# A model of neutrino mass and dark matter with an accidental symmetry 

Amine Ahriche ${ }^{\mathrm{a}, \mathrm{b}}$, Kristian L. McDonald ${ }^{\mathrm{c}, *}$, Salah Nasri ${ }^{\mathrm{d}}$, Takashi Toma ${ }^{\mathrm{e}}$<br>${ }^{\text {a }}$ Department of Physics, University of Jijel, PB 98 Ouled Aissa, DZ-18000 Jijel, Algeria<br>${ }^{\mathrm{b}}$ The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34014, Trieste, Italy<br>${ }^{\text {c }}$ School of Physics, The University of Sydney, NSW 2006, Australia<br>${ }^{\text {d }}$ Physics Department, UAE University, POB 17551, Al Ain, United Arab Emirates<br>${ }^{e}$ Laboratoire de Physique Théorique, CNRS UMR 8627, Université de Paris-Sud 11, F-91405 Orsay Cedex, France

## A R T I C L E I N F O

## Article history:

Received 30 April 2015
Accepted 14 May 2015
Available online 19 May 2015
Editor: J. Hisano


#### Abstract

We present a model of radiative neutrino mass that automatically contains an accidental $Z_{2}$ symmetry and thus provides a stable dark matter candidate. This allows a common framework for the origin of neutrino mass and dark matter without invoking any symmetries beyond those of the Standard Model. The model can be probed by direct-detection experiments and $\mu \rightarrow e+\gamma$ searches, and predicts a charged scalar that can appear at the TeV scale, within reach of collider experiments.

Crown Copyright © 2015 Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP3.


## 1. Introduction

The existence of massive neutrinos provides concrete evidence for physics beyond the Standard Model (SM). Similarly, the explanation of observed galactic rotation curves in terms of gravitating dark matter (DM) further suggests the SM is incomplete. Efforts to explain these two key evidences for new physics are varied, though an interesting approach is to seek a common or unified framework that simultaneously solves both puzzles. For example, if small neutrino masses are realized via radiative effects [1], it is conceivable that DM plays a role in generating the masses, allowing a type of unified description for massive neutrinos and DM. This is the motivation for the models of Krauss, Nasri and Trodden (KNT) [2-4] and Ma [5,6]. Both models extend the SM so that neutrino masses are generated radiatively with DM propagating in the loop diagram. In order to ensure DM stability (and preclude tree-level neutrino mass) a $Z_{2}$ symmetry is also imposed.

There are a number of generalizations of this basic idea which similarly extend the SM to allow radiative neutrino mass via couplings to DM [7-12]. In common with the KNT and Ma models, the generalized models also require the imposition of a new symme-

[^0]try to render the DM stable. ${ }^{1}$ However, it is interesting to consider models where DM stability instead results from an accidental symmetry, in accordance with our experience from the SM, where proton stability manifests the accidental baryon number symmetry.

In this work we present a model of radiative neutrino mass that automatically contains an accidental $Z_{2}$ symmetry and thus admits a stable DM candidate. The model realizes a simple unified framework for the origin of neutrino mass and DM while imposing only a minimal symmetry structure, namely that of the SM. Neutrino mass appears at the three-loop level via a diagram with the same topology as the KNT model, while the DM is a neutral fermion with a non-trivial charge under the accidental $Z_{2}$ symmetry. The model requires heavy DM ( $M_{\mathrm{DM}} \sim 20 \mathrm{TeV}$ ) and may be probed via DM direct-detection experiments and future $\mu \rightarrow e+\gamma$ searches. It also predicts a charged scalar that can appear at the TeV scale.

The layout of this paper is as follows. The model is introduced in Section 2. We calculate neutrino masses and discuss important constraints in Section 3. Relevant information regarding the DM is discussed in Section 4 while our main numerical analysis and results appear in Section 5. Conclusions are drawn in Section 6.

[^1]
## 2. The model

### 2.1. Field content

We extend the SM to include a charged singlet scalar, $S^{+} \sim$ $(1,1,2)$, a scalar septuplet, $\phi \sim(1,7,2)$, and three real septuplet fermions, $\mathcal{F}_{i} \sim(1,7,0)$, where $i=1,2,3$ labels generations. We adopt the symmetric-matrix notation for the septuplets, writing the scalar as $\phi_{a b c d e f}$, with $a, b, \ldots \in\{1,2\}$. The components are given by
$\phi_{111111}=\phi^{++++}, \phi_{111112}=\frac{\phi^{+++}}{\sqrt{6}}, \phi_{111122}=\frac{\phi^{++}}{\sqrt{15}}$,
$\phi_{111222}=\frac{\phi^{+}}{\sqrt{20}}, \phi_{112222}=\frac{\phi^{0}}{\sqrt{15}}$,
$\phi_{122222}=\frac{\phi^{-}}{\sqrt{6}}, \phi_{222222}=\phi^{--}$,
where $\phi^{++}$and $\phi^{--}$are distinct fields, $\phi^{--} \neq\left(\phi^{++}\right)^{*}$, and similarly $\phi^{-} \neq\left(\phi^{+}\right)^{*}$. For the septuplet fermions, denoted as $\mathcal{F}_{\text {abcdef }}$, we have
$\mathcal{F}_{111111}=\mathcal{F}_{L}^{+++}, \mathcal{F}_{111112}=\frac{\mathcal{F}_{L}^{++}}{\sqrt{6}}, \mathcal{F}_{111122}=\frac{\mathcal{F}_{L}^{+}}{\sqrt{15}}$,
$\mathcal{F}_{111222}=\frac{\mathcal{F}_{L}^{0}}{\sqrt{20}}, \mathcal{F}_{112222}=\frac{\left(\mathcal{F}_{R}^{+}\right)^{c}}{\sqrt{15}}$,
$\mathcal{F}_{122222}=\frac{\left(\mathcal{F}_{R}^{++}\right)^{c}}{\sqrt{6}}, \mathcal{F}_{222222}=\left(\mathcal{F}_{R}^{+++}\right)^{c}$.
The superscript " $c$ " denotes charge conjugation and the numerical factors ensure the kinetic terms are canonically normalized. With these fields, the Lagrangian contains the terms

$$
\begin{align*}
\mathcal{L} \supset & \mathcal{L}_{\mathrm{SM}}-\frac{1}{2} \overline{\mathcal{F}_{i}^{c}} \mathcal{M}_{i j} \mathcal{F}_{j} \\
& +\left\{g_{i \alpha} \overline{\mathcal{F}}_{i} \phi e_{\alpha R}+f_{\alpha \beta} \overline{L_{\alpha}^{c}} L_{\beta} S^{+}+\text {H.c. }\right\}-V(H, S, \phi) \tag{3}
\end{align*}
$$

where lepton flavors are labeled by lower-case Greek letters, $\alpha, \beta \in\{e, \mu, \tau\}$, and $L\left(e_{R}\right)$ is a SM lepton doublet (singlet). The scalar potential is denoted as $V(H, S, \phi)$. Note that the exotics $\phi$ and $\mathcal{F}$ do not couple directly to the SM neutrinos, though they shall play a key role in generating neutrino mass.

