the *second quantized operator* $\Gamma(R) : L^2(\mathbb{P}_\Pi) \mapsto L^2(\mathbb{P}_\Pi)$ putting

$$\Gamma(R)F(\Pi) := \sum_{n=0}^{\infty} \frac{1}{n!} I_n\big(\rho_R^n(T_n F)\big). \tag{9}$$

Obviously, $\Gamma(R)$ is a contraction on $L^2(\mathbb{P}_\Pi)$.

5. Lévy–Ornstein–Uhlenbeck equation

Assume that:

(H.1) E is densely and continuously imbedded into $\tilde{E}$.

(H.2) For any $t > 0$, the semigroup e^{At} has an extension to a bounded linear map, denoted also by e^{At}, from $\tilde{E}$ into E, and that e^{At}, $t \geqslant 0$, is stable on E; that is $|e^{At}x|_E \to 0$ as $t \uparrow +\infty$ for any $x \in E$.

(H.3) L is a pure jump process; that is

$$\mathbb{E}\, e^{i\langle x, L(t)\rangle_{\tilde{E}}} = e^{-t\Psi(x)}, \quad x \in \tilde{E},$$

where the co-called *Lévy exponent*

$$\Psi(x) := \int_{\tilde{E}} \big(1 - e^{i\langle x,y\rangle_{\tilde{E}}} + i\langle x, y\rangle_{\tilde{E}}\, \chi_{\{|y|_{\tilde{E}} \leqslant 1\}}\big)\, \nu(dy),$$

and ν, called the *Lévy measure* of L, is a non-negative measure on $\tilde{E}$ satisfying $\int_{\tilde{E}} (|x|^2_{\tilde{E}} \wedge 1)\, \nu(dx) < \infty$.

(H.4)
$$\int_0^\infty \int_{\tilde{E}} \big|e^{As} y\big|_E \big|\chi_{\{|y|_{\tilde{E}} \leqslant 1\}} - \chi_{\{|e^{As}y|_E \leqslant 1\}}\big|\, \nu(dy)\, ds < \infty.$$

(H.5)
$$\int_0^\infty \int_{\tilde{E}} \big(\big|e^{As} y\big|^2_E \wedge 1\big)\, \nu(dy)\, ds < \infty.$$

For $t \in [0, +\infty]$, define

$$m_t := \int_0^t \int_{\tilde{E}} e^{As} y (\chi_{\{|y|_{\tilde{E}} \leqslant 1\}} - \chi_{\{|e^{As}y|_E \leqslant 1\}})\, \nu(dy)\, ds, \tag{10}$$

$$\nu_t := \int_0^t \nu \circ \big(e^{As}\big)^{-1}\, ds. \tag{11}$$

Clearly, by (H.4) and (H.5), for each $t \in [0, +\infty]$, the integral appearing in (10) converges, $m_t \in E$, and ν_t is a measure on E satisfying

$$\int_E \left(|y|_E^2 \wedge 1\right) \nu_t(\mathrm{d}y) < \infty.$$

Let

$$\Psi_t(x) := \mathrm{i}\langle x, m_t\rangle_E + \int_E \left(1 - \mathrm{e}^{\mathrm{i}\langle x,y\rangle_E} + \mathrm{i}\langle x, y\rangle_E \chi_{\{|y|_E \leqslant 1\}}\right) \nu_t(\mathrm{d}y). \tag{12}$$

Proposition 5.1. *Eq.* (1) *defines a Markov family* $(X^x,\ x \in E)$ *on* E*, with a unique invariant measure* μ*. Moreover,* $X^x(t) = \mathrm{e}^{At}x + Y_t$*, where* Y_t *is given by* (4)*, and its distribution* μ_t *is infinitely divisible with the Lévy exponent* Ψ_t *and Lévy measure* ν_t*. Finally* μ *is the distribution of* Y_∞ *defined by* (2)*, it is infinitely divisible with the Lévy exponent* Ψ_∞ *and Lévy measure* ν_∞*.*

Proof. This result is a rather standard generalization of [6]. Namely, the identity $X^x(t) = \mathrm{e}^{At}x + Y_t$ is the so-called mild formula for the solution. The question is only, if the process X^x or equivalently Y takes values in E. To see this note that the law of Y is infinitely divisible in $\tilde{E}$, with the jump measure ν_t. Then, by (H.5), ν_t is a jump measure of an infinitely-divisible law on E. For more details see [6]. □

