Correction terms for the thermodynamics of a black Saturn

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Mir Faizal [a],*, Behnam Pourhassan [b]

[a] *Department of Physics and Astronomy, University of Waterloo, Waterloo, ON N2L 3G1, Canada*
[b] *School of Physics, Damghan University, Damghan, Iran*

A B S T R A C T

In this paper, we will analyze the effects of thermal fluctuations on the stability of a black Saturn. The entropy of the black Saturn will get corrected due to these thermal fluctuations. We will demonstrate that the correction term generated by these thermal fluctuations is a logarithmic term. Then we will use this corrected value of the entropy to obtain bounds for various parameters of the black Saturn. We will also analyze the thermodynamical stability of the black Saturn in presence of thermal fluctuations, using this corrected value of the entropy.

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1. Introduction

If entropy is not associated with a black hole, then the entropy of the universe will spontaneous reduce whenever an object with a finite entropy crosses the horizon. Thus, entropy has to be associated with a black hole to prevent the violation of the second law of thermodynamics [1,2]. In fact, black holes have more entropy than any other object of the same volume [3,4]. This prevents the violation of second law of thermodynamics. This maximum entropy of the black holes is proportional to the area of the horizon [5]. Thus, if S is the entropy associated with a black hole, and A is the area of the horizon, then the relation between S and A can be expressed as $S = A/4$. The observation that the entropy scales with the area of the black hole, instead of its volume, has motivated the development of the holographic principle [6,7]. The holographic principle states that the degrees of freedom in a region of space are the same as the degrees of freedom on the boundary surrounding that region of space. The geometry of black holes will undergo quantum fluctuations. These quantum corrections will lead to thermal fluctuations. These thermal fluctuations will in turn generate correction terms for various thermodynamical quantities associated with black holes [8,9]. Thus the holographic principle can get modified near Planck scale [10,11]. It may be noted that even though the thermodynamics of black holes is expected to get corrected due to thermal fluctuations, we can neglect such correction terms for large black holes. This is because these thermal fluctu-

ations occur because of quantum fluctuations of the geometry of space–time, and such quantum fluctuations can be neglected for large black holes. However, as the black holes radiate Hawking radiation, they tend to evaporate in course of time. Then the size of the black holes reduces in course of time due to the Hawking radiation. As the black holes become smaller the quantum fluctuations give more dominating contribution to the geometry of space–time. Thus, the thermal fluctuations cannot be neglected for small black holes, or for black hole at the last stages of their evaporation. The correction terms to the entropy of black holes coming from thermal fluctuations have been calculated. It has been demonstrated that these correction terms are expressed as logarithmic functions of the original thermodynamic quantities.

The corrections to the thermodynamics of black holes have also been calculated using the density of microstates for asymptotically flat black holes [12]. This analysis has been done in the framework of non-perturbative quantum general relativity. Here conformal blocks of a well defined conformal field theory are associated with the density of states for a black hole. This density of states is then used to calculate the relation between the entropy of a black hole and the area of its horizon. The leading order relation between the entropy of a black hole and the area of its horizon is observed to be the standard Bekenstein entropy area relation for the large black holes. However, this relation between the area and entropy of a black hole gets corrected in this analysis. The leading order correction terms to the entropy of the black hole are demonstrated to be logarithmic corrections. It may be noted that such correction terms have also been calculated using the Cardy formula [13]. In fact, it has been demonstrated using this formula that such logarithmic correction terms will be generated for all

black holes whose microscopic degrees of freedom are described by a conformal field theory. The correction terms to the entropy of a BTZ black hole have been calculated using such logarithmic exact partition function [14]. It has been again observed that these correction terms can be expressed using logarithmic functions. It has also been possible to obtain logarithmic correction terms for the entropy of a black hole by analyzing matter fields in backgrounds of a black hole [15–17].

The correction terms generated from string theoretical effects can also be expressed using logarithmic functions [18–21]. The logarithmic correction terms for the entropy of a dilatonic black holes have been calculated [22]. Finally, the expansion of the partition function has also been used to calculate the correction terms for the entropy of a black hole [23]. Such correction terms obtained by using the expansion of the partition function again are logarithmic correction terms. The correction to the thermodynamics of black holes from generalized uncertainty principle has also been studied [24]. In this analysis the thermodynamics of the black holes gets modifies due to the generalization of the usual Heisenberg uncertainty principle. It has been demonstrated this modified thermodynamics of the black holes predicts the existence of a remnant for black holes. The existence of such remnants for black holes can have important phenomenological consequences [25].

As the quantum fluctuations can occur in all black hole geometries, we expect that the thermodynamics of all black objects will get corrected due to thermal fluctuations. Thus, we can use the modified relation between the entropy and area to analyze the corrections for the thermodynamics of any black object. In this paper, we will analyze such correction terms for the thermodynamics of black Saturn. The black Saturns are solutions to Einstein equations in higher dimensions. They are described by a black hole surrounded by a black ring [26,27]. This black ring is in thermodynamical equilibrium with a spherical black hole. The thermodynamics of black Saturn has been studied [28]. The thermodynamic equilibrium is obtained because of the rotation of the black ring. It is also possible to construct a black Saturn with a static black ring [29,30]. In this case, the system remains in thermodynamic equilibrium because of an external magnetic field. It may be noted that conditions for meta-stability of a black Saturn have also been studied [31]. It has been demonstrated that the black Saturn is causal stably on the closure of the domain of outer communications [32]. The relation between the black Saturn and Myers–Perry black hole has also been analyzed [33]. It may be noted that the thermodynamics of a charged dilatonic black Saturn has also been studied [34]. It is expected that both the black hole and black ring in a black Saturn will reduce in size due to the Hawking radiation. Thus, at a certain stage quantum fluctuations in the geometry of a black Saturn will also become important. To analyze the effect of these quantum fluctuations in the geometry of a black Saturn, we will need to analyze the thermal fluctuations in the thermodynamics of black Saturn. So, we will study the corrections to the thermodynamics of a black Saturn by considering thermal fluctuations around the equilibrium.

