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Excitations of Limited-Diffraction Waves Approaching the Classical 0-order X-Wave by Rectangular Waveforms

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Abstract

In this paper, an approach for simplifying the experimental arrangement, needed to generate limited diffracting waves through annular ultrasonic arrays, is analyzed in terms mainly of the subsequent acoustic field. The main idea is to approximate the theoretical X-wave electrical excitations to rectangular driving signals in each array annulus, by means of the L2 curve criterion. The differences between theoretical X-wave signals and these approximate signals, related to real excitation effects, were minimized by using the transition times and amplitudes of the rectangular signals as fitting parameters.

Acoustic field simulations, based on the impulse response technique, are applied for evaluating the agreement degree between both emitted ultrasonic fields, whit the calculated classical X wave and with the new approximation method proposed here for low-cost limited-diffraction wave generation. In addition, source vibration and ultrasonic field simulated signals were compared with those of the classic x wave under an exact driving, with the purpose of validating the method.

The good agreement between the two vibration signals and resulting field distributions, obtained from the classical X wave excitations and those provided by the drastic simplification presented here, can be justified by the filtering effects induced by the transducer elements bands in frequency domain. These results suggest the possibility of achieving limited diffraction waves with relatively simple driving waveforms, which can be implemented with a moderate cost in analogical electronics.

Key words: 0-order X wave;; limited diffraction wave; electrical excitation; rectangular waveforms.

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1. Introduction

A first theoretical limited-diffraction solution, for the scalar wave equation in isotropic/homogeneous media, was found in 1941 by J. A. Stratton [1], where a Bessel beam solution was obtained. In 1987, J. Durnin et al. [2]

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produced experimentally the Bessel beams approximation in optics, which was extended to acoustics by D. K. Hsu et al., in 1989 [3].

The X wave was presented first by J.-Y. Lu and J. F. Greenleaf [4], with successfully applications in the ultrasonic field in 1991; it is a particular case of limited diffracting waves. In the practice, the generation of X wave ultrasonic fields in real-time needs from an expensive and laborious technology, because it is necessary to arrange a voluminous multi-pulser system that allows generate the rather complex-in-shape driving waveforms related to the classical X-wave, with precise and specific time distributions in voltage for the excitation of each distinct array element, and maintaining the driving pulse shapes under reactive loading conditions (strongly capacitive) with all transducers electrically connected to the multi-pulser system.

In this paper, an excitation approach is proposed in order to drastically simplify the technology associated to such complex experimental arrangement, being necessary nowadays to generate limited diffraction X waves.

In our approach, theoretical X-wave electrical excitations are approximated with rectangular pulses, by means of applying the criterion of the L2 curve [5, 6]. This allows to carry out an optimization of the approaching of the X-wave signal patterns (by certain simpler driving waveforms), in order to minimize their differences.
This is performed by means of the adjustment of the main pulse parameters, choosing finally values that provide a minimum in the logarithm of the root medium square error.

2. Basic Theory

2.1. Broadband 0-order X wave solution

The solution in terms of acoustic field, for the 0-order X-wave family, that is produced by an infinite aperture, in broadband conditions, can be expressed by:

$$V_{X_0} = \frac{a_0}{\sqrt{(r\sin\zeta)^2 + [a_0 - i(z\cos\zeta - ct)]^2}} \quad (1)$$

where, for z = 0, V_{X_0} represents the theoretical X wave excitations in the distinct *r* values (points) of the acoustic aperture, in accordance with J.-Y. Lu *et al.* [4], $a_0 > 0$ is a constant, *r* is the radial coordinate, ζ ($0 < \zeta < \pi/2$) is the Axicon angle, *z* is the axial distance, *c* is the speed of sound in the medium, and *t* is the time. In this case, the values used for the parameters were: $a_0 = 0.05$ *mm*, $\zeta = 4°$ and c = 1.5 mm/μs.

2.2. Rectangular pulse excitation

A generic expression for the rectangular pulses, to be optimized for approximating the wave excitations, is:

$$V_R = \begin{cases} A & ,|t| \le t_0 \\ 0 & ,|t| > t_0 \end{cases} \quad (2)$$

where V_R are the rectangular excitation functions, and $t_0 > 0$ and *A* represent the half width time and the amplitude of the rectangular pulses, respectively.