Remark 5.2. Assume that $E = \tilde{E}$, and that the semigroup e^{At}, $t \geqslant 0$, is exponentially stable; that is there are $M, \omega > 0$ such that $\|\mathrm{e}^{At}\|_{L(E,E)} \leqslant M\mathrm{e}^{-\omega t}$, $t \geqslant 0$. Then (H.4) and (H.5) hold if and only if

$$\int_{\{|x|_E \geqslant 1\}} \log|x|_E\, \nu(\mathrm{d}x) < \infty,$$

see [31] for finite-dimensional case and [6] for infinite-dimensional case.

From now on $\mu = \mu_\infty$, $\lambda = \nu_\infty$, and Π is a Poisson random measure on E with the intensity measure λ. Given a $\xi \in \mathbb{Z}_+(E)$ write

$$\bar{\xi}(\mathrm{d}x) = \xi(\mathrm{d}x)\chi_{\{|x|_E > 1\}} + \left(\xi(\mathrm{d}x) - \lambda(\mathrm{d}x)\right)\chi_{\{|x|_E \leqslant 1\}}.$$

Recall that by the Lévy–Khinchin decomposition theorem there is a vector $m \in E$ such that

$$Y_\infty = m + \int_E x\, \overline{\Pi}(\mathrm{d}x).$$

6. Main result

The main result of the present paper can be illustrated by the following diagram.

$$\begin{array}{ccc} L^2(\mu) & \xrightarrow{P_t} & L^2(\mu) \\ {\scriptstyle j}\big\downarrow & & {\scriptstyle j}\big\downarrow \\ L^2(\mathbb{P}_\Pi) & \xrightarrow{\Gamma(\mathrm{e}^{At})} & L^2(\mathbb{P}_\Pi) \\ {\scriptstyle \tau}\big\| & & {\scriptstyle \tau}\big\| \\ \bigoplus_{n=0}^\infty L^2_{(s)}(E^n,\lambda^n) & \xrightarrow{\bigoplus_{n=0}^\infty \rho^n_{\mathrm{e}^{At}}} & \bigoplus_{n=0}^\infty L^2_{(s)}(E^n,\lambda^n). \end{array} \tag{13}$$

More precisely, we have the following theorem:

Theorem 6.1. *Under assumptions* (H.1) *to* (H.5)*, for any* $t>0$*,* $\rho_{\mathrm{e}^{At}}$ *is a contraction on* $L^2(E,\lambda)$ *and* (13) *holds with*

$$(jf)(\xi) := f\left(m + \int_E x\,\bar{\xi}(\mathrm{d}x)\right), \qquad f \in L^2(\mu),\ \xi \in \mathbb{Z}_+(E),$$

and

$$\tau := \bigoplus_{n=0}^\infty \frac{1}{\sqrt{n!}} T^n.$$

Proof. Let us fix a $t>0$. The contractivity of $\rho_{\mathrm{e}^{At}}$ on $L^2(E,\lambda)$ follows from the fact that $\lambda = \nu_\infty$ is given by (11). We have

$$\begin{aligned}\int_E \left|\rho_{\mathrm{e}^{At}} f(x)\right|_E^2 \lambda(\mathrm{d}x) &= \int_0^\infty \int_E \left|f\left(\mathrm{e}^{At}x\right)\right|_E^2 \nu\circ\left(\mathrm{e}^{As}\right)^{-1}(\mathrm{d}x)\,\mathrm{d}s \\ &= \int_0^\infty \int_E \left|f\left(\mathrm{e}^{A(t+s)}x\right)\right|_E^2 \nu(\mathrm{d}x)\,\mathrm{d}s \\ &\leqslant \int_0^\infty \int_E \left|f\left(\mathrm{e}^{As}x\right)\right|_E^2 \nu(\mathrm{d}x)\,\mathrm{d}s = \int_E \left|f(x)\right|_E^2 \lambda(\mathrm{d}x).\end{aligned}$$

