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An efficient muscle fatigue model for forward and inverse dynamic analysis of human movements

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Abstract

The aim of this work is to present the integration of a simple and yet efficient dynamic muscle fatigue model in a multibody formulation with natural coordinates. The fatigue model considers the force production history of each muscle to estimate its fitness level by means of a three-compartment theory approach. The model is easily adapted to co-operate with standard Hill-type muscle models, allowing the simulation and analysis of the redundant muscle forces generated in the presence of muscular fatigue. This has particular relevance in the design of orthotic devices to support human locomotion and manipulation.

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Keywords: Redundant muscle forces; Muscular fatigue model; Hill-type muscle model; Natural coordinates; Forward dynamics; Inverse Dynamics

1. Introduction

Computational simulation and analysis of the human movement is becoming a fundamental tool in many medical and biomedical applications, providing accurate quantitative results that can be used to support medical decision and diagnosis or in the integrated design of ortho-prosthetic devices for manipulation and locomotion [1,2]. In particular, multibody dynamics methodologies have emerged and rapidly evolved through the last decades as an accurate and well adapted computational technique, naturally fitted to deal with the dynamics of biomechanical systems, due to their unique characteristic of allowing the calculation of complex biomechanical data in a non-invasive way [3,4]. This feature proved

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to be of stellar importance when dealing with living systems, such as the human body, as it opened the way for the calculation of joint reaction forces and for the estimation of individual muscle activation and force production without the use of indwelled body sensing [5].

For the calculation of the muscle forces, in particular, it was deemed necessary to include proper muscle models in the multibody methodologies that were able to account with the dynamics of the skeletal muscle tissue such as the relationship between the production of force and the muscle state in terms of its length and contraction velocity. These models provide means of obtaining a physiological solution for the force sharing problem using static optimization tools to deal with the redundant and indeterminate nature of the equations of motion involved. However in these models the muscle fatigue effects are never considered. This might not be relevant when analyzing normal everyday activities in normal subjects, such as gait or climbing stairs, but might not be sufficiently accurate if high levels of muscular activity are required to perform the movement or if the subject under analysis presents some level of pathology. In such circumstances muscular fatigue is certain to occur even for small analysis periods and should, therefore, be taken in consideration in the muscular model.

The muscular fatigue model presented in this work if often referred to as the three-compartment theory model and is based on the model proposed by Xia and Law [6], now adapted to the multibody dynamics framework with natural coordinates. It's a straightforward model with low computational costs and high numerical efficiency that requires a small set of parameters to perform.

The presented methodologies are applied to a simple case test, being the results obtained discussed in face of the methodological assumptions considered.

2. Methods

2.1. Multibody Dynamics with Natural Coordinates and Redundant Muscle Forces

The multibody formulation presented in this work employs natural coordinates, as introduced by Jalon and Bayo [7] and applied by Silva [8], to simulate and analyze the human movement and to calculate the associated redundant muscle forces. With this type of formulation, rigid bodies are defined using the Cartesian coordinates of points and unit vectors located in relevant anatomical landmarks, such as joint centers and extremities, as depicted in the example provided in Fig. 1.

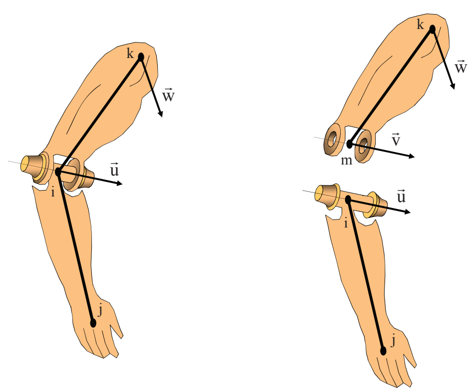


Fig. 1. Definition of a rigid body using a multibody formulation with natural coordinates [8].

