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The role of the SGT Density with Conditional Volatility, Skewness and Kurtosis in the Estimation of VaR: A Case of the Stock Exchange of Thailand

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Abstract

One of primary tools used to assess the financial risk is Value-at-Risk (VaR). It turns to be a standard measure of downward risk among financial intermediaries and regulators recently as it summarized the risk into just a single and easy-to-understand number. Despite the simplicity of VaR's concept, an accurate calculation of VaR is still challenging. This paper aims to propose an alternative approach which is believed to provide more accurate VaR rather than the traditional ones. Instead of the conventional Gaussian distribution, the more flexible skewed generalized t (SGT) density function is assumed for return series. Its volatility is characterized by eight types of GARCH process. Meanwhile, conditional skewness and kurtosis is modeled to exhibit time-varying feature by their past information set and autoregressive term. Daily returns on the SET index will be used to explore the performance of estimated VaR. The finding shows that this new approach can provide more accurate and robust estimates of the actual VaR threshold, especially with TS-GARCH model, than any other approaches that have been applied earlier.

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Keywords: Conditional value at risk; GARCH; Skewed generalized t distribution; Conditional skewness and kurtosis

1. Introduction

One of primary tools used to assess financial risk is Value-at-Risk (VaR). It is defined as the worst loss over a target horizon such that there is a low, pre-specified probability the actual loss will be larger. Its greatest advantage can be easily seen that it summarizes the risk in consideration into a single and easy-to-understand number. Despite the simplicity of VaR's concept, an accurate calculation of conditional VaR is still statistically challenging. Many earlier applications of VaR assume that the asset returns are normally distributed. Hence, the returns standardized by the conditional mean and conditional standard deviation are standard normal. This assumption simplifies the computation of VaR quite considerably. However, there are many empirical studies on return distributions since 1960s suggesting that they are not characterized by normality but by the stylized facts of fat tails, high peakedness and skewness (Kon, 1984; Badrinath and Chatterjee, 1988; and Mittnik and Rachev, 1993).

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This implies that extreme events are much more likely to occur in practice than that of the prediction from the symmetric thinner-tailed normal distribution. Although subsequent research proposes many alternative distributions that are more flexible than the normal for the standardized returns, they are all assumed to be iid, implying that the only features of the conditional return distribution, which depend upon the conditioning information, are the mean and variance (Kadir et al., 2011). In fact, it seems more sensible that other features of distribution such as skewness and kurtosis will depend on the conditioning information as well.

In this paper, the conditional density approach initiated by Hansen, 1994; and extended by Jondeau and Rockinger, 2003; and Bali et al., 2008 will be adopted to estimate the VaR threshold. In order to provide an accurate characterization of the shape and tails of the standardized return distribution, the more flexible skewed generalized *t* (SGT) distribution, which consists of mean, variance, skewness, tail-thickness and peakedness parameters, is used in place of the normality assumption. The estimation of the conditional mean and volatility of returns is based on AR(1) and eight variations of the GARCH(1,1) process, respectively. Besides the first two moments (conditional mean and variance), the higher-order moments of the SGT distribution are allowed to depend on the past information set by defining the skewness, tail-thickness, and peakedness parameters of the density as an autoregressive process similar to the ARCH model of Engle, 1982.

The empirical analyses are based on the daily returns on the Stock Exchange of Thailand (SET) value-weighted index during January 1976 to December 2010 (8,605 observations). The performance of the SGT-GARCH models with time-varying parameters in the estimation of VaR will be assessed through the unconditional coverage test of Kupiec, 1995; and the conditional coverage test of Christoffersen, 1998. The finding shows that the conditional SGT-VaR approach introduced in this paper provides quite accurate and robust estimates of the actual VaR threshold.

This paper is organized as follows. Section 2 presents the conditional SGT-VaR models. Section 3 describes assessment of performance of the conditional SGT-VaR models. Section 4 discusses the in-sample and out-of-sample performance of the conditional SGT -VaR models. Section 5 concludes the paper.