2. Black Saturn

In this section, we will review the thermodynamics of black Saturn. The metric for black Saturn can be written as [26]

$$ds^2 = -\frac{H_y}{H_x}\left[dt + \left(\frac{\omega_\psi}{H_y} + q\right)d\psi\right]^2$$
$$+ H_x\left[k^2 P(d\rho^2 + dz^2) + \frac{G_y}{H_y}d\psi^2 + \frac{G_x}{H_x}d\varphi^2\right], \quad (1)$$

where q and k are constants, and

$$G_x = \frac{\mu_4}{\mu_3\mu_5}\rho^2$$
$$G_y = \frac{\mu_3\mu_5}{\mu_4}. \quad (2)$$

Here we have used

$$P = (\mu_3\mu_4 + \rho^2)^2(\mu_1\mu_5 + \rho^2)(\mu_4\mu_5 + \rho^2), \quad (3)$$

and

$$\mu_i = \sqrt{\rho^2 + (z - a_i)^2} - (z - a_i) = R_i - (z - a_i). \quad (4)$$

The real constant parameters a_i ($i = 1, \ldots, 5$) satisfy the following condition,

$$a_1 \leq a_5 \leq a_4 \leq a_3 \leq a_2. \quad (5)$$

Furthermore, we also have

$$H_x = \frac{M_0 + c_1^2 M_1 + c_2^2 M_2 + c_1 c_2 M_3 + c_1^2 c_2^2 M_4}{F}$$

$$H_y = \frac{1}{F}\frac{\mu_3}{\mu_4}\left[\frac{\mu_1}{\mu_2}M_0 - c_1^2 M_1 \frac{\rho^2}{\mu_1\mu_2} - c_2^2 M_2 \frac{\mu_1\mu_2}{\rho^2} + c_1 c_2 M_3\right.$$
$$\left. + c_1^2 c_2^2 M_4 \frac{\mu_2}{\mu_1}\right], \quad (6)$$

where c_1 and c_2 are real constants, and

$$M_0 = \mu_2\mu_5^2(\mu_1 - \mu_3)^2(\mu_2 - \mu_4)^2(\rho^2 + \mu_1\mu_2)^2$$
$$\times (\rho^2 + \mu_1\mu_4)^2(\rho^2 + \mu_2\mu_3)^2,$$
$$M_1 = \mu_1^2\mu_2\mu_3\mu_4\mu_5\rho^2(\mu_1 - \mu_2)^2(\mu_2 - \mu_4)^2$$
$$\times (\mu_1 - \mu_5)^2(\rho^2 + \mu_2\mu_3)^2,$$
$$M_2 = \mu_2\mu_3\mu_4\mu_5\rho^2(\mu_1 - \mu_2)^2(\mu_1 - \mu_3)^2$$
$$\times (\rho^2 + \mu_1\mu_4)^2(\rho^2 + \mu_2\mu_5)^2,$$
$$M_3 = 2\mu_1\mu_2\mu_3\mu_4\mu_5(\mu_1 - \mu_3)(\mu_1 - \mu_5)$$
$$\times (\mu_2 - \mu_4)(\rho^2 + \mu_1^2)(\rho^2 + \mu_2^2)$$
$$\times (\rho^2 + \mu_1\mu_4)(\rho^2 + \mu_2\mu_3)(\rho^2 + \mu_2\mu_5),$$
$$M_4 = \mu_1^2\mu_2\mu_3^2\mu_4^2(\mu_1 - \mu_5)^2(\rho^2 + \mu_1\mu_2)^2(\rho^2 + \mu_2\mu_5)^2, \quad (7)$$

with

$$F = \mu_1\mu_5(\mu_1 - \mu_3)^2(\mu_2 - \mu_4)^2(\rho^2 + \mu_1\mu_3)$$
$$\times (\rho^2 + \mu_2\mu_3)(\rho^2 + \mu_1\mu_4)(\rho^2 + \mu_2\mu_4)(\rho^2 + \mu_2\mu_5)$$
$$\times (\rho^2 + \mu_3\mu_5)(\rho^2 + \mu_1^2)(\rho^2 + \mu_2^2)(\rho^2 + \mu_3^2)$$
$$\times (\rho^2 + \mu_4^2)(\rho^2 + \mu_5^2). \quad (8)$$