Using natural coordinates, the number of generalized coordinates applied in the definition of rigid bodies and joints is usually higher than the number of the degrees-of-freedom of the system, conferring these coordinates with a dependent nature. As in other multibody formulations with dependent coordinates, these dependencies are expressed by means of kinematic constraint equations, which in this case, due to the nature of the relations to impose (*e.g.* constant distances between points, constant angles between segments, unit vectors or superposition of points [8]), are algebraic equations that are always quadratic or linear with the generalized coordinates. This important characteristic, together with the fact that angular variables are not required for the definition of the spatial orientation of rigid bodies, makes this formulation, from the computational point of view, very interesting due to is simplicity, robustness and accuracy.

Considering $\mathbf{q}$ as the vector of generalized coordinates, containing the Cartesian coordinates and components of points and vectors, the vector of kinematic constraints ($\boldsymbol{\Phi}$) is represented by the following expression:

$$\boldsymbol{\Phi}(\mathbf{q},t) = \{\boldsymbol{\Phi}_1(\mathbf{q}) \cdots \boldsymbol{\Phi}_{ns}(\mathbf{q}) \boldsymbol{\Phi}_{ns+1}(\mathbf{q},t) \cdots \boldsymbol{\Phi}_{ns+nr}(\mathbf{q},t)\}^T = \mathbf{0} \tag{1}$$

where *ns* and *nr* represents respectively the number of scleronomic and rheonomic constraints used to construct and drive the multibody system. Using natural coordinates, the position $\mathbf{r}$ of any point of the system can be expressed in terms of the vector of generalized coordinates $\mathbf{q}$ by means of the following coordinate transformation [8]:

$$\mathbf{r} = \mathbf{Cq} \tag{2}$$

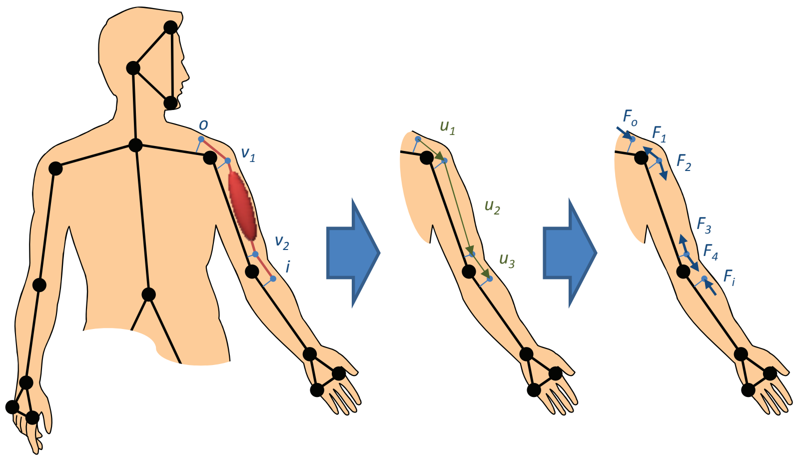
where matrix $\mathbf{C}$ is a 3×12 transformation matrix that is calculated from the local coordinates of the point in consideration, in the reference frame of the rigid body to witch it is attached. It should be noticed that since this matrix is constant in time, it is also used for the calculation of the velocity and acceleration of the considered point as a function of the generalized velocity and acceleration vectors of the system. Its use is further required not only for the determination of the mass matrix $\mathbf{M}_i$ but also to calculate the generalized external force vector $\mathbf{g}_i$ associated to each rigid body, as it will be seen later in this work.

Having these coordinates' specific characteristics in mind, the rest of the equations, associated with this formulation, are in all perspectives similar to the ones obtained in other types of multibody formulations. Hence, the equations of motion (EoM) of a multibody system defined with natural coordinates are given by:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \mathbf{g} \\ \boldsymbol{\Phi}_{\mathbf{q}} \ddot{\mathbf{q}} = \boldsymbol{\gamma} \end{cases} \tag{3}$$

where $\mathbf{M}$ represents the global mass matrix of the system, $\boldsymbol{\Phi}_{\mathbf{q}}$ is the Jacobian matrix of the system that holds the partial derivatives of the kinematic constraint equations in order to the generalized coordinates, $\ddot{\mathbf{q}}$ is the vector of generalized accelerations and $\boldsymbol{\gamma}$ is the vector of the right-end side of the acceleration constraint equations. Vector $\mathbf{g}$ is the vector of generalized external forces in which, among others, muscle forces are also included.