The explicit expansion for the fermion mass term is:

$$
\begin{align*}
& -\frac{1}{2}\left(\overline{\mathcal{F}_{i}^{c}}\right)_{a b c d e f} \mathcal{M}_{i j}\left(\mathcal{F}_{j}\right)_{p q r s t u} \epsilon^{a p} \epsilon^{b q} \epsilon^{c r} \epsilon^{d s} \epsilon^{e t} \epsilon^{f u}+\text { H.c. } \\
= & -\mathcal{M}_{i j}\left\{\overline{\mathcal{F}_{i R}^{+++}} \mathcal{F}_{j L}^{+++}-\overline{\mathcal{F}_{i R}^{++}} \mathcal{F}_{j L}^{++}+\overline{\mathcal{F}_{i R}^{+}} \mathcal{F}_{j L}^{+}-\frac{1}{2} \overline{\left(\mathcal{F}_{i L}^{0}\right)^{c}} \mathcal{F}_{j L}^{0}\right\} \\
& + \text { н.c. } \\
= & -\mathcal{M}_{i j}\left\{\overline{\mathcal{F}_{i}^{+++}} \mathcal{F}_{j}^{+++}+\overline{\mathcal{F}_{i}^{++}} \mathcal{F}_{j}^{++}+\overline{\mathcal{F}_{i}^{+}} \mathcal{F}_{j}^{+}+\frac{1}{2} \overline{\mathcal{F}_{i}^{0}} \mathcal{F}_{j}^{0}\right\}, \tag{4}
\end{align*}
$$

where we defined:
$\mathcal{F}^{+++}=\mathcal{F}_{L}^{+++}+\mathcal{F}_{R}^{+++}, \quad \mathcal{F}^{++}=\mathcal{F}_{L}^{++}-\mathcal{F}_{R}^{++}$,
$\mathcal{F}^{+}=\mathcal{F}_{L}^{+}+\mathcal{F}_{R}^{+}, \quad \mathcal{F}^{0}=\mathcal{F}_{L}^{0}-\left(\mathcal{F}_{L}^{0}\right)^{c}$.
Clearly $\mathcal{F}^{0}$ is a Majorana fermion, while the other six components of $\mathcal{F}$ partner-up to give three massive charged fermions (per generation). Without loss of generality, we choose a diagonal basis for the fermions, such that $\mathcal{M}_{i j}=\operatorname{diag}\left(M_{1}, M_{2}, M_{3}\right)$, with the masses ordered as $M_{1}<M_{2}<M_{3}$. We shall see below that $\mathcal{F}$ does not
mix with the SM leptons, to all orders of perturbation theory, so Eq. (5) describes the mass eigenstates, which should be used in the Yukawa terms in Eq. (3). The lightest neutral fermion will play the role of DM, and we denote its mass as $M_{\mathrm{DM}} \equiv M_{1}$.

### 2.2. An accidental symmetry

The model contains an exact accidental $Z_{2}$ symmetry with action:

$$
\begin{equation*}
\{\phi, \mathcal{F}\} \rightarrow\{-\phi,-\mathcal{F}\} \tag{6}
\end{equation*}
$$

To see this, note that the potential can be written as

$$
\begin{align*}
V(H, S, \phi)= & V(H)+V(\phi)+V(S) \\
& +V_{m}(H, S)+V_{m}(H, \phi)+V_{m}(S, \phi) \tag{7}
\end{align*}
$$

The first four terms trivially preserve the discrete symmetry, while the explicit forms for the last two mixing potentials are ${ }^{2}$

$$
\begin{align*}
V_{m}(H, \phi)= & \lambda_{H \phi 1}\left(H^{*}\right)^{a^{\prime}} H_{a^{\prime}}\left(\phi^{*}\right)^{a b c d e f} \phi_{a b c d e f} \\
& +\lambda_{H \phi 2}\left(H^{*}\right)^{a^{\prime}} H_{a}\left(\phi^{*}\right)^{a b c d e f} \phi_{a^{\prime} b c d e f} \tag{8}
\end{align*}
$$

and

$$
\begin{align*}
V_{m}(S, \phi)= & \lambda_{s \phi}|S|^{2}\left(\phi^{*}\right)^{a b c d e f} \phi_{a b c d e f} \\
& +\frac{\lambda_{s}}{4}\left(S^{-}\right)^{2} \phi_{a b c d e f} \phi_{a^{\prime} b^{\prime} c^{\prime} d^{\prime} e^{\prime} f^{\prime}} \epsilon^{a a^{\prime}} \epsilon^{b b^{\prime}} \epsilon^{c c^{\prime}} \epsilon^{d d^{\prime}} \epsilon^{e e^{\prime}} \epsilon^{f f^{\prime}} \\
& + \text { H.c. } \tag{9}
\end{align*}
$$

These potentials also preserve the symmetry defined by Eq. (6). Note that there appears to be a third distinct way to contract the $S U(2)$ indices in the mixing potential $V_{m}(S, \phi)$, namely

$$
\begin{equation*}
S^{-}\left(\phi^{*}\right)^{a b c d e f} \phi_{a b c a^{\prime} b^{\prime} c^{\prime}} \phi_{d e f d^{\prime} e^{\prime} f^{\prime}} \epsilon^{a^{\prime} d^{\prime}} \epsilon^{b^{\prime} e^{\prime}} \epsilon^{c^{\prime} f^{\prime}} \tag{10}
\end{equation*}
$$

This would explicitly break the $Z_{2}$ symmetry. However, this term is odd under the simultaneous interchange of the sets of dummy indices $\{a, b, c\} \leftrightarrow\{d, e, f\}$ and $\left\{a^{\prime}, b^{\prime}, c^{\prime}\right\} \leftrightarrow\left\{d^{\prime}, e^{\prime}, f^{\prime}\right\}[10]$, and thus vanishes identically. The full theory therefore preserves the accidental $Z_{2}$ symmetry defined by Eq. (6) and the model automatically contains an absolutely stable particle that is a DM candidate. The $Z_{2}$ symmetry also prevents mixing between $\mathcal{F}$ and the SM leptons. To the best of our knowledge this is the first such model of radiative neutrino mass with an accidental symmetry that automatically gives a DM candidate.