2. Conditional SGT -VaR models

To compute the precise conditional VaR, this literature builds on Bali et al., 2008. The aforementioned conditional SGT-VaR models are defined as follows:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + u_t = \mu_t + \sigma_t z_t \tag{1}$$

$$g(\sigma_t) = \sigma_t, \sigma_t^2 \text{ or } \ln(\sigma_t) \tag{2}$$

$$g(\sigma_t) = h(\sigma_{t-1}, z_{t-1} | \beta_0, \beta_1, \gamma) + \beta_2 \cdot g(\sigma_{t-1}) \tag{3}$$

where r_t is returns at time t ; μ_t and σ_t are, respectively the conditional mean and conditional standard deviation r_t based on past information set Ω_{t-1} up to time $t-1$; $\mu_t = \sigma_t z_t$ is the returns innovation at time t ; $z_t = (r_t - \mu_t) / \sigma_t$ is standardized returns, which its density function is given as:

$$f_Z(z_t | \lambda_t, \eta_t, \kappa_t) = C \cdot \left[1 + \frac{|z_t + \delta|^{\kappa_t}}{((\eta_t - 2) / \kappa_t)(1 + \text{sign}(z_t + \delta)\lambda_t)^{\kappa_t} \varphi^{\kappa_t}} \right]^{-\left(\frac{\eta_t + 1}{\kappa_t}\right)} \tag{4}$$

$$C = 0.5\kappa_t \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{-\frac{1}{\kappa_t}} \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \varphi^{-1} \tag{5}$$

$$\varphi = \frac{1}{\sqrt{1 - \rho^2}} \tag{6}$$

$$\rho = 2\lambda_t \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{\frac{1}{\kappa_t}} \cdot B\left(\frac{\eta_t - 1}{\kappa_t}, \frac{2}{\kappa_t}\right) \tag{7}$$

$$\nu = (1 + 3\lambda_t^2) \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{\frac{2}{\kappa_t}} \cdot B\left(\frac{\eta_t - 2}{\kappa_t}, \frac{3}{\kappa_t}\right) \tag{8}$$

$$\delta = \rho\varphi \tag{9}$$

The conditional standard deviation σ_t is assumed to follow various GARCH(1,1)-type models through the functional form $g(\sigma_t)$ as in expression (2) and (3). The conditional volatility equations $g(\sigma_t)$ for eight variations of GARCH(1,1) models are as follows:

$$\text{GARCH Model: } \sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{10}$$

$$\text{IGARCH Model: } \sigma_t^2 = \beta_0 + (1 - \beta_2) \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{11}$$

$$\text{EGARCH Model: } \ln(\sigma_t^2) = \beta_0 + \beta_1[|z_{t-1}| - E|z_{t-1}|] + \gamma z_{t-1} + \beta_2 \ln(\sigma_{t-1}^2) \quad (12)$$

$$\text{GJR-GARCH Model: } \sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \gamma S_{t-1}^- \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \quad (13)$$

$$\text{QGARCH Model: } \sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \gamma \sigma_{t-1} z_{t-1} + \beta_2 \sigma_{t-1}^2 \quad (14)$$

$$\text{TGARCH Model: } \sigma_t = \beta_0 + \beta_1 \sigma_{t-1} |z_{t-1}| + \gamma S_{t-1}^- \sigma_{t-1} z_{t-1} + \beta_2 \sigma_{t-1} \quad (15)$$

$$\text{TS-GARCH Model: } \sigma_t = \beta_0 + \beta_1 \sigma_{t-1} |z_{t-1}| + \beta_2 \sigma_{t-1} \quad (16)$$

$$\text{APGARCH Model: } \sigma_t^2 = \beta_0 + \beta_1 [\text{sign}(z_{t-1}) - \gamma]^2 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \quad (17)$$

where $S_{t-1}^- = 1$ for $\sigma_{t-1} z_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise. The conditional high-order moment parameters of the SGT density λ_t , η_t , κ_t are modeled as follows:

