Simple realization of inflaton potential on a Riemann surface

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ABSTRACT

The observation of the B-mode in the cosmic microwave background radiation combined with the so-called Lyth bound suggests the trans-Planckian variation of the inflaton field during inflation. Such a large variation generates concerns over inflation models in terms of the effective field theory below the Planck scale. If the inflaton resides in a Riemann surface and the inflaton potential is a multivalued function of the inflaton field when it is viewed as a function on a complex plane, the Lyth bound can be satisfied while keeping field values in the effective field theory within the Planck scale. We show that a multivalued inflaton potential can be realized starting from a single-valued Lagrangian of the effective field theory below the Planck scale.

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1. Introduction

Cosmic inflation scenario [1] has succeeded in not only solving the flatness and the horizon problems in the standard cosmology, but also providing the origin of the large scale structure of the Universe and the fluctuation of the cosmic microwave background (CMB) radiation [2]. Precise observations of the CMB have revealed the nature of inflation [3,4], and the observations can be well-explained by slow-roll inflation [5,6].

Recently, the BICEP2 Collaboration reported a large tensor fraction in the CMB, $r = O(0.1)$ [7]. Such a large tensor fraction is known to require a variation of the inflaton field value larger than the Planck scale during inflation (the so-called Lyth bound [8]). Thus, the observed tensor fraction apparently indicates a trans-Planckian inflaton field value during inflation. The chaotic inflation model [9], which shows a perfect fit with the BICEP2 results, is an excellent example in which the inflaton field value is much larger than the Planck scale.

For obvious reasons, however, models with such a large field value seem to be highly sensitive to the physics beyond the Planck scale including the theory of quantum gravity such as string theory. Without having any knowledge on the quantum gravity, the field theory is at the best considered to be an effective theory whose Lagrangian is given by a series expansion in fields. In particular, higher dimensional operators suppressed by the Planck scale encode the effects of the quantum gravity over which we have no control without knowing the nature of the fundamental theory.

One of the way out to tame the higher dimensional operators is to assume an approximate shift symmetry in the fundamental theory, so that the potential is almost unchanged by the shift of the inflaton field [10–13]. The shift symmetry also guarantees the flatness of the inflaton potential.

In this letter, we discuss an alternative way out where fields appearing in a series expansion of the effective field theory never exceed the Planck scale while the inflaton field satisfies the Lyth bound. There, the inflaton resides in a Riemann surface and the inflaton potential is a multivalued function of the inflaton field when it is viewed as a function on a complex plane. In this case, we can realize a field variation much larger than the Planck scale while keeping its amplitude within the Planck scale during inflation. This viewpoint has been considered in the context of the axion monodromy [14–16] motivated in string theory and its field theoretical approaches in Refs. [17–20]. As we will see, we show that such a multivalued inflaton potential and an inflaton field residing in a Riemann surface can be realized starting from a single-valued Lagrangian, where the appearance of the effectively enhanced field space is obtained by charge assignments of fields which break a $U(1)$ symmetry.[1]

2. Lyth bound

Here, let us briefly review the so-called Lyth bound [8]. The magnitude of the tensor perturbation depends only on the infla-

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[1] See also alternative possibilities to realize the effective trans-Planckian inflaton in terms of fields within the Planck scale by aligning several potentials of natural inflation [21] or by using the collective behavior of multi-inflatons [22,23].
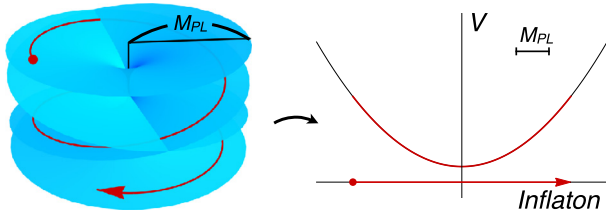
Fig. 1. A schematic picture of the inflaton potential on a Riemann surface. Left: A Riemann surface for the function $\phi^{1/4}$. We assume that the inflaton field takes a value on this surface. Right: Inflaton potential ($V \propto \phi^{1/4}$) on the Riemann surface.

tion scale during inflation. On the other hand, the magnitude of the scalar perturbation not only depends on the inflation scale but also on the slow-roll-ness of the inflaton field during inflation. As the inflaton rolls faster, the effect of the quantum fluctuation of the inflaton on the scalar perturbation is suppressed, since the scalar perturbation is essentially the fluctuation of the inflationary period. Therefore, for a given inflation scale, the tensor fraction becomes larger for a faster rolling of the inflaton.