In this work muscle structures are not modeled as muscle actuator drivers, responsible to prescribe the kinematics of the system [8], but instead, using Newton's method [7], they are regarded as a set of concentrated loads whose lines-of-action are defined by direction unit vectors ($\mathbf{u}_1$, $\mathbf{u}_2$, ... , $\mathbf{u}_d$). These vectors are constructed from the coordinates of the origin (subscript o), insertion (subscript *i*) and eventual via-points (subscripts v_1, v_2, ... , v_{vp}, with *vp* representing the number of via-points), as schematically indicated in Fig. 2. This means that the direction of the muscle forces is know from the kinematics of the system, being their magnitude the variable to prescribe, in case of a simulation in a forward dynamics perspective, or the unknown to calculate, in the case of an analysis in an inverse dynamics perspective. In either cases, the muscle forces will ensue as represented in Fig. 2.c).



(a) (b) (c)

Fig. 2. Example of a biarticular muscle structure with a) origin, insertion and two via points; b) direction unit vectors $\mathbf{u}_i$ and c) associated muscle forces $\mathbf{F}_i$.

It should be noted that some important geometrical assumptions are made that do not correspond to human skeletal muscle anatomy: it is considered that muscles have rectilinear orientations between via points and that they hold a constant cross-sectional area. These assumptions are important as they allow, from the force production point of view, to consider that the force exerted by the muscle has the same magnitude in all rectilinear sections of the muscle, changing only is direction in accordance with the kinematic structure of via points used. Hence for a given muscle *m*, the force produced by the muscle and applied at the p^{th} via point has the following general structure:

$$\mathbf{F}_p^m = \left(\mathbf{u}_{p+1} - \mathbf{u}_p\right) F^m \tag{4}$$

where F^m is the force magnitude of the muscle under consideration. This force, expressed in Cartesian coordinates, needs to be transformed in an equivalent generalized force before being included in the generalized force vector **g**. This transformation is accomplished using the transformation matrix **C**, evaluated from the local coordinates of the p^{th} via point, in the local reference frame of the rigid body *e* to which this point is attached to [8-10]:

$$\mathbf{g}_{p,e}^m = \mathbf{C}_{p,e}^T \mathbf{F}_p^m = \mathbf{C}_{p,e}^T \left(\mathbf{u}_{p+1} - \mathbf{u}_p\right) F^m \tag{5}$$

It should be noted that previous expression represents the generalized force vector associated to the p^{th} via point, calculated in the scope of the coordinates describing rigid body *e* and, therefore, it is not yet a final contribution to the global generalized force vector $\mathbf{g}_p^m$. In order to obtain that, a trivial vector assemblage is required that maps the components of $\mathbf{g}_{p,e}^m$ into their right locations in $\mathbf{g}_p^m$.

Muscle forces applied at the origin (*o*) and insertion (*i*) points are in all aspects similar to those presented for a general via point, with the exception that in these initial and terminal points only one direction unit vector is considered. The calculation of the global force vector $\mathbf{g}^m$ associated to muscle *m* is only concluded when the contributions of all the forces $\mathbf{g}_p^m$, applied to all points defining the muscle, are assembled in this vector:

$$\mathbf{g}^m = \sum_{p=1}^{vp+2} \mathbf{g}_p^m \tag{6}$$

Muscle forces defined as presented before can now be introduced in the EoM of the system and solved in a forward or inverse dynamics perspective. However it should be noted that such muscle forces lack physiological meaning as they do not account for the dynamics of the skeletal muscle tissue and in particular the force-length and force-velocity relationships. This relationships relate the dynamic state of the muscle with its ability to produce force. In order to include such characteristics a proper muscle model needs to be introduced in the numerical formalism. Archibald Hill introduced an adaptation of the Kelvin model, including an additional contractile element [11] to simulate the macroscopic action of the cross-bridge cycle theory of skeletal muscle force production described by Huxley [12]. The model used in this work is a Hill-type muscle model composed by a passive element (PE) and a contractile element (CE). Muscular electromechanical delay associated with activation dynamics is not accounted for in the present model and for the movement under analysis the action of tendon units was also omitted. The model presents a non-linear behavior with the muscle state (length and velocity) and a linear behavior with the muscle activation (the rate of fibers in contraction). The PE models the non-linear passive elastic properties of the muscle tissue, while the CE accounts for both the contractile structures and the viscous forces produced by intercellular and intracellular fluids enclosed in the muscle [8,13].