Here ω_ψ is expressed as

$$\omega_\psi = \frac{2}{F\sqrt{G_x}}\left[c_1 R_1\sqrt{M_0 M_1} - c_2 R_2\sqrt{M_0 M_2}\right.$$
$$\left. + c_1^2 c_2 R_2\sqrt{M_1 M_4} - c_1 c_2^2 R_1\sqrt{M_2 M_4}\right], \quad (9)$$

where R_1 and R_2 are given in the relation (4). Free parameters of the model are fixed by [27] as

$$L^2 = a_2 - a_1, \quad (10)$$

and

$$c_1 = \pm\sqrt{\frac{2(a_3 - a_1)(a_4 - a_1)}{a_5 - a_1}}. \quad (11)$$

We also have

$$c_2 = \sqrt{2}(a_4 - a_2)$$
$$\times \frac{\sqrt{(a_1 - a_3)(a_4 - a_2)(a_2 - a_5)(a_3 - a_5)} \pm (a_2 - a_1)(a_3 - a_4)}{\sqrt{(a_1 - a_4)(a_2 - a_4)(a_1 - a_5)(a_2 - a_5)(a_3 - a_5)}}, \tag{12}$$

and

$$k = \frac{2(a_1 - a_3)(a_2 - a_4)}{2(a_1 - a_3)(a_2 - a_4) + (a_1 - a_5)c_1 c_2} = \frac{2k_1 \hat{k}_2}{2k_1 \hat{k}_2 + c_1 c_2 k_3}. \tag{13}$$

Here we have used $\hat{k}_i = 1 - k_i$ and,

$$k_i = \frac{a_{i+2} - a_1}{L^2}, \tag{14}$$

with $i = 1, 2, 3$. The variable q can be written as

$$q = \frac{2k_1 c_2}{2k_1 - 2k_1 k_2 + c_1 c_2 k_3}. \tag{15}$$

So, we can see that all parameters can be written in terms of a_i with $i = 1, 2, 3, 4, 5$.

The Hawking temperatures for the black Saturn are obtained by combination of the Hawking temperatures for the black hole with the Hawking temperatures for the black ring. These combined Hawking temperatures can be expressed as [5],

$$T = \frac{1}{2\pi L} \sqrt{\frac{\hat{k}_2 \hat{k}_3}{2\hat{k}_1}} \left(\frac{(1 + k_2)^2}{1 + \frac{k_1 k_2 \hat{k}_2 \hat{k}_3}{k_3 \hat{k}_1} c^2} \right)$$
$$+ \frac{1}{2\pi L} \sqrt{\frac{k_1 \hat{k}_3 (k_1 - k_3)}{2k_2 (k_2 - k_3)}} \left(\frac{(1 + k_2)^2}{1 - (k_1 - k_2)c + \frac{k_1 k_2 \hat{k}_1}{k_3} c^2} \right), \tag{16}$$

where

$$c = \frac{1}{k_2} \left(\varepsilon \frac{k_1 - k_2}{\sqrt{k_1 \hat{k}_2 \hat{k}_3 (k_1 - k_3)}} - 1 \right). \tag{17}$$

Here $\varepsilon = \pm 1$, while $\varepsilon = 0$ gives a naked singularity. The entropy of the black Saturn, in absence of thermal fluctuations, can also be expressed as

$$S_0 = \frac{\pi^2 L^3}{(1 + k_2)^2} \sqrt{\frac{2\hat{k}_1^3}{\hat{k}_2 \hat{k}_3}} \left(1 + \frac{k_1 k_2 \hat{k}_2 \hat{k}_3 c^2}{k_3 \hat{k}_1} \right)$$
$$+ \frac{\pi^2 L^3}{(1 + k_2)^2} \sqrt{\frac{2k_2 (k_2 - k_3)^3}{k_1 (k_1 - k_3) \hat{k}_3}} \left(1 - (k_1 - k_2)c + \frac{k_1 k_2 \hat{k}_3 c^2}{k_3} \right). \tag{18}$$

This is the entropy of the combination of the black hole and black ring. The ADM mass of black Saturn, which can be interpreted as enthalpy $H = M_{ADM}$, is given by [5,35,36],

$$M_{ADM} = \frac{3\pi L^2}{4k_3 (1 + k_2 c)^2} \left(k_3 (\hat{k}_1 + k_2) - 2k_2 k_3 (k_1 - k_2)c \right.$$
$$\left. + k_2 [k_1 - k_2 k_3 (\hat{k}_2 + k_1)]c^2 \right). \tag{19}$$

We will use this expression to study enthalpy and therefore PV diagram.

3. Corrected thermodynamics

In this section, we will analyze the corrections to the thermodynamics of a black Saturn because of thermal fluctuations. These thermal fluctuations will become important as the black Saturn reduces in size due to the Hawking radiation. We can now write the partition function for this system as

$$Z = \int DgDA \exp(-I), \tag{20}$$

where $I \to -iI$ is the Euclidean action for this system. This partition function can also be written as [8,9]

$$Z = \int_0^\infty dE \, \rho(E) e^{-\beta E}, \tag{21}$$

where β is the inverse of the temperature. We can write an expression for the density of states as

$$\rho(E) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\beta \, e^{S(\beta)}, \tag{22}$$

where

$$S = \beta E + \ln Z. \tag{23}$$

The quantum fluctuations in the geometry of space–time, will lead to the thermal fluctuations in the thermodynamics of black Saturn. Thus, if the $S(\beta)$ is inverse of the corrected temperature, then we can expand it around the equilibrium temperature β_0,

$$S = S_0 + \frac{1}{2} (\beta - \beta_0)^2 \left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta = \beta_0} + \cdots. \tag{24}$$