To see (13) it is enough to show that for all $n \in \mathbb{N}$ and $f \in L^2(\mu)$,

$$T^n(jP_t f) = \rho^n_{\mathrm{e}^{At}} T^n(jf).$$

To do this fix an $f \in L^2(\mu)$. Given a $t \in [0, +\infty)$ denote by $\tilde{Y}_t$ a copy of Y_t, independent of Y_s, $s \in [0, +\infty]$, and by $\tilde{\mathbb{E}}$ the expectation with respect to $\tilde{Y}_t$, $t \in [0, +\infty)$. Note that for any t, the random variable $\mathrm{e}^{At} Y_\infty + \tilde{Y}_t$ has the law μ. Finally, by Proposition 5.1, the Mehler formula (3) reads

$$P_t f(x) = \tilde{\mathbb{E}} f\big(\mathrm{e}^{At} x + \tilde{Y}_t\big). \tag{14}$$

If $n = 0$, then, by (14),

$$\begin{aligned} T^0(jP_t f) &= \mathbb{E}(jP_t f)(\Pi) = \mathbb{E}P_t f(Y_\infty) = \mathbb{E}\tilde{\mathbb{E}} f\big(\mathrm{e}^{At} Y_\infty + \tilde{Y}_t\big) = \mathbb{E} f(Y_\infty) \\ &= \mathbb{E} jf(\Pi) = T^0(jf). \end{aligned}$$

Assume now that $n \geqslant 1$. Then by (7),

$$\begin{aligned} T^n(jP_t f)(y_1, \ldots, y_n) &= \mathbb{E} D^n_{y_1,\ldots,y_n} jP_t f(\Pi) \\ &= \mathbb{E} \sum_I (-1)^{n-|I|} P_t f\bigg(m + \int_E y \overline{\bigg(\sum_{i\in I} \delta_{y_i} + \Pi\bigg)}(\mathrm{d}y)\bigg) \\ &= \mathbb{E} \sum_I (-1)^{n-|I|} P_t f\bigg(\sum_{i\in I} y_i + Y_\infty\bigg). \end{aligned}$$

Hence, again by (14),

$$\begin{aligned} T^n(jP_t f)(y_1, \ldots, y_n) &= \sum_I (-1)^{n-|I|} \mathbb{E}\tilde{\mathbb{E}} f\bigg(\sum_{i\in I} \mathrm{e}^{At} y_i + \mathrm{e}^{At} Y_\infty + \tilde{Y}_t\bigg) \\ &= \sum_I (-1)^{n-|I|} \mathbb{E} f\bigg(\sum_{i\in I} \mathrm{e}^{At} y_i + Y_\infty\bigg) \\ &= \sum_I (-1)^{n-|I|} \mathbb{E} f\bigg(m + \int_E y \overline{\bigg(\sum_{i\in I} \delta_{\mathrm{e}^{At} y_i} + \Pi\bigg)}(\mathrm{d}y)\bigg) \\ &= \sum_I (-1)^{n-|I|} \mathbb{E} jf\bigg(\sum_{i\in I} \delta_{\mathrm{e}^{At} y_i} + \Pi\bigg) \\ &= \mathbb{E} D^n_{\mathrm{e}^{At} y_1,\ldots,\mathrm{e}^{At} y_n} jf(\Pi) \\ &= T^n(jf)\big(\mathrm{e}^{At} y_1, \ldots, \mathrm{e}^{At} y_n\big). \qquad \square \end{aligned}$$

As a direct consequence of Theorem 6.1 we have the following decomposition formula.

Corollary 6.2. *For all $t \geqslant 0$ and $f \in L^2(\mu)$,*

$$P_t f(Y_\infty) = \sum_{n=0}^{\infty} \frac{1}{n!} I_n\big(\rho^n_{\mathrm{e}^{At}} T^n(jf)\big), \tag{15}$$

where the series converges in $L^2(\Omega, \mathfrak{F}, \mathbb{P})$.

7. One-dimensional case with density

Assume that $E=\mathbb{R}$, and that μ is absolutely continuous with respect to Lebesgue measure. Let $q:=\mathrm{d}\mu/\mathrm{d}x$. We start this section with two simple general observations. We will use them to the study of the equation driven by an α-stable noise.