The total muscle force produced by this model is expressed as the sum of the force developed by the passive (F_{PE}^m) and contractile (F_{CE}^m) elements. The CE force can be further factorized into the maximum contractile force $\hat{F}_{CE}^m$ and the muscle activation a^m:

$$F^m = F_{PE}^m + F_{CE}^m = F_{PE}^m + \hat{F}_{CE}^m a^m \tag{7}$$

For the analytical expressions used in the contraction dynamics model and for further details about this implementation, the reader is referred to [8,10,13]. Substituting Eq. 7 in Eqs. 5 and 6, the following result is obtained for the expression of the generalized muscle force, factorized in the muscle model components:

$$\mathbf{g}^m = \mathbf{g}_{PE}^m + \mathbf{g}_{CE}^m = \mathbf{g}_{PE}^m + \hat{\mathbf{g}}_{CE}^m a^m \tag{8}$$

where $\mathbf{g}_{PE}^m$ and $\mathbf{g}_{CE}^m$ are respectively the generalized force vectors for the passive and contractile elements contributions, and $\hat{\mathbf{g}}_{CE}^m$ corresponds to the generalized force vector of the maximum available contractile force ($\hat{F}_{CE}^m$). Considering a general musculoskeletal multibody system defined with *nm* muscles, it comes that the generalized external force vector $\mathbf{g}$ is given by:

$$\mathbf{g} = \mathbf{g}^{ext} + \sum_{m=1}^{nm} \mathbf{g}^m = \mathbf{g}^{ext} + \sum_{m=1}^{nm} \left(\mathbf{g}_{PE}^m + \hat{\mathbf{g}}_{CE}^m a^m \right) \tag{9}$$

where $\mathbf{g}^{ext}$ holds the remaining external forces (including velocity-dependent forces such as centrifugal and Coriolis). Substituting Eq. 9 in Eq. 3, it becomes:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \mathbf{g}^{ext} + \sum_{m=1}^{nm} \left(\mathbf{g}_{PE}^m + \hat{\mathbf{g}}_{CE}^m a^m \right) \\ \mathbf{\Phi}_{\mathbf{q}} \ddot{\mathbf{q}} = \boldsymbol{\gamma} \end{cases} \tag{10}$$

2.2. The forward dynamics perspective

The solution of the EoM of the multibody system, in a forward dynamics analysis (FDA) perspective, follows, in this work, the method proposed in [7,14], i.e. the equations are solved as an initial value problem (IVP), giving an initial kinematic consistent state of the system and using the direct integration (DI) algorithm to integrate velocities and accelerations of the current time step into positions and velocities of the next time step. This implementation is straightforward even in the presence of muscle forces due to the fact that these are considered as known external forces to the system, being the activation patterns of each muscle unit required *a priori*.

It should be noted, however, that the second branch of Eq. 10 is unstable by definition and therefore, stabilization methods such as the Baumgarte stabilization, based on the feedback control theory [15], or the augmented Lagrangian formulation, which a penalty based iterative method [7,8,16], are required for a successful solution. The latter method presents a major advantage which is the ability to deal with redundant constraints and singular configurations of the system, although a direct calculation of the Lagrange multipliers is not possible unless a post-calculation step is carried out.