Neglecting the higher order corrections, we obtain

$$\rho(E) = \frac{e^{S_0}}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\beta \, \exp \left(\frac{1}{2} (\beta - \beta_0)^2 \left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta = \beta_0} \right). \tag{25}$$

We can write this expression as

$$\rho(E) = \frac{e^{S_0}}{\sqrt{2\pi}} \left[\left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta = \beta_0} \right]^{-1/2}. \tag{26}$$

Now, we obtain

$$S = S_0 - \frac{1}{2} \ln \left[\left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta = \beta_0} \right]^{1/2}. \tag{27}$$

It may be noted that this second derivative of entropy is actually a fluctuation squared of the energy, so we can write [8,9]

$$\left[\left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta = \beta_0} \right]^{1/2} = C_0 T^2 \tag{28}$$

Hence,

$$S = S_0 - \frac{1}{2} \ln C_0 T^2. \tag{29}$$

Here, S_0 is the original entropy of the combination of the black hole and the black ring given by the equation (18). Thus, the entropy of both black ring and black hole will get corrections to it because of thermal fluctuation. Now, we will introduce a variable α

to parameterize the effect of thermal fluctuations on the thermodynamics of black Saturn. Thus, we will write the expression for the entropy as

$$S = S_0 - \frac{\alpha}{2} \ln C_0 T^2,$$ (30)

where

$$C_0 = T \frac{\partial S_0}{\partial T}.$$ (31)

We can infer from relation (14) that all thermodynamics quantities depend on a_1, a_2, a_3, a_4 and a_5. Therefore, we can calculate thermodynamics quantities in terms of a_i with $i = 1, \ldots, 5$. First of all, for simplicity we can fix four of them according to condition given by the equation (5), and obtain thermodynamics quantities in terms of only one free parameter. Therefore we will consider five different cases. Using the equations (16), (18) and (30), we can study logarithmic corrected entropy. For the five cases of fixed parameters, we can analyze the behavior of entropy in the plots of Fig. 1. We fix parameters as $a_1 = 1, a_2 = 5, a_3 = 4, a_4 = 3$ and $a_5 = 2$, which satisfy condition (5).

In Fig. 1(a), we vary a_1 with the condition $a_1 \leq 2$, and in Fig. 1(e), we vary a_5, with the condition $1 \leq a_5 \leq 3$. It is observed that both are in agreement with the condition (5). We can see that both solutions are only valid in absence of logarithmic correction, so we are not allowed to use fixed parameters as illustrated by Fig. 1(a) and (e). In Fig. 1(b), we vary a_2 with the condition $a_2 \geq 4$ and observe that presence of logarithmic correction fixes $a_2 = a_3$. Similar result is obtained by Fig. 1(c). Here we vary a_3 with the condition $a_3 \leq 5$, and see that presence of logarithmic correction fixes $a_3 = a_2$. In Fig. 1(d), we vary a_4 with the condition $2 \leq a_4 \leq 4$, and observe that it is only possible to consider the case of $a_4 = a_5$ in presence of logarithmic corrections. Therefore, the logarithmic correction fixes free parameters $(a_1, a_2, a_3, a_4, a_5)$. Thus, we can consider the following example $(1, 4, 4, 3, 2)$, $(1, 5, 5, 3, 2)$ or $(1, 5, 4, 2, 2)$. It implies that if we fix four parameters, then the last parameter also should be fixed. When, all parameters are fixed then, temperature and entropy and all thermodynamics quantities are constants while we need them as a function of parameters.

So, we will only fix three of these parameters and keep two of them as variable. From Fig. 1, we observe that by considering a_1 and a_5 as variable, we do not obtain a physical result. Therefore, we will fix $a_1 = 1$ and $a_5 = 2$, and vary other parameters (according to the condition (5)). In the plots of Fig. 2, we plot the entropy corrected by the logarithmic term (30), and ordinary entropy (18) as a function of two a_i. In plots of (a), we fix a_2 and vary a_3 and a_4. This is done for both $\alpha = 0$ and $\alpha = 1$. In plots of (b), we fix a_3 and vary a_2 and a_4. This is again done for $\alpha = 0$ and $\alpha = 1$. In plots of (c), we fix a_4 and vary a_3 along with a_2. Here again we consider both the values of α, i.e., $\alpha = 0$ and $\alpha = 1$. The allowed values of a_3 and a_4 can be inferred from the plot of Fig. 2 (a). This is done by considering those values that satisfy the condition (5). Thus, we consider $a_3 \geq 2$ and $a_4 \geq a_3$, and observe that there is no entropy in this region. The same fact can be obtained by using plot of Fig. 2(c). This is done by fixing a_4 and varying a_3 and a_2 according to the condition (5). Here we again obtain nothing. Therefore the only choice we have is to selected values like in Fig. 2(b). However, for these values the entropy is negative.