As a result, there is a lower-bound on the variation of the inflaton field during inflation $\Delta\phi$, the so-called Lyth bound [8];

$$\frac{\Delta\phi}{M_{PL}} \gtrsim 1.6 \times \left(\frac{r}{0.2}\right)^{1/2}, \tag{1}$$

where $M_{PL} \simeq 2.4 \times 10^{18}$ GeV denotes the reduced Planck scale.[2] Therefore, the observed tensor fraction, $r \simeq 0.2$, indicates the inflaton field value much larger than the Planck scale. As discussed above, such a large field value generates concerns over inflation models based on the effective field theory where the Lagrangian is given by a series expansion in fields.

3. Multivalued inflaton potential

A possible way around the argument of the Lyth bound is to assume that the inflaton potential is a multivalued function of the inflaton when the inflaton is considered to reside in a complex plane. As a simple example, let us imagine that the inflaton potential is given by

$$V \propto \phi^{1/N} + \text{h.c.}, \tag{2}$$

where N is an arbitrary integer. Obviously, this potential is multivalued when it is viewed as a function on a complex plane. Then, let us further assume that the inflaton field ϕ is a complex scalar field which takes a value not on a simple complex plane but on a Riemann surface for $\phi^{1/N}$, i.e.

$$\phi = |\phi| \times e^{i\alpha} \quad (\alpha = 0 - 2N\pi). \tag{3}$$

With this assumption, the inflaton potential is now a single-valued function on the inflaton Riemann surface, and hence, we can achieve a field variation much larger than the field amplitude itself, i.e.

$$\Delta\phi = \left|\int d\phi\right| \gg |\phi|, \tag{4}$$

for $N \gg 1$.[3]

In Fig. 1, we show a schematic picture to illustrate the inflaton field residing in a Riemann surface for $N = 4$. As the figure shows,

the variation of the inflaton field can be much larger than the amplitude. On the Riemann surface, the multivalued inflaton potential becomes single-valued. In this way, we can construct a large field inflation model effectively.

One of serious drawbacks of this idea is, however, that the multivalued potential has a singularity at the origin of the field space. Such a singularity is far from acceptable from the viewpoint of the effective field theory below the Planck scale, where we assume that the effective Lagrangian is given by a series expansion in terms of regular fields. Therefore, the question is whether we can construct a model with a multivalued inflaton potential starting from a single-valued field theory.

4. Effective multivalued inflaton potential

To construct a model with a multivalued inflaton potential out of a single-valued potential, let us consider two complex scalar fields ϕ and S which reside in complex planes. We also assume a continuous global $U(1)$ symmetry[4] with the charge assignments

$$\phi : N, \quad S : 1, \tag{5}$$

where N is a sufficiently large integer. The scalar potential consistent with the $U(1)$ symmetry is given by

$$\begin{aligned} V &= V\left(\phi\phi^*, SS^*, \phi^*S^N\right) \\ &= -m_\phi^2\phi\phi^* + y_\phi\left(\phi\phi^*\right)^2 - m_S^2 SS^* + y_S\left(SS^*\right)^2 \\ &\quad + \left(\frac{c}{M_{PL}^{N-3}}\phi^*S^N + \text{h.c.}\right) + \cdots, \end{aligned} \tag{6}$$

where m_ϕ^2, m_S^2, y_ϕ, y_S and c are constants and $\cdots$ denotes higher dimensional terms irrelevant for our discussion.

We assume that ϕ and S obtain non-vanishing vacuum expectation values (VEVs), which is realized by negative masses-squared of ϕ and S around the origin. Then the radial directions of ϕ and S, and one linear combination of the phase directions of them obtains large masses from the scalar potential in Eq. (6). However, another linear combination of the phase directions of ϕ and S, which corresponds to a Nambu–Goldstone boson (NGB) associated with spontaneous breaking of the global $U(1)$ symmetry, remains massless.

Along the NGB direction, the field values of ϕ and S are related by

$$S = \lambda\phi^{1/N}, \tag{7}$$

due to the $U(1)$ symmetry. Here, λ is a constant determined by the scalar potential. It should be emphasized here that the phase of S is "multivalued" function of the phase of ϕ, which plays a crucial role in the following discussion. We note that since the charge of ϕ is larger than that of S, the NGB is mostly the phase direction of ϕ if the VEVs of ϕ and S are of the same order, which we assume in the following.