2.3. The inverse dynamics perspective

In an Inverse Dynamics Analysis (IDA), the movement of the system is know from the start and the objective is to calculate the internal forces, that hold together the topology and structure of the system, and the external forces that give rise to the observed movement. Considering the EoM presented in Eq. 10 and the paradigm of Inverse Dynamics (ID), the unknowns to calculate are the vector of Lagrange multipliers $\boldsymbol{\lambda}$ and the vector of muscle activations **a**, such as:

$$\mathbf{a} = \left\{ a^1 \;\ldots\; a^m \;\ldots\; a^{nm} \right\}^T \tag{11}$$

where $a^m = a^m(t)$ represents the activation of muscle m at instant t of the analysis. Rearranging the first branch of Eq. 10, considering Eq. 11, and isolating the unknowns in the left-hand side of the equation, yields:

$$\begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{q}}^T & -\boldsymbol{\chi}^T \end{bmatrix} \begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix} = \mathbf{g}^{ext} + \mathbf{g}_{PE} - \mathbf{M}\ddot{\mathbf{q}} \tag{12}$$

with

$$\boldsymbol{\chi} = \begin{bmatrix} \hat{\mathbf{g}}_{CE}^1 \\ \vdots \\ \hat{\mathbf{g}}_{CE}^{nm} \end{bmatrix} \quad \text{and} \quad \mathbf{g}_{PE} = \sum_{m=1}^{nm} \mathbf{g}_{PE}^m \tag{13}$$

It should be noted that the force produced by the PE is independent of the activation levels of the muscle, depending only on its actual state. Since the kinematics are known, these forces can be promptly calculated, appearing this way in the right-end side of the EoM of the system. The second branch of Eq. 10 is trivial since the movement of the system is known and, therefore, it will not be further referred in the scope of IDA. The final configuration of the EoM presented in Eq. 12 is indeterminate since there are more unknowns to solve than available equations. These additional unknowns are the muscle activations, fact that is physiologically understood and referred as muscular redundancy, *i.e.*, an infinite number of muscle force configurations can produce a specific motion or posture, although the central nervous system (CNS) will only chose one. From the numerical point of view, finding such solution in indeterminate IDA problems involves the use of static optimization tools that are able to choose, from the infinite set of possible solutions, the one that minimizes a specified cost function or physiological objective.

2.4. Solving the EoM of the system with static optimization tools

The optimization procedure that unfolds under this section is of general application and can be used in other types of multibody formulations to calculate the redundant muscle forces and not only in the present one with natural coordinates. However, in natural coordinates, due to the fact angular coordinates are not used, there are no explicit EoM involving the net articular moments and the associated muscle forces. This means that for solving this optimization problem with natural coordinates, additional optimization constraints are required, which correspond to all the equations of motion of the system. This can be seen as a drawback of this approach, since the size of the problem increases significantly, considering that to the unknowns associated to muscle activations, one must add those associated to all other Lagrange multipliers of the system. However, from other perspective, this allows for an integrated solution of the problem in terms of redundant muscle forces and in terms of associated reaction forces at the joints, which is seen as a very interesting feature of the present approach.

The control variables of the optimization problem to solve are in this case the unknowns of the IDA, which are grouped in vector $\mathbf{x}$ such that:

$$\mathbf{x} = \begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix} \tag{14}$$

The solution of $\mathbf{x}$ must respect the limitations imposed by the constraints of the multibody system. The first type of constraints are the equality constraints $\mathbf{f}_{eq}$, that arise from the EoM of the musculoskeletal system, now rewritten in their homogeneous form:

$$\mathbf{f}_{eq} = \begin{Bmatrix} f_1 \\ \vdots \\ f_{nc} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{q}}^T & -\boldsymbol{\chi}^T \end{bmatrix} \begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix} + \mathbf{M}\ddot{\mathbf{q}} - (\mathbf{g}^{ext} + \mathbf{g}_{PE}) = \mathbf{0} \tag{15}$$

The gradient of $\mathbf{f}_{eq}$ is usually required by the optimization routines for the calculation of the optimal solution. In many optimization problems the evaluation of this matrix is usually cumbersome, requiring additional computational efforts and often a numerical approximation by finite differences is the only available option. In this case, this gradient is available analytically from the multibody formulation and, therefore, no additional calculations are further required:

$$\nabla \mathbf{f}_{\begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix}} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{q}}^T & -\boldsymbol{\chi}^T \end{bmatrix} \tag{16}$$