2.3. Band-limited signals

Both broadband signals, related to the expressions: (1) for $z = 0$, and (2), were limited in bandwidth by employing (3):

$$u_{BL} = F^{-1}[B(f)] * V \tag{3}$$

$$B(f) = \begin{cases} b[0.42 - 0.5\cos\left(\frac{\pi f}{f_0}\right) + 0.08\cos\left(\frac{2\pi f}{f_0}\right)], 0 \le f \le 2f_0 \\ 0, otherwise \end{cases} \tag{4}$$

where: u_{BL} is related to the band-limited wave excitations derived from the broadband wave excitations V, and could represent the particle velocity in the emitter transducer face; $B(f)$ represents the voltage-velocity emitting transfer function; $b > 0$ is a constant, and f_0 is the central frequency in transducer band. In this case: $f_0 = 2.5$ MHz and $b = 0.05$ *V-mm/s*.

2.4. L2 criterion

The band-limited expressions derived from (1) and (2) for each annulus, by means of (3), are employed in the expression (5) describing the evaluation index considered to find the optimum amplitude and half width time values of the rectangular pulses (using the L2 criterion):

$$L_2^k(A, t_0) = 20\log_{10}\sqrt{\frac{\sum_{j=1}^{M}[u_{X_0 BLk}(t_j) - u_{R\ BLk}(A, t_0, t_j)]^2}{\sum_{j=1}^{M} u_{X_0 BLk}^2(t_j)}} \tag{5}$$

where: $u_{X_0\ BLk}(t_j)$ is the band-limited 0-order X-wave excitation, and $u_{R\ BLk}(t_j)$ is the band-limited rectangular excitation pulse, for the annulus k at the time t_j.

2.5. Spatial impulse response method

The spatial impulse response method [6] was applied to obtain the pressure field under a process of diffraction. For this case, the pressure response can be written as:

$$p(x,t) = \rho\sum_{k=1}^{N}\frac{\partial(u_{BLk})}{\partial t} * h_k(x,t) \tag{6}$$

where p is the acoustic pressure, x is the vector of position, ρ is the density of medium, u_{BLk} is the band-limited wave excitation for the annulus k, h_k is the spatial impulse response of the annulus k and * represent the time convolution operator.

3. Excitation and Field Results

All simulations have been made for an eight-rings 2.5 MHz array transducer with 4 mm wide for each annulus.

The table 1 shows, for each annulus of the array transducer, the values of separation position, the radium of evaluation of the 0-order X wave equation, and the obtained values for the amplitude and half width time of the rectangular excitation pulses, after the criterion of the curve L2 was applied for each 0-order X-wave electrical excitation of the annular array.

Table 1. Values obtained for the excitation rectangular waveforms of the annular array transducer

Number of Element	Separation position [mm]	Radius of evaluation [mm]	Normalized Driving Amplitude	Half width time of the rectangular pulse t_0 [ns]
1	4	0	0.56	70.95
2	9.18	3.39	0.203	218.5
3	14.4	5.995	0.179	322.8
4	19.62	8.605	0.1465	448.68
5	24.8	11.205	0.1291	574.59
6	30	13.8	0.1159	700.5
7	35.2	16.4	0.1073	812.42
8	40.6	19.05	0.0994	938.33

The figure 1 shows the X wave electrical excitations and the rectangular pulse excitations obtained by means of an optimization process with the criterion L2. For this case, the parameters to be optimized were the width and the amplitude of the rectangular pulses.

The signal used as a pattern, in the criterion of the curve L2, was the velocity in the emitter face of the transducer calculated from the X-wave solution; whereas the signal to be fitted was the velocity generated by our rectangular driving approach.

The signals of velocity obtained with expression (3) for, X wave excitations and rectangular pulsed excitations, are shown in the figure 2. This figure shows a notable similarity between the signals of velocity obtained with X wave excitations, and those produced from the approach presented here (with rectangular driving pulses). This interesting circumstance could be explained by the filtering effect exerted by the transducer elements.

In addition to this initial comparative verification in the transducers vibration behaviors, the related ultrasonic field patterns, effectively radiated from these excitations in the annular aperture, were compared with the objective of validating the method.