$$\lambda_t = -1 + 2 / (1 + \exp(-\tilde{\lambda}_t)), \quad \tilde{\lambda}_t = \lambda_0 + \lambda_1 z_{t-1} + \lambda_2 \tilde{\lambda}_{t-1} \quad (18)$$

$$\eta_t = 2 + \exp(\tilde{\eta}_t), \quad \tilde{\eta}_t = \eta_0 + \eta_1 z_{t-1} + \eta_2 \tilde{\eta}_{t-1} \quad (19)$$

$$\kappa_t = \exp(\tilde{\kappa}_t), \quad \tilde{\kappa}_t = \kappa_0 + \kappa_1 z_{t-1} + \kappa_2 \tilde{\kappa}_{t-1} \quad (20)$$

Note that λ_t is restricted skewness parameter; η_t and κ_t are restricted kurtosis parameters according to the SGT definition that $|\lambda_t| < 1$, $\eta_t > 2$, and $\kappa_t > 0$. $\tilde{\lambda}_t$, $\tilde{\eta}_t$, and $\tilde{\kappa}_t$ are unrestricted ones which has time-varying form. The conditional SGT-GARCH parameters are obtained from the maximization of the sample log-likelihood function $L = \sum [\ln(f_z(z_t | \lambda_t, \eta_t, \kappa_t)) - \ln(\sigma_t)]$ with respect to α_0 , α_1 , β_0 , β_1 , β_2 , λ_0 , λ_1 , λ_2 , η_0 , η_1 , η_2 , κ_0 , κ_1 , κ_2 and/or γ depending on each GARCH(1,1) specification and subject to positivity and stationary constraints associating with each GARCH(1,1) specification. After all conditional parameters of the return distribution are estimated, the r_t^* , which is the corresponding conditional threshold for the return r_t at a given coverage probability ϕ , can be obtained firstly from solving a_t from the equation $\int_{-\infty}^{\infty} f_z(z_t) dz_t = \phi$ and then substitute back into equation $r_t^* = \mu_t + a_t \sigma_t$.

3. Assessment of the performance of conditional VaR

3.1. Unconditional coverage test

Given independence, Kupiec, 1995 constructed the unconditional coverage test (LR^{UC}) under the null hypothesis that the actual and expected numbers of observations falling below VaR threshold (called exceedence) are statistically the same as $LR^{UC} = 2 \cdot [\tau \ln(\frac{\tau}{\phi N}) + (N - \tau) \ln(\frac{N - \tau}{N - \phi N})]$, where N is the number of sample observations, ϕ is the coverage probability, ϕN and τ are the expected and actual number of observations falling below the VaR threshold a_t . The LR^{UC} is distributed by $\chi^2(1)$. The acceptance of null hypothesis refers that the computed conditional VaR threshold provides a good assessment of risk exposure.

3.2. Conditional coverage test

Christoffersen, 1998 argued that the unconditional coverage test is insufficient to assess the VaR threshold when the assumption of serial independence is violated. The author developed the conditional coverage test to examine the serial independence of VaR estimates by defining the indicator I_t as $I_t = 1$ if exceedence occurs and $I_t = 0$ otherwise. The conditional coverage test statistic is constructed under null hypothesis of serial independence against the alternative of explicit first-order Markov dependence as $LR^{IND} = 2 \cdot [n_{00} \ln(\frac{\Pi_{00}}{1 - \Pi}) + n_{01} \ln(\frac{1 - \Pi_{00}}{\Pi}) + n_{10} \ln(\frac{\Pi_{10}}{1 - \Pi}) + n_{11} \ln(\frac{1 - \Pi_{10}}{\Pi})]$, where n_{ij} is the number of observations of indicator variable I_t with value i followed by j , $\Pi_{00} = n_{00} / (n_{00} + n_{01})$, $\Pi_{10} = n_{10} / (n_{10} + n_{11})$, $\Pi = (n_{01} + n_{11}) / N$, and $N = n_{00} + n_{01} + n_{10} + n_{11}$. The LR^{IND} is distributed by $\chi^2(1)$. The acceptance of null hypothesis indicates that the serial independence assumption is held and it suffices to use the unconditional coverage test to assess the VaR threshold.