If we obtain the third solution, we can relate all parameters to each other and rewrite them in terms of only one free parameter as a variable. Therefore all of them are considered as variables. Even though there are several choices, we will consider the following choices,

$$a_2 = 5a_3$$

$$a_3 = 4a_4$$

$$a_4 = 3a_5$$

$$a_5 = 2a_1^2.$$ (32)

Therefore the only free parameter of model is a_1. Using ansatz (32) in the corrected entropy (30) gives us the result which is illustrated by Fig. 3. As was expected, the parameters for black hole (a_1 and hence other parameters) get restricted in presence of logarithmic corrections ($\alpha = 1$). The values we selected can be used to obtain, $a_1 \geq 4$ and $a_5 \geq 32$, $a_4 \geq 66$, $a_3 \geq 264$ and $a_2 \geq 1320$. On the other hand, it is also possible to consider

$$0.1 < a_1 < 0.5,$$

$$0.5 < a_1 < 0.9.$$ (33)

There is a singularity at $a_1 = 0.5$. It is possible to use specific heat for choosing one of the above mentioned solutions. It is possible to choose other possibilities and obtain similar results.

It is possible to demonstrate that

$$\frac{\partial S}{\partial a_1} = \frac{\partial S_0}{\partial a_1} - \frac{\alpha}{2}(1 + \frac{1}{T})\frac{\partial T}{\partial a_1}.$$ (34)

Thus, we can write the internal energy as,

$$E = \int T dS = E_0 - \frac{\alpha}{2}T - \frac{\alpha}{4}T^2,$$ (35)

where

$$E_0 = \int T dS_0.$$ (36)

So we can study behavior of $\Delta E = E - E_0$ in terms of T, and this in turn can be expressed in terms of a_1 (see Fig. 4). There are two special choices of a_1, where the logarithmic correction does not contribute to the result. The first one being $a_1 = 0.5$, and this corresponds to a singularity in the entropy (see Fig. 3), and $a_1 \approx 20$ and this is not in appropriate domain of $0.1 < a_1 < 0.5$ and $0.5 < a_1 < 0.9$. This will be demonstrated to be a requirement of stability. So, we can infer that, in the case of $0.1 < a_1 < 0.5$ corrected energy is bigger than ordinary energy ($E > E_0$). On the other hand, for the case of $a_1 > 0.5$, we have $E < E_0$.

Then, using the relation (19) and,

$$H = E + PV,$$ (37)

we can investigate PV diagram. We can also analyze the behavior of Gibbs free energy because

$$G = F + PV,$$ (38)

where

$$F = E - TS,$$ (39)

is Helmholtz free energy. It is easy to find that,

$$F = F_0 - \frac{\alpha}{2}T(1 - \ln C_0 T^2) - \frac{\alpha}{4}T^2,$$ (40)

where

$$F_0 = E_0 - TS_0.$$ (41)

So, we can study $\Delta F = F - F_0$ in terms of a_1 (see Fig. 5). Fig. 5 shows that the Helmholtz free energy exists only for selected values of a_1 as,