At tree-level the components of $\mathcal{F}$ are mass-degenerate, while the components of $\phi$ experience a mild splitting due to the $\lambda_{H \phi 2}$-term in $V_{m}(H, \phi)$. For $M_{\phi} \gtrsim \mathcal{O}(\mathrm{TeV})$ this mass-splitting is not significant and is essentially negligible for $\lambda_{H \phi 2} \lesssim 0.1$. Thus, to good approximation the components of $\mathcal{F}_{i}$ are degenerate at tree-level, with masses $M_{i}$, as are the components of $\phi$ (with masses $M_{\phi}$ ). Radiative corrections remove these mass degeneracies; loops containing SM gauge bosons give small mass-splittings for the components of $\mathcal{F}$, leaving $\mathcal{F}^{0}$ as the lightest exotic fermion. Similar splittings are induced for the components of $\phi$ which are readily calculated with the results of Ref. [14]. For most purposes in this work these tiny splittings can be ignored.

The model contains two distinct $Z_{2}$-odd DM candidates, namely $\mathcal{F}_{1}^{0}$ and $\phi^{0}$. However, $\phi^{0}$ has degenerate real and imaginary components and also couples to the $Z$ boson. This leads to tree-level $Z$ boson exchanges that are incompatible with direct detection constraints. Thus, $\phi^{0}$ can be excluded as a DM candidate, leaving $\mathcal{F}_{1}^{0}$

[^2]as the sole DM candidate in the model and restricting one to the parameter space with $M_{\mathrm{DM}}=M_{1}<M_{\phi}$. The SM Higgs develops a nonzero vacuum value, $\langle H\rangle \neq 0$, breaking the electroweak symmetry in the usual way. Furthermore, in the parameter space with $\langle\phi\rangle=0$, which preserves the discrete symmetry, the $\rho$-parameter retains its standard tree-level value. ${ }^{3}$

Note that the inclusion of non-renormalizable dimension 5 operators ( $D=5$ ) can break the accidental symmetry. In particular, the $D=5$ operator $H H \phi^{\dagger} \phi^{\dagger} \phi$ would allow the DM to decay. This feature is not specific to the present model; one expects global symmetries to be broken by gravitational effects [15] so nonrenormalizable operators will, in general, break global symmetries. This is true even in related models which impose e.g. a global $Z_{2}$ or $U(1)$ symmetry. In our model, the fate of the accidental $Z_{2}$ symmetry is analogous to the fate of the accidental baryon number symmetry in the SM. The latter is broken by $D=5$ operators, leading to proton decay. However, proton longevity can be ensured by the details of the UV completion, giving either a long-lived or absolutely stable proton. Unsurprisingly, the situation is similar for our DM candidate. We focus on the renormalizable theory here but note that a cutoff that adequately suppresses proton decay also ensures DM longevity.

We note that a number of works have studied larger multiplets in connection with neutrino mass [10,12,16] (for related phenomenology see Ref. [17]). In particular, Ref. [12] recently considered stable quintuplet fermionic DM in a three-loop model of neutrino mass. ${ }^{4}$

## 3. Three-loop neutrino mass and lepton flavor violating constraints

The combination of the Yukawa Lagrangian and the terms

$$
\begin{aligned}
& V(H, S, \phi) \supset \frac{\lambda_{\mathrm{S}}}{4}\left(S^{-}\right)^{2} \phi_{a b c d e f} \phi_{a^{\prime} b^{\prime} c^{\prime} d^{\prime} e^{\prime} f^{\prime}} \epsilon^{a a^{\prime}} \epsilon^{b b^{\prime}} \epsilon^{c c^{\prime}} \epsilon^{d d^{\prime}} \epsilon^{e e^{\prime}} \epsilon^{f f^{\prime}} \\
& =\frac{\lambda \mathrm{s}}{2}\left(S^{-}\right)^{2}\left\{\phi^{++++} \phi^{--}-\phi^{+++} \phi^{-}+\phi^{++} \phi^{0}-\frac{1}{2} \phi^{+} \phi^{+}\right\}
\end{aligned}
$$

in the scalar potential, are sufficient to explicitly break lepton number symmetry. Consequently SM neutrinos are Majorana particles that acquire radiative masses at the three-loop level, as shown in Fig. 1. In the limit where the mass-splittings among components of $\phi$ and $\mathcal{F}$ are neglected, the calculation of the loop-diagram gives
$\left(\mathcal{M}_{\nu}\right)_{\alpha \beta}=\frac{7 \lambda_{\mathrm{S}}}{\left(4 \pi^{2}\right)^{3}} \frac{m_{\gamma} m_{\delta}}{M_{\phi}} f_{\alpha \gamma} f_{\beta \delta} g_{\gamma i}^{*} g_{\delta i}^{*} \times F\left(\frac{M_{i}^{2}}{M_{\phi}^{2}}, \frac{M_{S}^{2}}{M_{\phi}^{2}}\right)$,
where the function $F$ encodes the loop integrals [4] and $M_{\mathrm{S}}$ is the charged-singlet mass.

Neutrino masses calculated via Eq. (11) must satisfy the data from neutrino oscillation experiments and reproduce the following best-fit regions for the mixing angles and mass-squared differences: $s_{12}^{2}=0.320_{-0.017}^{+0.016}, s_{23}^{2}=0.43_{-0.03}^{+0.03}, s_{13}^{2}=0.025_{-0.003}^{+0.003}$, $\Delta m_{21}^{2}=7.62_{-0.19}^{+0.19} \times 10^{-5} \mathrm{eV}^{2}$, and $\left|\Delta m_{13}^{2}\right|=2.55_{-0.09}^{+0.06} \times 10^{-3} \mathrm{eV}^{2}$ [18]. Matching to these experimental values reveals the regions of parameter space where the model gives viable neutrino masses.