Now let us explicitly break the $U(1)$ symmetry softly by introducing a potential,

$$\Delta V = \Lambda^3 S + \text{h.c.}, \tag{8}$$

where Λ is an order parameter of the explicit breaking of the $U(1)$ symmetry. We assume that the scale Λ is sufficiently smaller than the VEVs of ϕ and S. Then the low energy effective theory is

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[2] Here, we have taken the e-folding number to be $\Delta N_e \simeq 10$.

[3] The usage of the multivalued inflaton potential is crucial since otherwise the inflaton potential shows a trivial periodicity during the large variation $\Delta\phi$, which has no physical meaning.

[4] We may also replace the continuous symmetry with a discrete symmetry such as Z_M with a large integer M.
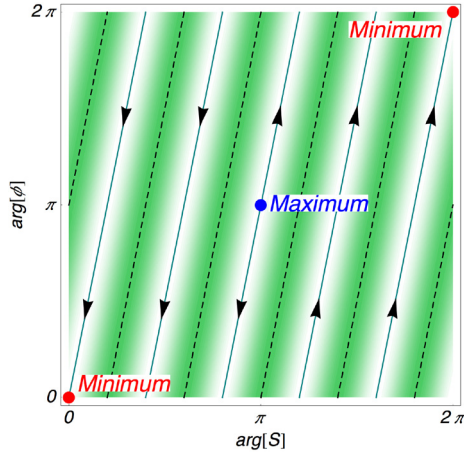
Fig. 2. An inflaton trajectory on phases of S and ϕ for $N = 5$. The inflaton potential in Eq. (9) is realized along the trajectory shown by the solid line. Arrowheads indicate the height of the potential along the trajectory, and the field values which maximize and minimize the potential in Eq. (9) are denoted by points. Here, we have shifted the phases of ϕ and S so that $\arg \phi = \arg S = 0$ is the minimum of the potential. The shaded regions have large potentials by the last term in Eq. (6).

well-described by the (now pseudo-)NGB with the explicit breaking term in Eq. (8). Since the pseudo-NGB is mostly composed of the phase direction of ϕ, the dynamics of the NGB is approximately identified with the dynamics of the phase of ϕ. Expressing the potential by ϕ using Eq. (7), the resulting low energy effective potential along the pseudo-NGB is given by

$$\Delta V_{\text{eff}} = \lambda \Lambda^3 \phi^{1/N} + \text{h.c.}, \tag{9}$$

which is nothing but the multivalued inflaton potential discussed in the previous section. The phase direction (i.e. the pseudo-NGB) plays the role of the inflaton.

The inflaton dynamics with the multivalued potential can be understood in the following way. Due to the relation in Eq. (7), when ϕ rotates 2π, S rotates only $2\pi/N$. Then, since the inflaton potential is provided for the phase of S as in Eq. (8), the periodicity of the potential for the phase of ϕ, which is the main component of the inflaton, is effectively enlarged to $2\pi N$. This non-trivial periodicity is the origin of the effective multivalued nature and the trans-Planckian variation of the inflaton field during inflation. In Fig. 2, we show the potential on the phases of S and ϕ for $N = 5$. The inflaton trajectory corresponds to the bottom of the valley along which the potential is very flat over the range of the $[0, 2\pi N]$.

To show the inflaton potential quantitatively, let us extract a canonically normalized pseudo-NGB, a, given by an identification:

$$\phi \to \langle \phi \rangle \exp\left[iN \frac{a}{f_a} \right], \qquad S \to \langle S \rangle \exp\left[i \frac{a}{f_a} \right],$$

$$f_a \equiv \sqrt{2N^2 |\langle \phi \rangle|^2 + 2 |\langle S \rangle|^2}. \tag{10}$$

The scalar potential of a is given by

$$V(a) = 2 |\Lambda^3 \langle S \rangle| \left(1 - \cos \frac{a}{f_a} \right), \tag{11}$$

where we have eliminated a constant phase and a sign inside the cosine by shifting a. Here, we have added a constant term to the potential so that the cosmological constant vanishes at the vacuum.

As a result, we obtain the potential of the so-called natural inflation [10], which is consistent with the results of the Planck [4]

and the BICEP2 [7] experiments for $f_a \gtrsim 5 M_{PL}$ [24].[5] If N is sufficiently large, f_a can be as large as $5 M_{PL}$ while keeping ϕ and S within the Planck scale, which is possible due to the multivalued nature of the effective potential in Eq. (9).