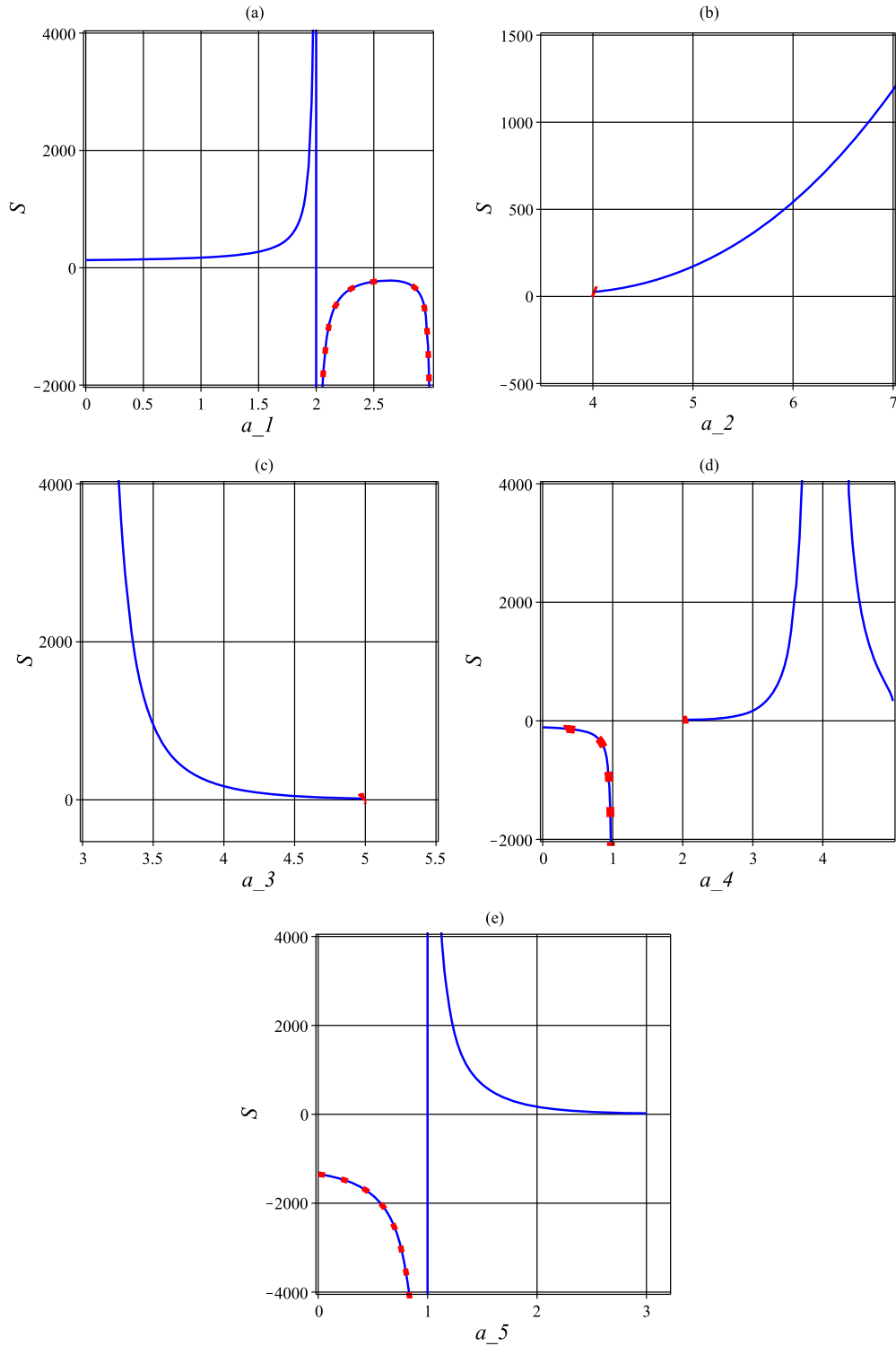$$0.2 \leq a_1 < 0.5.$$

$$0.65 < a_1 < 1.$$ (42)

Fig. 1. Entropy in terms of a_i, solid blue lines denote the case of $\alpha = 0$ (ordinary entropy) and dashed red lines denote the case of $\alpha = 1$ (logarithmic corrected entropy). (a) $a_2 = 5$, $a_3 = 4$, $a_4 = 3$ and $a_5 = 2$, (b) $a_1 = 1$, $a_3 = 4$, $a_4 = 3$ and $a_5 = 2$, (c) $a_1 = 1$, $a_2 = 5$, $a_4 = 3$ and $a_5 = 2$, (d) $a_1 = 1$, $a_2 = 5$, $a_3 = 4$ and $a_5 = 2$, (e) $a_1 = 1$, $a_2 = 5$, $a_3 = 4$ and $a_4 = 3$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

This is more restricted than previous result. It is clear that there are different values of a_1, where $F = F_0$, such that the effects from the logarithmic corrections are canceled by effect of a_1. In this case, we have $a_1 = 0.2, 0.275, 0.5, 0.78$. It may be noted that bounds on the parameters of a black Saturn have been obtained from the existence of Helmholtz free energy. We will demonstrate this to be related to the stability condition, i.e., it is related to the