The figure 3 shows the acoustic field simulations, based on impulse response technique and array diffraction analysis [6-8], that have been performed for evaluating the concord between both emitted ultrasonic fields: a) from the approximate limited - diffracting wave proposed here, and b) from the calculated classical X-wave solution with z set to 0 mm.

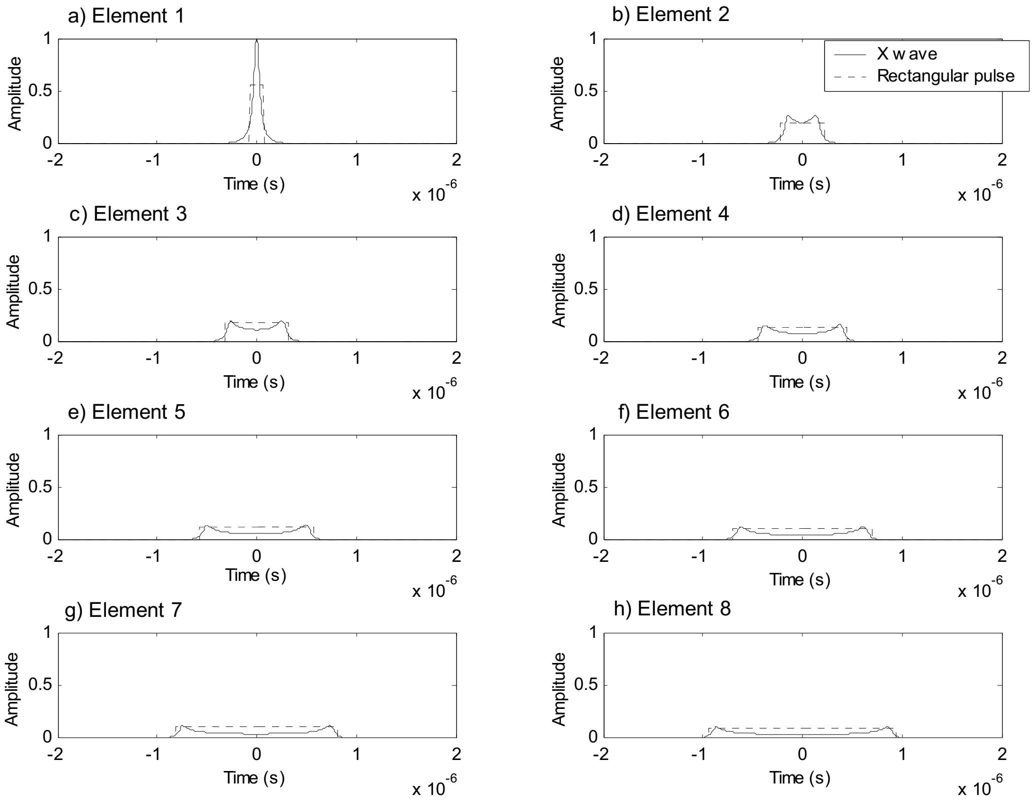

Fig.1 Rectangular pulse approach and classical X-wave driving waveforms evaluated in z = 0 mm. Both signals were normalized with respect to the maximum of all the X-wave waveforms. The time range for evaluation in the horizontal axis was limited from -2 µs to 2 µs.

The light differences observed between them are due to the following fact: the distribution in the domain of the frequencies, of the rectangular pulsed excitations, filtered by the transducer emitting transfer function, do not coincide completely with the composition in frequencies of the filtered 0-order X waves excitations.

This circumstance provokes some little discrepancies in the pressure beam formation (figure 3), especially in the region of the side lobes.