Note that

$$\kappa : L^2(\mathbb{R},\mathrm{d}x)\ni f\mapsto fq^{-1/2}\in L^2(\mu)$$

is a linear isometry. Let $\tilde{T}^n := T^n\circ j\circ\kappa : L^2(\mathbb{R},\mathrm{d}x)\mapsto L^2_{(s)}(\mathbb{R}^n,\lambda^n)$. By Theorem 6.1 we have the following fact.

Lemma 7.1. *Let $t>0$. If P_t is a compact operator on $L^2(\mu)$, then for any n, $\rho^n_{\mathrm{e}^{At}}\circ\tilde{T}^n$ is a compact operator from $L^2(\mathrm{d}x)$ to $L^2_{(s)}(\mathbb{R}^n,\lambda^n)$.*

For $f:\mathbb{R}\mapsto\mathbb{R}$ write

$$\nabla^n_{y_1,\ldots,y_n} f(x)=\sum_{I\subset\{1,\ldots,n\}}(-1)^{n-|I|} f\left(x-\sum_{i\in I} y_i\right),\quad n\in\mathbb{N}.$$

Lemma 7.2. *For any n, $f:\mathbb{R}\mapsto\mathbb{R}$ and $y=(y_1,\ldots,y_n)\in\mathbb{R}^n$,*

$$\tilde{T}^n f(y)=\int_{\mathbb{R}} G_n(x,y)f(x)\,\mathrm{d}x,$$

where

$$G_n(x,y):=q^{-1/2}(x)\nabla^n_y q(x).$$

Proof. We have

$$\begin{aligned}\tilde{T}^n f(y_1,\ldots,y_n)&=\mathbb{E}D^n_{y_1,\ldots,y_n} j\big(fq^{-1/2}\big)(\Pi)\\&=\sum_{I\subset\{1,\ldots,n\}}(-1)^{n-|I|}\int_{\mathbb{R}} f\left(z+\sum_{i\in I}y_i\right)q^{-1/2}\left(z+\sum_{i\in I}y_i\right)q(z)\,\mathrm{d}z\\&=\int_{\mathbb{R}} G_n(x,y)f(x)\,\mathrm{d}x.\qquad\square\end{aligned}$$

7.1. α-Stable case

Assume that L is a symmetric α-stable process taking values in $E=\mathbb{R}$. Then its Lévy exponent is $\Psi(x)=|x|^\alpha$ and the Lévy measure is $\lambda(\mathrm{d}x)=c_\alpha|x|^{-1-\alpha}\,\mathrm{d}x$. Given $\gamma>0$, consider the following one-dimensional Lévy–Ornstein–Uhlenbeck equation

$$\mathrm{d}X=-\gamma X\,\mathrm{d}t+(\alpha\gamma)^{1/\alpha}\,\mathrm{d}L. \tag{16}$$

Note that the invariant measure μ for the Markov family given by (16) is symmetric α-stable. Our main result of this section is the following theorem.

Theorem 7.3. *Let $\alpha \in (0,2)$. Then for any $t > 0$, P_t is neither compact nor self-adjoint. Moreover, the adjoint semigroup is given by*

$$P_t^* g(Y_\infty) = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-n\gamma\alpha t}}{n!} I_n\big(\rho^n_{\mathrm{e}^{\gamma t}}\big(T^n(jg)\big)\big), \quad \forall t \geqslant 0,\ \forall g \in L^2(\mu), \tag{17}$$

and (P_t) satisfies the following spectral gap property

$$\int_{\mathbb{R}} \left| P_t f(x) - \int_{\mathbb{R}} P_t f(z)\,\mu(\mathrm{d}z)\right|^2 \mu(\mathrm{d}x) \leqslant \mathrm{e}^{-\gamma\alpha t} |f|^2_{L^2(\mu)} \tag{18}$$

for all $t \geqslant 0$ and $f \in L^2(\mu)$.