Also from note, is the fact that in the optimization procedure now proposed, joint rotational drivers coexist with muscle actuators. The coexistence of such drivers allows for a more versatile guidance of the system and, at the same time, it introduces an alternative way of calculating the muscle activations by controlling the boundaries of the control variables associated with the Lagrange multipliers of these driver equations, hereafter designated by $\boldsymbol{\lambda}^*$. When muscles are considered in the model, it is desired to eliminate the contribution from the Lagrange multipliers associated to joint drivers, forcing the optimizer to fill the muscle activation variables with the optimal values. To induce this effect, the Lagrange multipliers associated to these joint driving equations must be kept within a bound of ε. This will be the first inequality type of constraints:

$$|\boldsymbol{\lambda}^*| \leq \varepsilon \qquad (17)$$

The optimizer, constrained by Eq. 17, will numerically tend assign values for the muscle activations $\mathbf{a}$, in order to hold the prescribed motion. Activation values are in addition constrained to be positive and in a range between 0 and 1 as:

$$0 \leq a^m \leq 1 \text{, for } m = 1,\ldots,nm \qquad (18)$$

The remaining Lagrange multipliers in the control variables, associated with other kinematic constraints of the system such as rigid body constraints and joint reaction forces, hereafter identified as $\boldsymbol{\lambda}^R$, will not have any special limitations:

$$-\infty \leq \boldsymbol{\lambda}^R \leq +\infty \qquad (19)$$

Optimization procedures act as to minimizing a cost function F_0 that describes the muscle system energy depletion. These functions may include quantities like muscle force, muscle geometric and physiological properties, and metabolic energy expenditure. For the study cases presented next, the cost function used calculates the sum of the cube of the muscle stresses:

$$F_0 = \sum_{m=1}^{nm} \left(\sigma_{CE}^m \right)^3 \qquad (20)$$

This function was originally proposed by Crowninshield and Brand [17] but other cost functions can also be implemented [18,19]. The optimizer used in the program is the DNCONG routine, the double precision version of the NCONG routine, available in the FORTRAN version of the IMSL Library developed by Visual Numerics, Inc [20].

2.5. Introducing fatigue dynamics in the optimization procedure

Muscle fatigue is the degradation of the capacity of a muscle to exert contractile force, and is divided in central and peripheral muscle fatigue. Central fatigue is associated with altered neural signal transmission after muscle action, narrowing its output. Peripheral muscle fatigue is understood as the consumption of metabolites (intramuscular glycogen, ATP, etc.) interfering in the contraction process, ranging from the generation of the action potential until the cross-bridge cycle. Although central fatigue plays a major role in muscular performance, with constant brain regulation of motor activity to ensure homeostatis [21,22], the goal in this work is to include only a peripheral muscle fatigue model, and test its compatibility with multibody algorithms.

Initial empirical models were developed by Rohmert [23], subsequently extended by models including physiological parameters and fatigue dynamic laws [24,25], and later by new theoretical models that derive the muscle force function from simple biophysical principles. This work uses such a model, adapted from the work by Xia and Law [6], which derives from a motor unit (MU)-based fatigue model proposed by Liu [26].

The model described here considers that muscles are compartmentalized in MUs, *i.e.*, groups of muscle fibers and the motor neuron that innervates them. When a motor neuron is stimulated above the recruitment threshold by the CNS, an action potential is triggered and the muscle fibers in that MU are activated, producing tensile force in a discrete way. The model considers the compartment theory approach to describe the fatigue dynamics of skeletal muscle, *i.e.*, MUs are assumed to be ideally in one of three states: activated, resting or fatigued. To each one of the referred states is associated the respective compartment. A single muscle fatigue condition is then given by the combination of the states of the MUs, although, in reality, MUs may be in a continuous state that ranges from fully fatigued to fully recovered.

Motor units exerting maximum force will fall in the activated compartment M_A, exhausted MUs in the fatigued compartment M_F, and resting MUs in the resting compartment M_R, as schematically depicted in Fig. 3.

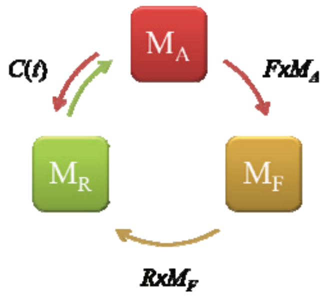


Fig. 3. Three-compartment theory flowchart [6].