4. Risk measurement of the conditional SGT-VaR models

4.1. Assessment of in-sample VaR performance

Table 1 presents statistics on the VaR threshold of all models for the coverage probabilities ϕ of 1%, 1.5%, 2%, 2.5%, and 5% using the sample between January 1976 and December 2010 for both

estimation and prediction (in-sample analysis). The first row for each coverage probability presents the average estimated VaR thresholds of eight GARCH(1,1) types. The second presents the actual and expected (Actl/Expt) number of returns that fall below each threshold. The third row presents the unconditional coverage test statistics (LR^{UC}) and the conditional coverage test statistics (LR^{IND}).

The LR^{IND} in all models and coverage probabilities cannot reject the null hypothesis of the serial independent assumption of the unconditional coverage test, indicating that the assessment of VaR threshold can rely on the LR^{UC} . The LR^{UC} shows that the APGARCh model is the most inaccurate for predicting the VaR threshold since they rejects the null hypothesis at all coverage probability levels. The GARCH, IGARCH, and QGARCH models are all accurate only at high coverage probability but not the low one (except the IGARCH model at 2% level). In contrast, the EGARCH, GJR-GARCH, and TGARCH models do poorly for the high coverage probabilities but become better when it goes further to the tails of the return distribution (low coverage probabilities). The TS-GARCH model provides the best assessment of the risk exposure of a portfolio mimicking the SET index returns since the null hypothesis cannot be rejected at all coverage probability levels. It implies that the VaR threshold obtained from the TS-GARCH model based on the SGT distribution with time-varying volatility, skewness and kurtosis is accurate and robust regardless of coverage probability chosen.

Table 1: In-sample VaR performance of the conditional SGT-GARCH models

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
1.0%	-2.9559	-3.0247	-3.4144	-3.4979	-2.9712	-3.4028	-3.2255	-2.2495
Actl/Expt	125/86	116/86	74/86	75/86	121/86	78/86	92/86	330/86
LR^{UC}/LR^{IND}	15.64/0.02**	9.50/0.24**	1.78*/0.18**	1.49*/1.32**	12.75/0.35**	0.78**/0.11**	0.41**/0.00**	406.41/0.88**
1.5%	-2.7121	-2.7716	-2.9152	-2.9845	-2.7229	-3.0781	-2.9463	-2.1693
Actl/Expt	175/129	169/129	128/129	126/129	172/129	107/129	127/129	359/129
LR^{UC}/LR^{IND}	14.96/0.05**	11.45/0.14**	0.01**/0.53**	0.07**/0.01**	13.15/0.09**	4.06*/0.09**	0.03**/0.01**	281.02/1.09**
2.0%	-2.5365	-2.5896	-2.5947	-2.6550	-2.5444	-2.8489	-2.7472	-2.1088
Actl/Expt	219/172	206/172	200/172	190/172	218/172	134/172	162/172	381/172
LR^{UC}/LR^{IND}	12.04/0.06**	6.43*/0.00**	4.40*/0.03**	1.85**/0.01**	11.55/0.05**	9.29/0.00**	0.61**/0.43**	193.09/1.57**
2.5%	-2.3984	-2.4465	-2.3631	-2.4169	-2.4041	-2.6714	-2.5918	-2.0590
Actl/Expt	273/215	256/215	280/215	269/215	272/215	177/215	204/215	402/215
LR^{UC}/LR^{IND}	14.76/0.62**	7.54/0.05**	18.38/0.00**	12.86/0.04**	14.28/0.23**	7.35/0.13**	0.59**/0.27**	133.22/0.98**
5.0%	-1.9546	-1.9878	-1.7236	-1.7602	-1.9545	-2.1184	-2.0988	-1.8789
Actl/Expt	475/430	450/430	617/430	620/430	474/430	385/430	391/430	523/430
LR^{UC}/LR^{IND}	4.77*/0.00**	0.95**/0.09**	75.75/0.14**	78.07/1.22**	4.56*/0.00**	5.16*/0.85**	3.86*/1.56**	19.79/0.29**

Note: *, ** denote that the null hypothesis cannot be rejected at 5% and 1%, respectively.