The Yukawa couplings $g_{i \alpha}$ generate flavor changing processes like $\mu \rightarrow e+\gamma$. Calculating the corresponding diagrams in the limit

[^3]

Fig. 1. Diagram for radiative neutrino mass, where $S$ and $\phi$ are new scalars and $\mathcal{F}$ is an exotic fermion. The lightest component of $\mathcal{F}$ is a stable dark matter candidate.
where the mass-splittings are neglected, and including the diagram containing the singlet $S$, gives

$$
\begin{align*}
\mathcal{B}(\mu \rightarrow e \gamma) \simeq & \frac{\alpha v^{4}}{384 \pi} \\
& \times\left\{\frac{1764}{M_{\phi}^{4}}\left|\sum_{i} g_{i \mu} g_{i e}^{*} F_{2}\left(M_{i}^{2} / M_{\phi}^{2}\right)\right|^{2}+\frac{\left|f_{\tau e}^{*} f_{\mu \tau}\right|^{2}}{M_{\mathrm{S}}^{4}}\right\}, \tag{12}
\end{align*}
$$

where $F_{2}(x)=\left[1-6 x+3 x^{2}+2 x^{3}-6 x^{2} \log x\right] /\left[6(x-1)^{4}\right]$ and $v$ is the vacuum expectation value of $H$. The related expression for $\mathcal{B}(\tau \rightarrow \mu+\gamma)$ is obtained by a simple change of flavor labels in Eq. (12). Replacing the final-state electrons with muons in the diagram for $\mu \rightarrow e+\gamma$ gives the one-loop contributions to the muon's anomalous magnetic moment. In the limit where the radiative mass-splittings are neglected these give
$\delta a_{\mu}=-\frac{m_{\mu}^{2}}{16 \pi^{2}}\left\{\sum_{i} \frac{7\left|g_{i \mu}\right|^{2}}{M_{\phi}^{2}} F_{2}\left(M_{i}^{2} / M_{\phi}^{2}\right)+\sum_{\alpha \neq \mu} \frac{\left|f_{\mu \alpha}\right|^{2}}{6 M_{S}^{2}}\right\}$.
The last term is due to the charged scalar $S$.
A further constraint of $\left(\mathcal{M}_{\nu}\right)_{e e} \lesssim 0.35 \mathrm{eV}$ follows from nullresults in searches for neutrino-less double-beta decay [19], though analysis shows that this constraint is readily satisfied in the model. This constraint is expected to improve after next generation experiments, with an anticipated precision of $\left(\mathcal{M}_{\nu}\right)_{e e} \lesssim 0.01 \mathrm{eV}$ [20].

## 4. Dark matter

### 4.1. Relic density

As mentioned already, the only viable DM candidate in the model is the lightest neutral fermion $\mathcal{F}_{1}^{0}$. There are two classes of interactions that can maintain thermal contact between the DM and the SM in the early universe. Interactions mediated by the scalar $\phi$ have the cross section
$\sigma\left(2 \mathcal{F}^{0} \rightarrow \ell_{\beta}^{+} \ell_{\alpha}^{-}\right)=\frac{\left|g_{1 \beta} g_{1 \alpha}^{*}\right|^{2}}{48 \pi} \frac{M_{\mathrm{DM}}^{2}\left(M_{\mathrm{DM}}^{4}+M_{\phi}^{4}\right)}{\left(M_{\mathrm{DM}}^{2}+M_{\phi}^{2}\right)^{4}} \times v_{r} \equiv \sigma_{0,0}^{\alpha \beta}$
where $v_{r}$ is the DM relative velocity, in the center-of-mass frame. Note that there are no $s$-wave annihilations when final-state lepton masses are neglected, as the DM is a Majorana fermion. There are no coannihilations mediated by $\phi$, though given the small radiative mass-splittings, one should include the annihilations of singly-charged fermions:

$$
\begin{align*}
\sigma\left(\mathcal{F}^{-} \mathcal{F}^{+} \rightarrow \ell_{\beta}^{+} \ell_{\alpha}^{-}\right) & =\frac{\left|g_{1 \beta} g_{1 \alpha}^{*}\right|^{2}}{48 \pi} \frac{M_{\mathrm{DM}}^{2}\left(M_{\mathrm{DM}}^{4}+M_{\phi}^{4}\right)}{\left(M_{\mathrm{DM}}^{2}+M_{\phi}^{2}\right)^{4}} \times v_{r} \\
& \equiv \sigma_{ \pm}^{\alpha \beta} \tag{15}
\end{align*}
$$

and similarly for the higher-charged fermions

$$
\begin{align*}
& \sigma\left(\mathcal{F}^{--} \mathcal{F}^{++}\right.\left.\rightarrow \ell_{\beta}^{+} \ell_{\alpha}^{-}\right) \equiv \sigma_{ \pm \pm}^{\alpha \beta}=\sigma_{ \pm}^{\alpha \beta}  \tag{16}\\
& \sigma\left(\mathcal{F}^{---} \mathcal{F}^{+++} \rightarrow \ell_{\beta}^{+} \ell_{\alpha}^{-}\right) \equiv \sigma_{ \pm \pm \pm}^{\alpha \beta}=\sigma_{ \pm}^{\alpha \beta} \tag{17}
\end{align*}
$$

There are also processes mediated by $S U(2)_{L}$ gauge bosons, which can be calculated in the limit of an exact $S U(2)$ symmetry. The corresponding cross sections can be obtained with the results of Ref. [21]. Due to the small mass-splitting among the components of $\mathcal{F}_{1}$, one should also include coannihilation processes. Adding annihilation and coannihilation channels together in the standard way gives [22]

$$
\begin{align*}
& \sigma_{e f f}(2 \mathcal{F} \rightarrow S M) \times v_{r} \\
& =\frac{1}{g_{\text {eff }}^{2}}\left[\sigma_{W} \times v_{r}+\sum_{\alpha, \beta}\left\{g_{0}^{2} \sigma_{0,0}^{\alpha \beta}+2 g_{ \pm} \sigma_{ \pm}^{\alpha \beta}\right.\right. \\
& \left.\left.+2 g_{ \pm \pm} \sigma_{ \pm \pm}^{\alpha \beta}+2 g_{ \pm \pm \pm} \sigma_{ \pm \pm \pm}^{\alpha \beta}\right\} \times v_{r}\right], \tag{18}
\end{align*}
$$

where the mass-splittings among fermion components are neglected and the $S U(2)_{L}$ channels give
$\sigma_{W} \equiv \frac{7 \pi \alpha_{2}^{2}}{2 M_{\mathrm{DM}}^{2} v_{r}}\left\{1392+526 v_{r}^{2}\right\}$.
In the above, $g_{\text {eff }}=g_{0}+2 g_{ \pm}+2 g_{ \pm \pm}+2 g_{ \pm \pm \pm}$, with $g_{0}=g_{ \pm}=$ $g_{ \pm \pm}=g_{ \pm \pm \pm}=2$.