The number of muscle fibers in a particular compartment is given in percentage of maximum voluntary contraction. The available contractile force will then be limited to the MUs available for force production, *i.e.*, those in the activated and resting compartments. Hence, the residual capacity *RC* of the muscle can be defined as:

$$RC(t) = M_A + M_R = 1.0 - M_F \tag{21}$$

The (fatigued) muscle force available $\hat{F}_{CE}^{f}$ will then be expressed by:

$$\hat{F}_{CE}^{f} = RC(t) \times \hat{F}_{CE}^{m} \tag{22}$$

Liu's model only considers three parameters (to ease fitting experimental data): a fatigue factor *F*, a recovery factor *R* and the total number of motor units in the muscle. The input factor is the brain effort (*BE*), analogous to the muscle activation a^m stated in the previous section. However this model fails to consider non-constant muscle efforts, a major disadvantage for its usage in multibody dynamics, where muscle efforts can be expected to vary in time with complex kinematics.

Xia and Law [6] responded to this drawback by including fluctuating muscle force intensities. They used the previous model's parameters and reformulated the rate at which MUs become fatigued and the rate at which MUs recover, and hence defining the passage of MUs between the different compartments:

$$\frac{dM_R}{dt} = -C(t) + R \times M_F(t) \tag{23}$$

$$\frac{dM_A}{dt} = C(t) - F \times M_A(t) \tag{24}$$

$$\frac{dM_F}{dt} = F \times M_A(t) - R \times M_F(t) \tag{25}$$

These equations are used when a force solicitation is made, updating the fitness state of the muscle structure. In Eqs. 23 and 24, $C(t)$ is the muscle's activation-deactivation driving controller, which is the term that gives the number of fibers used in the instantaneous effort, enabling this model to process the dynamics of muscle fatigue with variable efforts. To define the controller $C(t)$, two additional parameters are introduced: L_D and L_R, respectively the muscle force development and relaxation factors. $C(t)$ is a bounded controller that depends on the relation between the compartments' state and a target load TL, in this work referred to as the maximum contractile force F_{CE}^m :

$$C(t)=\begin{cases} L_R\times(F_{CE}^m - M_A\times\hat{F}_{CE}^m) & F^m\le M_A\hat{F}_{CE}^m \\ L_D\times(F_{CE}^m - M_A\times\hat{F}_{CE}^m) & M_A\hat{F}_{CE}^m < F^m \le (M_A+M_R)\hat{F}_{CE}^m \\ L_D\times M_R\times\hat{F}_{CE}^m & (M_A+M_R)\hat{F}_{CE}^m < F^m \end{cases} \tag{26}$$

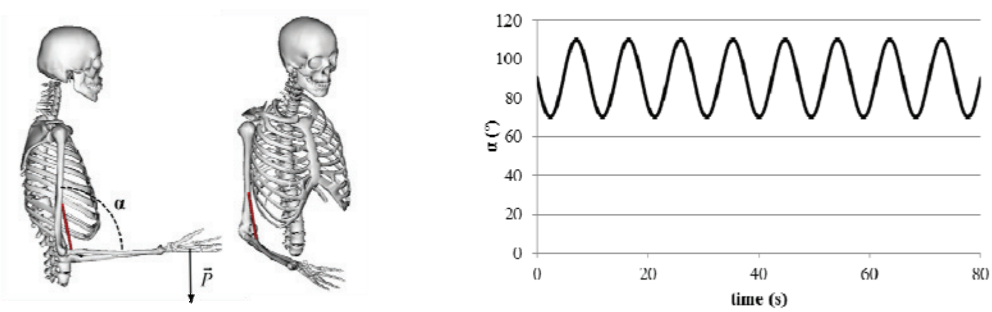
3. Results and Discussion

To test the model functionality with muscle fatigue, a simple right upper extremity model of the elbow joint is designed as represented in in Fig. 4. A concentrated force $P=100N$ is applied at $30cm$ from the elbow joint and, in addition, the whole biomechanical system is subjected to the gravitational force. The model is defined by three rigid bodies: torso, arm and a forearm-hand complex; and only the *brachialis* muscle is considered for the demonstration of the fatigue model. For the geometrical and inertial properties of the selected muscle and rigid bodies, the reader is referred to [10]. The values used for the fatigue parameters are the joint specific values obtained from [6] and are presented in Table 1. These values are considered for example purposes only. It is assumed that $M_R(t=0)$ = 100%.