Before closing this section, let us discuss how small VEVs are acceptable in this model. For that purpose, let us remember that the two scalar fields are connected via higher dimensional operators in Eq. (6),

$$V \supset \frac{c}{M_{PL}^{N-3}} \phi^* S^N + \text{h.c.}, \tag{12}$$

without which we have two $U(1)$ symmetries. When this operator is ineffective, the linear combination of the phase directions other then the NGB, b, becomes lighter than a. In this case, b plays a role of the inflaton, whose decay constant in the potential is given by f_a/N. Therefore, the variation of the inflaton field b cannot exceed the Planck scale. As a result, it is required that

$$\langle \phi \rangle \simeq \langle S \rangle \gg M_{PL} \left(\frac{2N^2}{c} \left(\frac{m_{\text{inf}}}{M_{PL}} \right)^2 \right)^{1/(N-1)}. \tag{13}$$

Here, we have expressed the condition in terms of the mass of the inflaton m_{inf}. It is determined by the normalization of the CMB fluctuation; $m_{\text{inf}} = \mathcal{O}(10^{13})$ GeV for $f_a \gg M_{PL}$. Thus, we find that the VEVs of S and ϕ are bounded from below,

$$\langle \phi \rangle \simeq \langle S \rangle \gg 10^{-10/(N-1)} M_{PL}. \tag{14}$$

5. Multivalued axion inflation

Instead of putting the explicit breaking of the $U(1)$ symmetry, the potential in Eq. (8), we may generate the breaking by non-perturbative dynamics. Let us assume QCD-like gauge dynamics which exhibits spontaneous breaking of chiral symmetries at a scale Λ_{dym} far below the VEVs of ϕ and S. We couple the field S to the gauge dynamics via a Yukawa coupling:

$$\mathcal{L}_{\text{int}} = y S Q \bar{Q}, \tag{15}$$

where Q and $\bar{Q}$ are fermion fields charged under the gauge symmetry, and y denotes a coupling constant.

Now that the $U(1)$ symmetry has an anomaly of the gauge symmetry, as is the case with the QCD-axion [26–28], the pseudo-NGB obtains a potential,

$$V \simeq m_f \Lambda_{\text{dyn}}^3 \left(1 - \cos(\arg S) \right)$$
$$= m_f \Lambda_{\text{dyn}}^3 \left(1 - \cos\left(\arg \phi^{1/N} \right) \right), \tag{16}$$

where we have eliminated constant phases. Here, we have assumed that there is a fermion charged under the gauge symmetry with a mass $m_f < \Lambda_{\text{dyn}}$. We have again obtained a multivalued potential of the field ϕ.

In terms of the canonically normalized NGB a, the scalar potential of a is given by

$$V(a) = \Lambda_{\text{dyn}}^4 \left(1 - \cos \frac{a}{f_a} \right), \tag{17}$$

which again leads to the potential of the natural inflation [10]. As emphasized above, the multivalued nature played a crucial role to realize $f_a \gg M_{PL}$ while keeping the VEVs of S and ϕ below the Planck scale.

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[5] For a consistency of more generalized natural inflation models with the BICEP2 and the Planck results, see e.g. [25].

6. Discussion

We have considered inflation models such that the inflaton potential is a multivalued function when it is viewed as a function on a complex plane while the inflaton field (effectively) resides in a Riemann surface. In this way, we can satisfy the Lyth bound while fields appearing in the effective field theory are sub-Planckian. We have shown some simple examples where the multivalued inflaton potential is realized starting from a single-valued Lagrangian of the effective field theory below the Planck scale.

It should be emphasized that the effectively enhanced inflaton field space originates from the charge assignments of fields which break a $U(1)$ symmetry. This mechanism should be contrasted with other attempts to realize the effectively trans-Planckian field variation by, for example, alignment between several potentials of natural inflation [21] or by using the collective behavior of multi-inflatons [22,23]. In our model, on the other hand, we can realize the trans-Planckian field variation using only two fields without having alignments.

As we have shown, the simple examples result in the natural inflation model. There, the shift symmetry, which is often imposed to control the inflaton potential [10–13], is understood as non-linear realization of a global $U(1)$ symmetry.

So far, we have imposed the global $U(1)$ symmetry. Since the symmetry is explicitly broken to generate the multivalued inflaton potential, the origin of the symmetry should be scrutinized in future works. It would be interesting to construct a model such that the $U(1)$ symmetry is an accidental one as a result of some other well-motivated symmetries.

Acknowledgements

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