positivity of the heat capacity. Hence, the black Saturns are stable only for certain values of parameters. Furthermore, we would like to clarify that it has been demonstrated that black Saturns are generally unstable. This is because higher entropy solution with the same charges always exists. However, it is possible for black Saturns to be metastable. We will use the metastability of black Saturns to find bounds on the parameters. So, basically we analyze
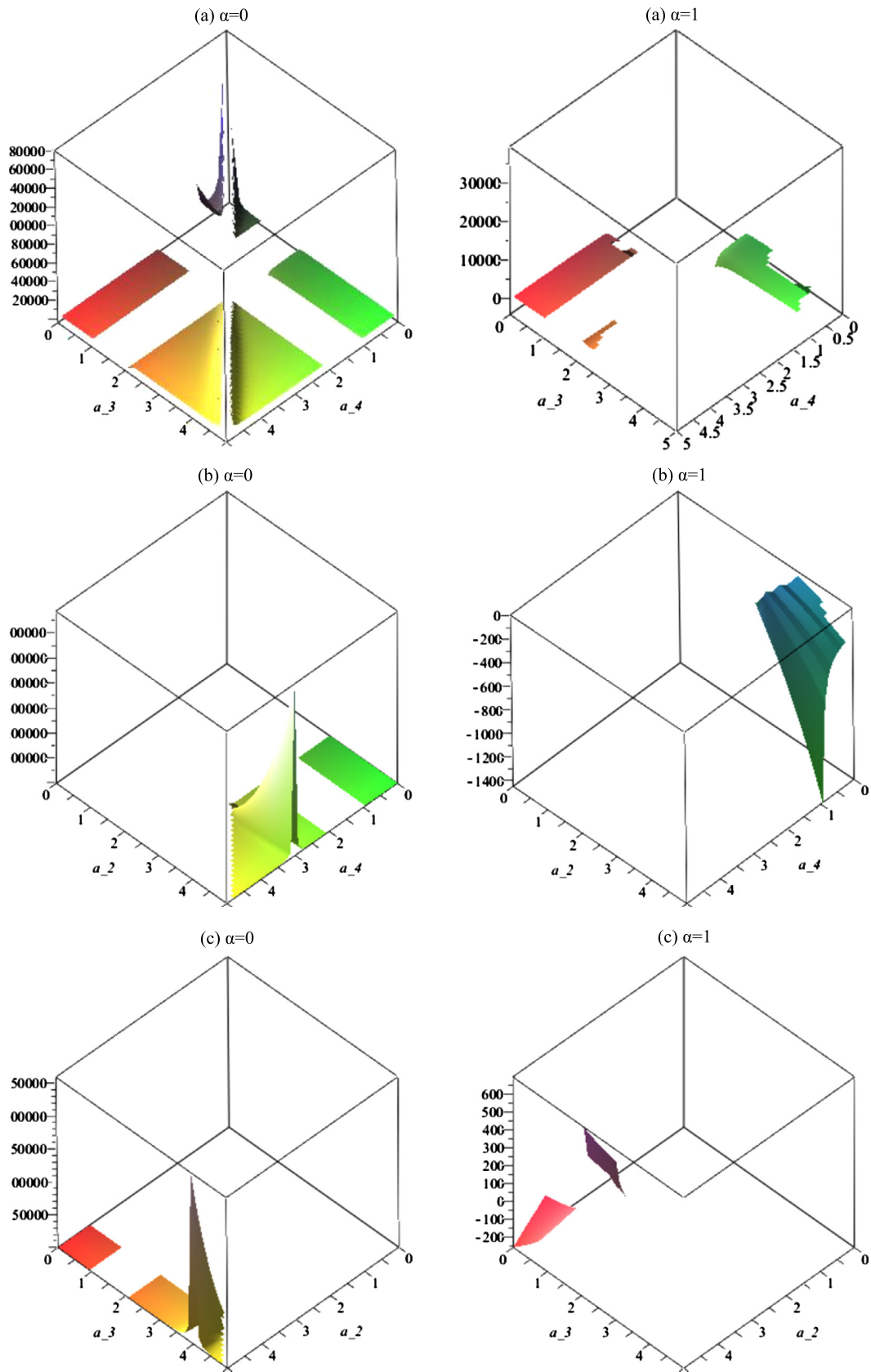
Fig. 2. Entropy in terms of a_2, a_3 and a_4 with $a_1 = 1$ and $a_5 = 2$. Left plots denote the case of $\alpha = 0$ (ordinary entropy) and right plots denote the case of $\alpha = 1$ (logarithmic corrected entropy). (a) $a_2 = 5$, (b) $a_3 = 3$, (c) $a_4 = 4$.
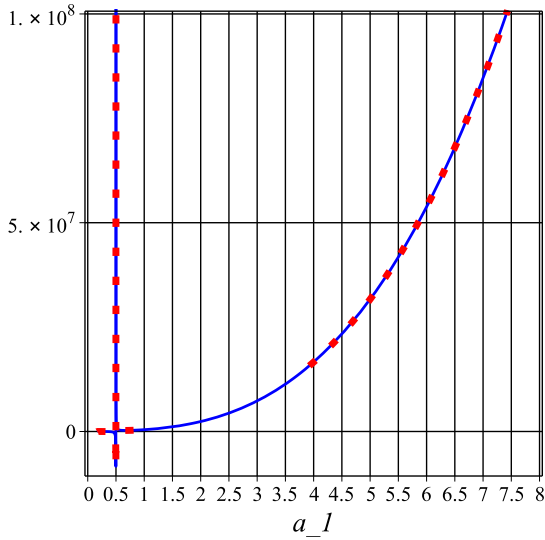
Fig. 3. Entropy in terms of a_1, with $\alpha = 0$ (solid blue) and $\alpha = 1$ (dotted red). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
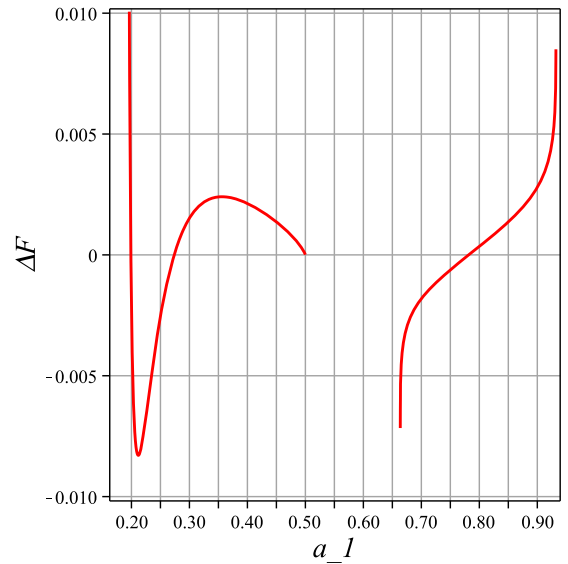

Fig. 5. Change of Helmholtz free energy (due to logarithmic correction) in terms of a_1.
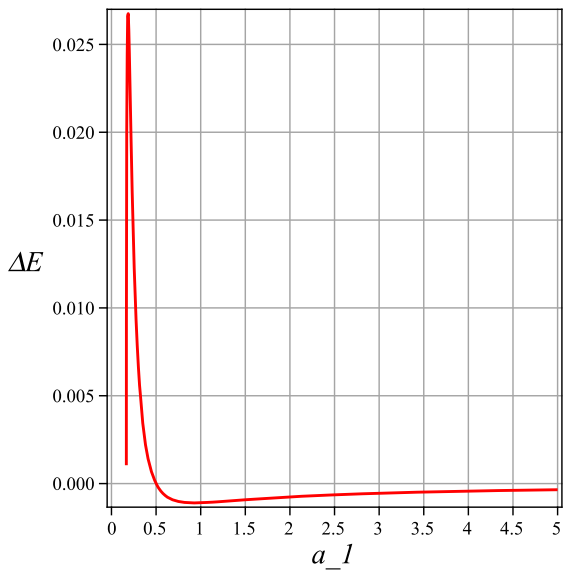