Proof. Note for any $z \in \mathbb{R}\setminus\{0\}$, ρ_z is a bounded bijection on $L^2(\mathbb{R},\lambda)$. Therefore, as far as compactness is concerned, then, taking into account Lemma 7.1, it is enough to show that $\tilde{T} := \tilde{T}^1$ is not compact from $L^2(\mathrm{d}x)$ into $L^2(\mathbb{R},\lambda)$. By Lemma 7.2, $\tilde{T}$ is given by the kernel

$$G_1(x,y) = \frac{q(x-y) - q(x)}{\sqrt{q(x)}}, \quad x, y \in \mathbb{R},$$

where q is the density of the α-stable law μ. We will use the fact, see e.g. [17], that $q \in C^1(\mathbb{R})$, $q(x) > 0$, and $q(x)$ decreases like $|x|^{-1-\alpha}$ as $|x| \to \infty$. Let $f_n(x) = \chi_{[n,n+1]}(x)$, $n \in \mathbb{N}$, $x \in \mathbb{R}$. Then

$$g_n(y) := \tilde{T} f_n(y) = \int_0^1 \frac{q(x+n-y) - q(x+n)}{\sqrt{q(x+n)}}\,\mathrm{d}x, \quad y \in \mathbb{R},\ n \in \mathbb{N}.$$

Hence there are constants $C_0, C_1 > 0$ and $n_0 \in \mathbb{N}$ such that for $m \geqslant n_0$,

$$g_m(y) \geqslant C_0 \int_0^1 q(x+m)^{-1/2}\,\mathrm{d}x \geqslant C_1 m^{(1+\alpha)/2}, \quad \forall y \in [m, m+1].$$

Next note that for any n there is an m_n such that for all $m \geqslant m_n$,

$$q(x+n-y) \leqslant q(x+n), \quad \forall x \in [0,1],\ \forall y \in [m, m+1].$$

Therefore, for all $n \in \mathbb{N}$ and $m \geqslant m_n$,

$$
\begin{aligned}
|g_m - g_n|^2_{L^2(\mathbb{R},\lambda)} &\geqslant \int_m^{m+1} \left|g_m(y) - g_n(y)\right|^2 \frac{c_\alpha\,\mathrm{d}y}{|y|^{1+\alpha}} \\
&\geqslant \int_m^{m+1} C_1^2 m^{1+\alpha} \frac{c_\alpha\,\mathrm{d}y}{|y|^{1+\alpha}} \\
&\geqslant \alpha C_1^2 c_\alpha m^{1+\alpha}\bigl(m^{-\alpha} - (m+1)^{-\alpha}\bigr),
\end{aligned}
$$

and consequently there is a constant $C > 0$ such that

$$
|g_m - g_n|^2_{L^2(\mathbb{R},\lambda)} \geqslant C, \quad \forall n \geqslant n_0,\ \forall m \geqslant m_n.
$$

Thus the sequence (g_n) is not relatively compact in $L^2(\mathbb{R},\lambda)$, and hence $\tilde{T}$ is not compact.

We will show that for $t > 0$, P_t is not symmetric. By Theorems 3.1 and 6.1, for $f, g \in L^2(\mu)$,

$$
\begin{aligned}
\langle P_t f, g\rangle_{L^2(\mu)} &= \mathbb{E} P_t f(Y_\infty) g(Y_\infty) = \mathbb{E}(j P_t f)(\Pi)(jg)(\Pi) \\
&= \sum_{n=0}^\infty \frac{1}{n!} \mathbb{E}\bigl\langle \rho^n_{\mathrm{e}^{-\gamma t}} T^n(jf), T^n(jg)\bigr\rangle_{L^2(\mathbb{R},\lambda^n)}.
\end{aligned}
$$

Note that for the operator $\rho^n_{\mathrm{e}^{-\gamma t}}$ considered on $L^2(\mathbb{R}^n,\lambda^n)$ we have

$$
\bigl(\rho^n_{\mathrm{e}^{-\gamma t}}\bigr)^* h(y_1,\ldots,y_n) = \mathrm{e}^{-n\gamma\alpha t} h\bigl(\mathrm{e}^{\gamma t} y_1,\ldots,\mathrm{e}^{\gamma t} y_n\bigr),
$$

and hence (17) holds. To show that P_t is not symmetric it is enough to find a $g \in L^2(\mu)$ such that

$$
I_1\bigl(\rho_{\mathrm{e}^{-\gamma t}}\bigl(T(jg)\bigr)\bigr) \neq \mathrm{e}^{-\gamma\alpha t} I_1\bigl(\rho_{\mathrm{e}^{\gamma t}}\bigl(T(jg)\bigr)\bigr). \tag{19}
$$