Table 1: Fatigue parameters used in the upper extremity model.

F [s^{-1}]	R [s^{-1}]	L_D [s^{-1}]	L_R [s^{-1}]
0.016	0.0024	10	10

This model is tested in an IDA perspective where a kinematic driver for the elbow joint is prescribed, imposing a sinusoidal variation of the angle between the humerus and the radius-ulna-hand complex, depicted in Fig 4.a) as α. The evolution of α with time is shown in Fig. 4.b). The analysis was performed until the fatigue state of the muscle causes the system to be incapable of performing the desired motion.



(a) (b)

Fig. 4. Simple upper extremity model with a constant force P applied in the hand: a) Representation of the multibody model and external loadings; b) The evolution of the elbow angle over time. (Image obtained using the GUI of OpenSim software [27]).

The calculated muscle force of the contractile element F_{CE}^{m} for the considered muscles is illustrated in Fig. 5. It can be seen that there is an accumulation of fatigue in the muscle due to the unceasing production of contractile force and in a similar way to what happens in the real biological system, the CNS has to increase the amount of activated MUs due to the deterioration of muscular fitness, in order to maintain the desired acceleration of the anatomical segments.

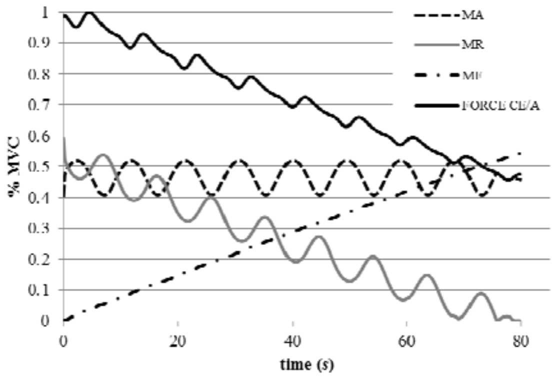


Fig. 5. Fatigue results for the brachialis muscle.

4. Conclusions

In this work a methodology was presented that allows for the calculation of the redundant muscle forces in a multibody environment defined with natural coordinates. Muscles were successfully introduced as concentrated forces applied over the origin, insertion and viapoints of the muscle structure under simulation or analysis. Physiological meaning was added to the solution of the redundant muscle force problem through the use of a proper Hill-type muscle model that considers the contraction dynamics of the muscle tissue, relating the available contractile force with the actual state of the muscle.

In the majority of multibody methodologies using other types of coordinates than natural coordinates, the calculation of the joint reaction forces is usually carried out in a post-processing step, and the Lagrange multipliers associated to these forces are not directly considered in the optimization. In this work, it was shown that the proposed multibody formulation is suitable to calculate muscle actuator efforts in a static optimization framework, considering also the joint reaction forces during the static optimization step. Since cost functions can include the Lagrange multipliers associated to reaction forces magnitudes, these can be used to simulate joint pathologies such as osteoarthritis or joint prosthetics, which is considered as an another advantage of the integrated calculation procedure.

Novel to most multibody approaches was the successful adaptation and implementation of a muscle fatigue model to the calculation of the redundant muscle forces. The model is straightforward, easy to implement computationally over any existing multibody methodology and can be used both in FDA or in IDA. These characteristics confer an added relevance to the proposed model, which produces results that can be compared to real physiological situations, *i.e.*, when a contractile output is desired to maintain and muscle fatigue is present, the CNS has to increase the number of recruited MUs in order to compensate the loss of force production capacity. This physical process is compatible with the results obtained as an increase of the amount of fatigue MUs led to an increase of muscle activation in order to keep the prescribed sinusoidal movement.

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