In principle one can calculate the mass range that gives a viable DM relic density using the above expressions. However, the cross section into gauge bosons may be significantly enhanced by the non-perturbative Sommerfeld correction [23-25]. One must solve the Schrödinger equation in terms of non-relativistic bound state of two DM particles in order to estimate the non-perturbative Sommerfeld correction. The calculation is somewhat involved, though the correction has been calculated for several $S U(2)_{L}$ multiplets in Ref. [21] and the effect is found to be important for larger multiplets. The enhancement of the cross section influences the DM mass required to give the observed relic density as the DM mass is the unique parameter that can control the cross section when the annihilation cross section is dominated by gauge interactions. ${ }^{5}$ For example, the DM mass is shifted from 3.8 TeV to 9.5 TeV for a fermion quintuplet with $Y=0$, from 5.0 TeV to 9.4 TeV for scalar quintuplet with $Y=0$, and from 8.5 TeV to 25 TeV for scalar septuplet with $Y=0$ [26]. A similar enhancement is expected for the fermion septuplet DM with $Y=0$ in our model, though a detailed calculation is beyond the scope of this work. Guided by the results listed in Ref. [26] we expect the Sommerfeld enhancement will increase the requisite DM mass by a factor of approximately 3. As we shall see, this suggests the required DM mass should be $\sim 20-25 \mathrm{TeV}$ when the Sommerfeld effect is taken into account.

The DM annihilation processes which induce monochromatic gamma-rays are also enhanced by the Sommerfeld correction in the present universe. This can be a significant signature of DM as an indirect detection signal. Since the DM mass is predicted around $M_{\mathrm{DM}}=20-25 \mathrm{TeV}$ in our model, after including Sommerfeld correction, monochromatic gamma-rays at $E_{\gamma}=M_{\mathrm{DM}}$ could be detected by future gamma-ray experiments such as CTA [27].

[^4]

Fig. 2. The DM and charged scalar masses versus the scalar septuplet mass for the case with no Sommerfeld enhancement. The blue line at $M_{\mathrm{DM}}=7.2 \mathrm{TeV}$ gives the best-fit value for $\Omega_{\mathrm{DM}} h^{2}$ in the limit $g_{i \alpha} \rightarrow 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

### 4.2. Direct detection

There is no tree-level coupling between DM and quarks. However, $W$ boson exchange gives three one-loop diagrams which can produce signals at direct-detection experiments [9]. There are both spin-dependent and spin-independent contributions to the scattering, however, spin-dependent contributions are suppressed by the heavy DM mass. As we consider relatively heavy values of $M_{\mathrm{DM}}>1 \mathrm{TeV}$, the spin-dependent contributions can be neglected. Therefore spin-independent scattering dominates and the cross section is determined by SM interactions:
$\sigma_{\mathrm{SI}}\left(\mathcal{F}^{0} N \rightarrow \mathcal{F}^{0} N\right) \simeq \frac{36 \pi \alpha_{2}^{4} M_{A}^{4} f^{2}}{M_{W}^{2}}\left[\frac{1}{M_{h}^{2}}+\frac{1}{M_{W}^{2}}\right]^{2}$.
The DM scatters from a target nucleus $A$ of mass $M_{A}$, and the standard parametrization for the nucleon is adopted:
$\langle N| \sum_{q} m_{q} \bar{q} q|N\rangle=f m_{N}$.
Here $m_{N}$ is the nucleon mass and $f=\sum_{q} f_{q}$ is subject to the standard QCD uncertainties. For $f \approx 0.3$, the cross section for the one-loop processes is $\sigma_{\mathrm{SI}} \simeq 4 \times 10^{-44} \mathrm{~cm}^{2}$, which is just beyond the sensitivity of LUX [28] for heavy DM with $M_{\mathrm{DM}} \sim 25 \mathrm{TeV}$. Note, however, that recent lattice simulations suggest a somewhat lower value of strange content $f_{s} \approx 0.043 \pm 0.011$ [29], which, when combined with cancellations from two-loop diagrams, gives a smaller cross section of $\sigma_{\mathrm{SI}} \approx 4 \times 10^{-46} \mathrm{~cm}^{2}$ [30]. ${ }^{6}$ In either case, the result is beyond the current sensitivity of LUX, though future discovery prospects for the DM candidate can be considered promising.

## 5. Numerical results and discussion

As already mentioned above, to determine the viable DM mass range one should include the Sommerfeld enhancement. However, as a first task we perform a numerical scan of the parameter space without the Sommerfeld enhancement, determining the favored DM mass range. We subsequently include a simple estimate of the effect.

For the numerical scan we seek regions of parameter space that satisfy the previously mentioned constraints, while simultaneously giving neutrino masses and mixings in agreement with

[^5]

Fig. 3. Numerical results for the case where the Sommerfeld enhancement is considered. Left: the Yukawa coupling range that satisfies all previous constraints. The dashed line represents the case where all couplings are close in absolute value, i.e., $\min (|f|)=\max (|f|)$. Center: the masses $M_{\mathrm{DM}}$ and $M_{\mathrm{S}}$ versus the septuplet scalar mass $M_{\phi}$. The blue line gives the $M_{\mathrm{DM}}$ best-fit value for $\Omega_{\mathrm{DM}} h^{2}$ when $g_{i \alpha} \rightarrow 0$. Right: The constraints from lepton flavor violating decays scaled by the experimental bounds versus the charged scalar mass. Here the muon anomalous magnetic moment is smaller than the experimental bound by more than one order of magnitude. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
the experimental values and a DM relic density within the range $\Omega_{\mathrm{DM}} h^{2} \sim 0.09-0.14$. We consider the free parameter values

$$
\begin{align*}
\left|f_{\alpha \beta}\right|^{2},\left|g_{i \alpha}\right|^{2} & \lesssim 9, \quad 500 \mathrm{GeV} \leq M_{\mathrm{DM}} \leq 10 \mathrm{TeV} \\
100 \mathrm{GeV} & \leq M_{S} \leq 10 \mathrm{TeV}, \quad M_{2,3}, M_{\phi} \gtrsim M_{\mathrm{DM}} . \tag{22}
\end{align*}
$$

The results for the values of $M_{\mathrm{DM}}, M_{\mathrm{S}}$ and $M_{\phi}$ are shown in Fig. 2. We find that viable neutrino masses can be obtained for a large region of parameter space, though the DM mass should be confined to the tidy range of $7.18-7.31 \mathrm{TeV}$ for the relic density to match the observed value. This region is somewhat tighter than the corresponding region for the related models with triplets [9] and quintuplets [10], due to the fact that the cross sections for annihilations mediated by the couplings $g_{i \alpha}$, namely Eqs. (14)-(17), are smaller compared to the contribution of $S U(2)_{L}$ gauge bosons (19). In the triplet and quintuplet cases $[9,10]$ the charged lepton contribution is non-negligible, allowing a greater spread for the DM mass interval.