Fig. 4. Change of internal energy (due to logarithmic correction) in terms of a_1.
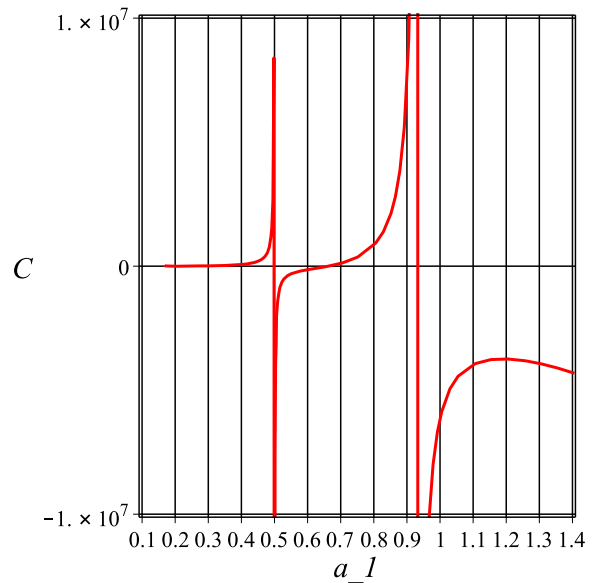

Fig. 6. Specific heat in terms of a_1.

the bounds from metastability of black Saturns, and also the effect of thermal fluctuations on such bounds.

4. Phase transition

In this section, we will analyze the phase transition in the black Saturn. Using the sign of specific heat at constant volume,

$$C = T \frac{\partial S}{\partial T}, \tag{43}$$

one can investigate phase transition of black hole. We will analyze the instability in presence of logarithmic corrections. Using the relation (34), above equation can be written as

$$C = C_0 - \frac{\alpha}{2}(1 + T). \tag{44}$$

It is easy to check that change of specific heat due to logarithmic correction is small as compared to 10^7. Therefore, we can observe

that logarithmic corrections do not effect the phase transition and thermodynamics stability of black Saturn. As these logarithmic corrections are generated from the thermal fluctuations, which in turn are generated from quantum corrections, we can infer that the black Saturn remains stable even in presence of quantum corrections. Thus, the black Saturn continues to remain stable as it get smaller due to the Hawking radiation. In Fig. 6, we have obtained phase transition points, which are $a_1 = 0.5, 0.65, 0.92$. Thus, there are two choices for black Saturn parameter

$$0.2 \leq a_1 \leq 0.5,$$

$$0.65 \leq a_1 \leq 0.9. \tag{45}$$

The black Saturn has thermodynamics stability for both these choices. There are two singular points, $a_1 = 0.5$ and $a_1 \approx 0.92$. Therefore, we can find two intervals for possible values of a_1. This

is given by overlap of the regions given by (33) and (42). Now, the equation (45) can be written as

$$0.2 \leq a_1 < 0.9, \qquad a_1 \neq 0.5. \tag{46}$$

However there is a condition given by (5). This condition uses (32) to infer that $a_5 = 2a_1^2$ and $a_5 \geq a_1$. Both these conditions are satisfied if

$$0.6 \leq a_1 < 0.9. \tag{47}$$

Thus, we have been able to analyze the effect of thermal corrections on the stability of black Saturn.

5. Conclusions

In this paper, we studied thermodynamics quantities of a black Saturn. We were able to analyze the effect of thermal fluctuations to the thermodynamics of black Saturn. The leading order correction terms to the entropy of black Saturn were the standard logarithmic corrections. It was not possible to fix all the free parameters for the black Saturn. However, we were able to use the stability condition to obtain a bound for these free parameters. The bound we obtain for these parameters could be expressed as $0.6 \leq a_1 < 0.9$, $0.72 \leq a_5 < 1.62$, $2.16 \leq a_4 < 4.86$, $8.64 \leq a_3 < 19.44$, $43.2 \leq a_2 < 97.2$. We also analyzed the phase transition for the black Saturn. We were able to explicitly calculate the points where phase transition can take place. It was demonstrated that the thermal fluctuations do not effect stability of the black Saturn, and so their effect can be neglected when analyzing the phase transition in the black Saturn. It will be interesting to explicitly calculate the partition function for density of states of the black Saturn. This can then be used to calculate the Helmholtz free energy and entropy for the black Saturn. Then, we can compare the results obtained to the analysis done in this paper. The thermodynamics of a charged dilatonic black Saturn has also been studied [34]. In this analysis a charged black ring along with a black Saturn has been studied using the Einstein–Maxwell-dilaton theory. This analysis was performed in five dimensions. This was done by embedding a neutral black ring and black Saturn solutions in six dimensions. They were then boosted with respect to the time coordinate and the sixth dimension. Then, the Kaluza–Klein reduction was used to obtain the charged solutions. The phase diagram

was also studied for this system. It would be interesting to analyze the effect of thermal fluctuations on the thermodynamics of this charged dilatonic black Saturn. The entropy of this charged dilatonic black Saturn is also expected to get logarithmic corrections due to these thermal fluctuations. We can then analyze the effect of such a corrected value of entropy on the stability of a charged dilatonic black Saturn.

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