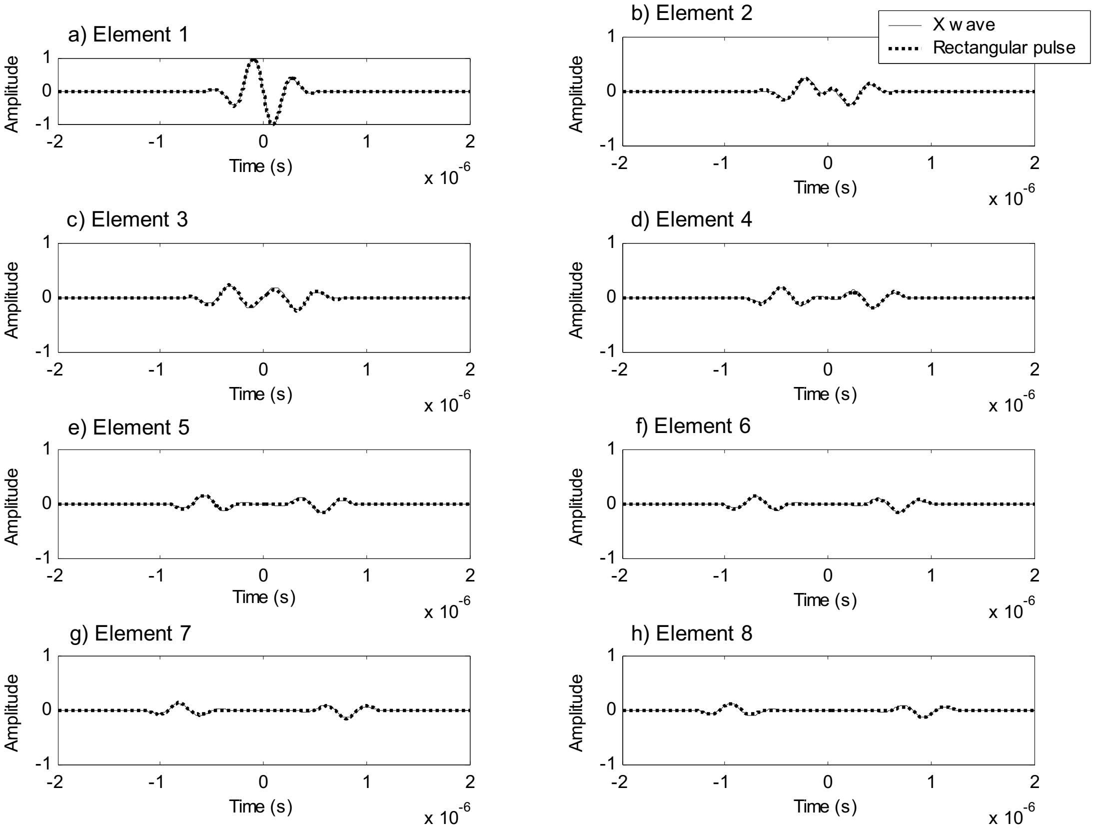

Fig.2 Signals of velocity produced by rectangular pulsed excitations and by the X wave electrical excitations. Both were normalized with respect to the maximum of signal of velocity produced by the X wave excitation evaluated in $r = 0$ mm. The time range of evaluation in the horizontal axis was limited from -2 μs to 2 μs.

4. Conclusions

The good agreement obtained between the ultrasonic field results of the classical 0-order X-wave solution for driving an annular array, and the rectangular simplification presented here for its pulsed electrical driving, can be justified by the filtering effect due to the transduction and propagation of the electrical driving waveform through the emitting transducer elements.

These results suggest the possibility of simplifying the technology commonly associated to the multi-channel electronic driving systems needed to generate limited diffracting waves.

Furthermore, it have been shown that other alternative types of waveforms, more simple than those classically considered for limited-diffraction excitations, can be employed for electrical excitation with this same purpose, because they produce a similar distribution in frequency domain (after the filtering process by the transducer elements) than in the said previous limited-diffraction approaches.