The Sommerfeld enhancement is expected to increase the required DM mass by a factor of roughly 3 . Therefore, in order to approximately take this effect into account, we redo the numerical scan with the DM mass in the relic density replaced by $M_{\mathrm{DM}} / 3$, searching for parameter space that gives viable neutrino masses and mixings and is consistent with low-energy constraints. This approach only provides a rough approximation for the value of the DM mass but, importantly, it allows us to discover if the requisite heavier values of $M_{\mathrm{DM}}$ and $M_{\phi}$ are compatible with the low-energy data. Note that, because the relic density calculation has a reduced sensitivity to the couplings $g_{i \alpha}$ (as DM annihilations in the early universe are dominated by $S U(2)_{L}$ annihilations), the key question is whether there is viable parameter space that achieves neutrino mass and satisfies the constraints, given the heaviness of the DM. Our approach allows us to answer this question and a small shift in $M_{\text {DM }}$ should not significantly affect the conclusion.

Performing the modified numerical scan produces the new results shown in Fig. 3. There is considerable parameter space that satisfies the constraints with the DM mass in the range $19.7-23.1 \mathrm{TeV}$, centered around the value of $M_{\mathrm{DM}}=21.7 \mathrm{TeV}$, which is preferred in the limit $g_{i \alpha}=0$. The scalar $\phi$ must now be heavier than 19.9 TeV , while the charged scalar singlet $S$ can remain as light $\sim 500 \mathrm{GeV}$, similar to the case without Sommerfeld enhancement effect. One observes that the branching ratio $B(\tau \rightarrow$ $\mu+\gamma)$ is smaller than the experimental bound by 4-6 orders of magnitude while the constraint of $B(\mu \rightarrow e+\gamma)<5.7 \times 10^{-13}$ is more severe. In particular, it is evident that improved measurements of $B(\mu \rightarrow e+\gamma)$ are capable of excluding the model.

Though not shown in the figure, the preferred regions of parameter space are not ruled out by the data on the anomalous magnetic moment of the muon; the extra contribution from the exotics can contribute to the observed discrepancy, though it cannot explain it entirely [31,32].

We note that with only two generations of fermions $\mathcal{F}_{i}$ ( $g_{3 \alpha}=0$ ), the bound on $B(\mu \rightarrow e+\gamma)$ is violated. Therefore three generations of $\mathcal{F}_{i}$ are required to remain consistent with constraints from lepton flavor violating processes. Also, the neutrino data prefers that one does not introduce large hierarchies between $M_{\mathrm{DM}}$ and the other exotic masses, $M_{2,3}$ and $M_{\phi}$, with $M_{\phi, 2,3} \sim \mathcal{O}(1-10) \times M_{\text {DM }}$ preferred. The exotics are therefore clustered near $M_{\mathrm{DM}}$. Finally, we emphasize that the preferred values of $M_{\mathrm{DM}}$ should only be taken as a guide, though our analysis clearly shows that one can satisfy the low-energy constraints with the required heavier values of $M_{\mathrm{DM}}$ and $M_{\phi}$.

## 6. Conclusion

We presented an original model of radiative neutrino mass that automatically contains an accidental $Z_{2}$ symmetry and thus provides a stable DM candidate. This gives a common description for neutrino mass and DM without invoking any symmetries beyond those present in the SM. The DM is the neutral component of a septuplet fermion $\mathcal{F} \sim(1,7,0)$, and should have mass $M_{\mathrm{DM}} \approx 20-25 \mathrm{TeV}$. The model can give observable signals via flavor-changing leptonic decays and DM direct-detection experiments. It also predicts a charged scalar $S$ that can be at the TeV scale and within reach of future colliders.

## Acknowledgements

AA is supported by the Algerian Ministry of Higher Education and Scientific Research under the CNEPRU Project No. D01720130042. KM is supported by the Australian Research Council. TT acknowledges support from P2IO Excellence Laboratory.