Since

$$
T(jg)(y) = \mathbb{E}\bigl(g(Y_\infty + y) - g(Y_\infty)\bigr) = \int_{\mathbb{R}} \bigl(g(x+y) - g(x)\bigr)\,\mu(\mathrm{d}x),
$$

(19) can be written in the equivalent form

$$
\begin{aligned}
K_L &:= \int_{\mathbb{R}}\int_{\mathbb{R}} \bigl(g\bigl(x + \mathrm{e}^{-\gamma t} y\bigr) - g(x)\bigr)\,\mu(\mathrm{d}x)\,\Pi(\mathrm{d}y) \\
&\neq K_R := \mathrm{e}^{-\gamma\alpha t} \int_{\mathbb{R}}\int_{\mathbb{R}} \bigl(g\bigl(x + \mathrm{e}^{\gamma t} y\bigr) - g(x)\bigr)\,\mu(\mathrm{d}x)\,\Pi(\mathrm{d}y).
\end{aligned}
$$

Take $g(x)=\mathrm{e}^{\mathrm{i}x}$. Then

$$K_L=\mathrm{e}^{-1}\int_{\mathbb{R}}\left(\mathrm{e}^{\mathrm{i}\mathrm{e}^{-\gamma t}y}-1\right)\Pi(\mathrm{d}y),\qquad K_R=\mathrm{e}^{-\gamma\alpha t-1}\int_{\mathbb{R}}\left(\mathrm{e}^{\mathrm{i}\mathrm{e}^{\gamma t}y}-1\right)\Pi(\mathrm{d}y).$$

Thus for any $z\in\mathbb{R}$,

$$\begin{aligned}\mathbb{E}\mathrm{e}^{\mathrm{i}z\mathrm{e}K_L}&=\mathbb{E}\exp\left\{\mathrm{i}z\int_{\mathbb{R}}\left(\mathrm{e}^{\mathrm{i}\mathrm{e}^{-\gamma t}y}-1\right)\Pi(\mathrm{d}y)\right\}\\&=\exp\left\{-\int_{\mathbb{R}}\left(1-\mathrm{e}^{\mathrm{i}z(\mathrm{e}^{\mathrm{i}\mathrm{e}^{-\gamma t}y}-1)}\right)\lambda(\mathrm{d}y)\right\}\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\mathrm{e}^{\mathrm{i}z\mathrm{e}K_R}&=\mathbb{E}\exp\left\{\mathrm{i}z\mathrm{e}^{-\gamma\alpha t}\int_{\mathbb{R}}\left(\mathrm{e}^{\mathrm{i}\mathrm{e}^{\gamma t}y}-1\right)\Pi(\mathrm{d}y)\right\}\\&=\exp\left\{-\int_{\mathbb{R}}\left(1-\mathrm{e}^{\mathrm{i}z\mathrm{e}^{-\gamma\alpha t}(\mathrm{e}^{\mathrm{i}\mathrm{e}^{\gamma t}y}-1)}\right)\lambda(\mathrm{d}y)\right\}.\end{aligned}$$

To see the spectral gap property note that by Theorems 3.1, 6.1, and 3.3, for $f\in L^2(\mu)$,

$$\begin{aligned}\int_{\mathbb{R}}\left|P_tf(x)-\int_{\mathbb{R}}P_tf(z)\,\mu(\mathrm{d}z)\right|^2\mu(\mathrm{d}x)&=\mathbb{E}\left|P_tf(Y_\infty)-\mathbb{E}P_tf(Y_\infty)\right|^2\\&=\sum_{n=1}^{\infty}\left|\rho^n_{\mathrm{e}^{-\gamma t}}T^n(jf)\right|^2_{L^2(\mathbb{R}^n,\lambda^n)}.\end{aligned}$$

By direct calculation for any $z>0$ and any $h\in L^2(\mathbb{R}^n,\lambda^n)$, we have $|\rho^n_z h|_{L^2(\mathbb{R}^n,\lambda^n)}=z^{n\alpha/2}|h|_{L^2(\mathbb{R}^n,\lambda^n)}$. Hence

$$\left\|\rho^n_z\right\|_{L(L^2(\mathbb{R}^n,\lambda^n);L^2(\mathbb{R}^n,\lambda^n))}=z^{n\alpha/2},$$

and consequently (18) holds. □

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