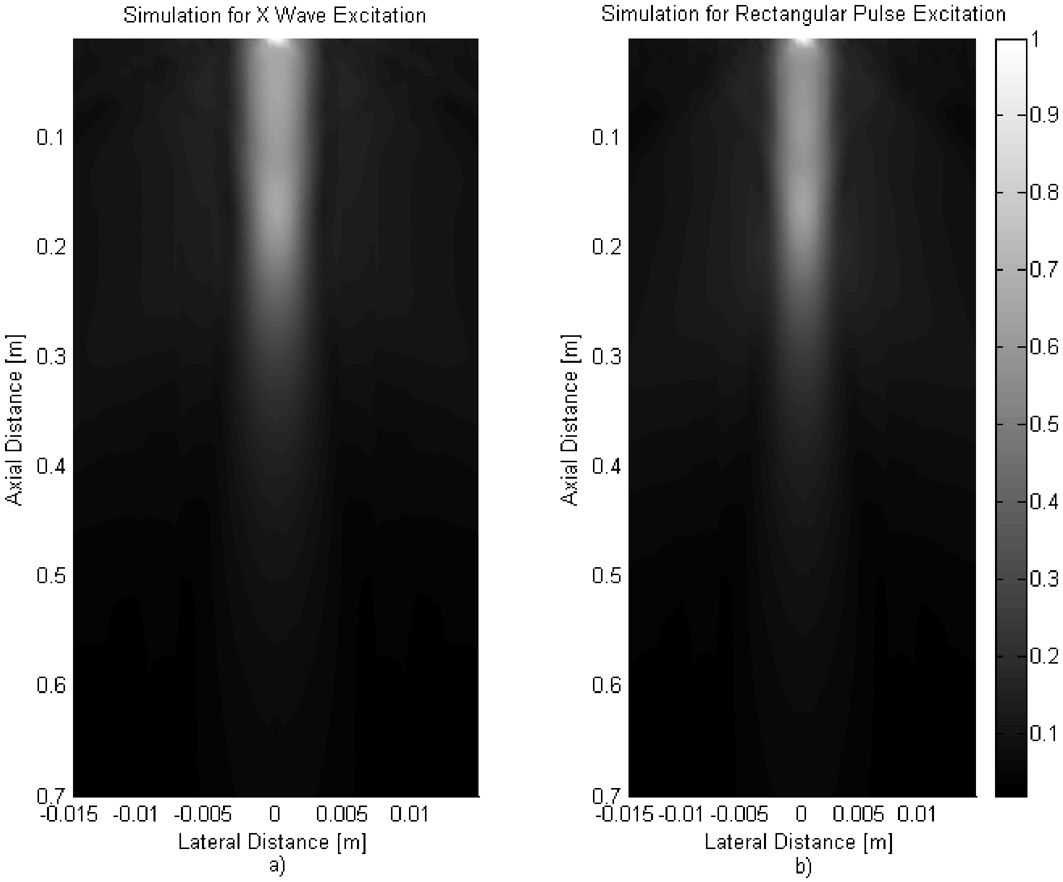

Fig.3 Plots of the peak maximum values of the normalized pressure field, calculated for a plane of symmetry. a) The classic X wave excitations and b) our rectangular pulsed excitations approach. Both pressure fields were normalized with respect to the maximum of each plot.

Acknowledgments

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References

[1] J. A. Stratton, “Electromagnetic Theory”, IEEE Press, 1941.

[2] J. Durnin; J. J. Miceli Jr., J. H. Eberly, “Diffraction-Free Beams”, Phys. Rev. Lett., Vol. 58 (15), p. 1499, 1987.

[3] D. K. Hsu, F. J. Margetan, D. O. Thompson, “Bessel Beam Ultrasonic Transducer: Fabrication Method and Experimental Results”, Appl. Phys. Lett., Vol. 55 (20), p. 2066, 1989.

[4] J.-Y. Lu, J. F. Greenleaf, “Theory and Acoustic Experiments of Nondiffracting X Waves”, IEEE Ultrason. Symp. Proc., p. 1155, 1991.

[5] A. Dutt, V. Rokhlin “Fast Fourier transform for nonequispaced data”, SIAM J. Sci. Comput., Vol. 14, p. 1368-1393, 1993.

[6] H. Calás, E. Moreno, J. A. Eiras, A. Aulet, J. Figueredo, L. Leija, “Non-uniformly polarized piezoelectric modal transducer: fabrication method and experimental results”, IOP Smart Materials and Structures, Vol. 15 (4), p. 904, 2006.

[7] P. R. Stepanishen, “Transient radiation from pistons in an infinite planar baffle”, J. Acoust. Soc. Am., Vol. 49 (5B), p. 1629, 1971.

[8] L. G. Ullate, A. Ramos, J. L. San Emeterio “Analysis of the ultrasonic field radiated by time – delay cylindrically focused linear arrays”. IEEE Trans. Ultrason. Ferroelec. Freq. Control. Vol. 41, nº 5, pp. 749-760, 1994.