## References

[^6][3] E.A. Baltz, L. Bergstrom, Phys. Rev. D 67 (2003) 043516, arXiv:hep-ph/0211325; K. Cheung, O. Seto, Phys. Rev. D 69 (2004) 113009, arXiv:hep-ph/0403003; A. Ahriche, S. Nasri, R. Soualah, arXiv: 1403.5694 [hep-ph].
[4] A. Ahriche, S. Nasri, J. Cosmol. Astropart. Phys. 1307 (2013) 035, arXiv:1304. 2055.
[5] E. Ma, Phys. Rev. D 73 (2006) 077301, arXiv:hep-ph/0601225.
[6] S.-Y. Ho, J. Tandean, arXiv:1303.5700 [hep-ph];
D. Schmidt, T. Schwetz, T. Toma, Phys. Rev. D 85 (2012) 073009, arXiv:1201. 0906 [hep-ph];
R. Bouchand, A. Merle, J. High Energy Phys. 1207 (2012) 084, arXiv:1205.0008 [hep-ph].
[7] M. Aoki, S. Kanemura, O. Seto, Phys. Rev. Lett. 102 (2009) 051805, arXiv:0807. 0361 [hep-ph];
M. Aoki, S. Kanemura, O. Seto, Phys. Rev. D 80 (2009) 033007, arXiv:0904.3829 [hep-ph];
M. Aoki, S. Kanemura, T. Shindou, K. Yagyu, J. High Energy Phys. 1007 (2010) 084;
M. Aoki, S. Kanemura, T. Shindou, K. Yagyu, J. High Energy Phys. 1011 (2010) 049, arXiv: 1005.5159 [hep-ph];
S. Kanemura, O. Seto, T. Shimomura, Phys. Rev. D 84 (2011) 016004, arXiv: 1101.5713 [hep-ph];
M. Aoki, S. Kanemura, K. Yagyu, Phys. Lett. B 702 (2011) 355, arXiv:1105.2075 [hep-ph];
M. Aoki, S. Kanemura, K. Yagyu, Phys. Lett. B 706 (2012) 495 (Erratum);
M. Lindner, D. Schmidt, T. Schwetz, Phys. Lett. B 705 (2011) 324, arXiv: 1105.4626 [hep-ph];
M. Lindner, D. Schmidt, T. Schwetz, Phys. Rev. D 85 (2012) 033004, arXiv: 1111.0599 [hep-ph];
Y.H. Ahn, H. Okada, Phys. Rev. D 85 (2012) 073010, arXiv:1201.4436 [hep-ph]; S.S.C. Law, K.L. McDonald, Phys. Lett. B 713 (2012) 490, arXiv:1204.2529 [hep-ph];
G. Guo, X.-G. He, G.-N. Li, J. High Energy Phys. 1210 (2012) 044, arXiv: 1207.6308 [hep-ph];
M. Gustafsson, J.M. No, M.A. Rivera, Phys. Rev. Lett. 110 (21) (2013) 211802, arXiv:1212.4806 [hep-ph];
M. Aoki, J. Kubo, H. Takano, Phys. Rev. D 87 (2013) 116001, arXiv:1302.3936 [hep-ph].
[8] Y. Kajiyama, H. Okada, K. Yagyu, Nucl. Phys. B 874 (2013) 198, arXiv:1303.3463 [hep-ph];
Y. Kajiyama, H. Okada, T. Toma, Phys. Rev. D 88 (1) (2013) 015029, arXiv: 1303.7356;
S.S.C. Law, K.L. McDonald, J. High Energy Phys. 1309 (2013) 092, arXiv: 1305.6467 [hep-ph];
D. Restrepo, O. Zapata, C.E. Yaguna, J. High Energy Phys. 1311 (2013) 011, arXiv:1308.3655 [hep-ph];
E. Ma, I. Picek, B. Radovcic, Phys. Lett. B 726 (2013) 744, arXiv:1308.5313 [hep-ph];
V. Brdar, I. Picek, B. Radovcic, Phys. Lett. B 728 (2014) 198, arXiv:1310.3183 [hep-ph];
H. Okada, K. Yagyu, Phys. Rev. D 89 (5) (2014) 053008, arXiv:1311.4360 [hep-ph];
S. Baek, H. Okada, T. Toma, arXiv:1312.3761 [hep-ph];
S. Baek, H. Okada, T. Toma, arXiv:1401.6921 [hep-ph];
H. Okada, arXiv:1404.0280 [hep-ph].
[9] A. Ahriche, C.S. Chen, K.L. McDonald, S. Nasri, Phys. Rev. D 90 (2014) 015024, arXiv:1404.2696 [hep-ph].
[10] A. Ahriche, K.L. McDonald, S. Nasri, J. High Energy Phys. 1410 (2014) 167, arXiv: 1404.5917 [hep-ph].
[11] J.N. Ng, A. de la Puente, Phys. Rev. D 90 (9) (2014) 095018, arXiv:1404.1415 [hep-ph];
C.S. Chen, K.L. McDonald, S. Nasri, Phys. Lett. B 734 (2014) 388, arXiv:1404. 6033 [hep-ph];
S. Kanemura, T. Matsui, H. Sugiyama, Phys. Rev. D 90 (2014) 013001, arXiv: 1405.1935 [hep-ph];
H. Okada, K. Yagyu, Phys. Rev. D 90 (2014) 035019, arXiv:1405.2368 [hep-ph]; S. Kanemura, N. Machida, T. Shindou, Phys. Lett. B 738 (2014) 178, arXiv: 1405.5834 [hep-ph];
M. Aoki, T. Toma, J. Cosmol. Astropart. Phys. 1409 (2014) 016, arXiv:1405.5870 [hep-ph];
H. Ishida, H. Okada, arXiv: 1406.5808 [hep-ph];
H. Okada, Y. Orikasa, Phys. Rev. D 90 (2014) 075023, arXiv:1407.2543 [hep-ph]; H. Okada, T. Toma, K. Yagyu, Phys. Rev. D 90 (2014) 095005, arXiv:1408.0961 [hep-ph];
H. Okada, Y. Orikasa, arXiv:1412.3616 [hep-ph];
H. Hatanaka, K. Nishiwaki, H. Okada, Y. Orikasa, arXiv:1412.8664 [hep-ph]; S. Baek, H. Okada, K. Yagyu, arXiv:1501.01530 [hep-ph];
L.G. Jin, R. Tang, F. Zhang, Phys. Lett. B 741 (2015) 163, arXiv:1501.02020 [hep-ph];
H. Okada, arXiv: 1503.04557 [hep-ph];
H. Okada, N. Okada, Y. Orikasa, arXiv:1504.01204 [hep-ph].
[12] P. Culjak, K. Kumericki, I. Picek, Phys. Lett. B 744 (2015) 237, arXiv:1502.07887 [hep-ph].
[13] P.S. Bhupal Dev, A. Pilaftsis, Phys. Rev. D 87 (2013) 053007, arXiv:1212.3808 [hep-ph].
[14] M. Cirelli, N. Fornengo, A. Strumia, Nucl. Phys. B 753 (2006) 178, arXiv:hep-ph/ 0512090.
[15] E. Witten, Nucl. Phys. Proc. Suppl. 91 (2001) 3, arXiv:hep-ph/ 0006332.
[16] K.S. Babu, S. Nandi, Z. Tavartkiladze, Phys. Rev. D 80 (2009) 071702, arXiv: 0905.2710 [hep-ph];
I. Picek, B. Radovcic, Phys. Lett. B 687 (2010) 338, arXiv:0911.1374 [hep-ph];
K. Kumericki, I. Picek, B. Radovcic, Phys. Rev. D 84 (2011) 093002, arXiv: 1106.1069 [hep-ph];
Y. Liao, J. High Energy Phys. 1106 (2011) 098, arXiv:1011.3633 [hep-ph];
K. Kumericki, I. Picek, B. Radovcic, Phys. Rev. D 86 (2012) 013006, arXiv: 1204.6599 [hep-ph];
K.L. McDonald, J. High Energy Phys. 1307 (2013) 020, arXiv:1303.4573 [hep-ph];
K.L. McDonald, J. High Energy Phys. 1311 (2013) 131, arXiv:1310.0609 [hep-ph];
G. Bambhaniya, J. Chakrabortty, S. Goswami, P. Konar, Phys. Rev. D 88 (7) (2013) 075006, arXiv:1305.2795 [hep-ph];
S.S.C. Law, K.L. McDonald, Phys. Rev. D 87 (2013) 113003, arXiv:1303.4887 [hep-ph];
C.-S. Chen, Y.-J. Zheng, arXiv:1312.7207 [hep-ph];
R. Ding, Z.-L. Han, Y. Liao, H.-J. Liu, J.-Y. Liu, arXiv:1403.2040 [hep-ph].
[17] K. Hally, H.E. Logan, T. Pilkington, Phys. Rev. D 85 (2012) 095017, arXiv:1202.5073 [hep-ph];
K. Earl, K. Hartling, H.E. Logan, T. Pilkington, Phys. Rev. D 88 (2013) 015002, arXiv:1303.1244 [hep-ph];
K. Earl, K. Hartling, H.E. Logan, T. Pilkington, Phys. Rev. D 90 (2014) 055029, arXiv:1311.3656 [hep-ph].
[18] D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D 86 (2012) 073012, arXiv: 1205.4018 [hep-ph];
M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, T. Schwetz, J. High Energy Phys. 1212 (2012) 123, arXiv:1209.3023 [hep-ph].
[19] F. Simkovic, A. Faessler, H. Muther, V. Rodin, M. Stauf, Phys. Rev. C 79 (2009) 055501, arXiv:0902.0331 [nucl-th].
[20] F.T. Avignone, G.S. King, Y.G. Zdesenko, New J. Phys. 7 (2005) 6; W. Rodejohann, Int. J. Mod. Phys. E 20 (2011) 1833, arXiv:1106.1334 [hep-ph].
[21] M. Cirelli, A. Strumia, New J. Phys. 11 (2009) 105005, arXiv:0903.3381 [hep-ph];
M. Cirelli, A. Strumia, M. Tamburini, Nucl. Phys. B 787 (2007) 152, arXiv: 0706.4071 [hep-ph].
[22] K. Griest, D. Seckel, Phys. Rev. D 43 (1991) 3191.
[23] J. Hisano, S. Matsumoto, M.M. Nojiri, Phys. Rev. Lett. 92 (2004) 031303, arXiv:hep-ph/0307216.
[24] J. Hisano, S. Matsumoto, M.M. Nojiri, O. Saito, Phys. Rev. D 71 (2005) 063528, arXiv:hep-ph/0412403.
[25] J. Hisano, S. Matsumoto, M. Nagai, O. Saito, M. Senami, Phys. Lett. B 646 (2007) 34, arXiv:hep-ph/0610249.
[26] M. Farina, D. Pappadopulo, A. Strumia, J. High Energy Phys. 1308 (2013) 022, arXiv:1303.7244 [hep-ph].
[27] K. Bernlöhr, A. Barnacka, Y. Becherini, O. Blanch Bigas, E. Carmona, P. Colin, G. Decerprit, F. Di Pierro, et al., Astropart. Phys. 43 (2013) 171, arXiv:1210.3503 [astro-ph.IM].
[28] D.S. Akerib, et al., LUX Collaboration, arXiv:1310.8214 [astro-ph.CO].
[29] P. Junnarkar, A. Walker-Loud, Phys. Rev. D 87 (11) (2013) 114510, arXiv: 1301.1114 [hep-lat].
[30] J. Hisano, K. Ishiwata, N. Nagata, Phys. Rev. D 82 (2010) 115007, arXiv: 1007.2601 [hep-ph];
R.J. Hill, M.P. Solon, Phys. Lett. B 707 (2012) 539, arXiv: 1111.0016 [hep-ph].
[31] G.W. Bennett, et al., Muon g-2 Collaboration, Phys. Rev. D 73 (2006) 072003, arXiv:hep-ex/0602035.
[32] F. Jegerlehner, A. Nyffeler, Phys. Rep. 477 (2009) 1, arXiv:0902.3360 [hep-ph].


[^0]:    * Corresponding author.

    E-mail addresses: aahriche@ictp.it (A. Ahriche), klmcd@physics.usyd.edu.au (K.L. McDonald), snasri@uaeu.ac.ae (S. Nasri), takashi.toma@th.u-psud.fr (T. Toma).

[^1]:    ${ }^{1}$ In some models the DM is merely sufficiently long-lived, rather than absolutely stable. This does not require a new symmetry but instead relies on technicallynatural parameter hierarchies (either among mass parameters [13] or dimensionless couplings [10]).

[^2]:    ${ }^{2}$ The second term is equivalent to the standard $\left(H^{\dagger} \tau_{i} H\right)\left(\phi^{\dagger} T_{i} \phi\right)$ term, where $\tau_{i}$ and $T_{i}$ denote $S U(2)$ generators for the distinct representations.

[^3]:    ${ }^{3}$ We shall see below that the septuplets must be heavier than the TeV scale; given the very small mass-splittings, relative to the weak scale, this should ensure that the new contributions to the oblique parameters are negligible. Also, similar to other models with large multiplets, the $S U(2)_{L}$ coupling constant encounters a Landau pole in the UV, due to the heavy septuplets.
    ${ }^{4}$ Interestingly, the model of Ref. [12] gives an accidental $Z_{2}$ symmetry after imposing a separate $Z_{2}^{\prime}$ symmetry.

[^4]:    ${ }^{5}$ This is expected in the present model, due to the relatively large value of $\sigma_{W}$ in Eq. (19).

[^5]:    ${ }^{6}$ We estimate the two-loop effect with a simple scaling of the results in Ref. [26].

[^6]:    [1] A. Zee, Phys. Lett. B 93 (1980) 389; A. Zee, Phys. Lett. B 95 (1980) 461; K.S. Babu, Phys. Lett. B 203 (1988) 132; For general discussion of radiative models see e.g. S.S.C. Law, K.L. McDonald, Int. J. Mod. Phys. A 29 (2014) 1450064, arXiv:1303.6384 [hep-ph]; For a review of neutrino mass models see e.g. S.M. Boucenna, S. Morisi, J.W.F. Valle, Adv. High Energy Phys. 2014 (2014) 831598, arXiv:1404.3751 [hep-ph].
    [2] L.M. Krauss, S. Nasri, M. Trodden, Phys. Rev. D 67 (2003) 085002, arXiv:hep-ph/ 0210389.