4.2. Assessment of out-of-sample VaR performance

Table 2 presents statistics on the VaR threshold of all models for the coverage probabilities ϕ of 1%, 1.5%, 2%, 2.5%, and 5% using the sample between January 2000 and December 2009 for estimation, and the last quarter of December 2010 for prediction (out-of-sample analysis).

The results from the LR^{IND} show that all unconditional coverage statistics are reliable and suffice to assess the performance of VaR threshold (except the TGARCH model at 1% level that the LR^{IND} cannot be calculated). The LR^{UC} in all models cannot be rejected the null hypothesis at all coverage probability levels. It strongly indicates that all models provide accurate and robust VaR threshold in case of out-of-sample analysis.

5. Conclusion

With complexity in the current financial market, Value-at-Risk (VaR) is one of primary tool used to assess the financial risk. Despite the simplicity of its concept, an accurate calculation of conditional VaR is still statistically challenging. This paper proposes an alternative to compute conditional VaR called conditional SGT-VaR approach. The traditional normality assumption has been relaxed to the more flexible skewed generalized t (SGT) distribution. The conditional volatility is assumed to follow 8 types of GARCH(1,1) process including symmetric and asymmetric ones. Furthermore, the conventional assumption in conditional VaR calculation that distribution of standardized return is iid is also relaxed. We allow higher-order moments of the SGT density to rely on the past information set by

defining the skewness, tail-thickness and peakedness parameters of the SGT density as an autoregressive form similar to the ARCH process.

The role of conditional skewness and kurtosis in the estimation of the conditional VaR is investigated by using the unconditional coverage test and conditional coverage test to evaluate the performance of the conditional SGT-VaR approach. The in-sample performance results indicate that the conditional SGT-VaR approach with time-varying skewness and kurtosis in case of the TS-GARCH provides very good prediction of market risks regardless of coverage probability chosen. However, the performance results for out-of-sample analysis are still unclear. The SGT-VaR approach with conditional volatility, skewness and kurtosis in all GARCH-type can provide accurate VaR threshold. There is no superior GARCH specification among others. Future research should extend the prediction sample size for the out-of-sample analysis.

Table 2: Out-of-sample VaR performance of the conditional SGT-GARCH models

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
1.0%	-2.7706	-2.7581	-2.9126	-2.7735	-2.7934	-2.7666	-2.7935	-2.8850
Actl/Expt	1/0.62	2/0.62	1/0.62	1/0.62	1/0.62	0/0.62	1/0.62	1/0.62
LR ^{UC} /LR ^{IND}	0.19**/0.03**	1.96**/0.14**	0.19**/0.03**	0.19**/0.03**	0.19**/0.03**	1.25**/NA	0.19**/0.03**	0.19**/0.03**
1.5%	-2.5020	-2.4589	-2.6330	-2.5049	-2.5213	-2.4929	-2.5196	-2.6077
Actl/Expt	1/0.93	2/0.93	1/0.93	1/0.93	1/0.93	1/0.93	1/0.93	1/0.93
LR ^{UC} /LR ^{IND}	0.01**/0.03**	0.94**/0.14**	0.01**/0.03**	0.01**/0.03**	0.01**/0.03**	0.01**/0.03**	0.01**/0.03**	0.01**/0.03**
2.0%	-2.3144	-2.2556	-2.4372	-2.3173	-2.3317	-2.3024	-2.3286	-2.4141
Actl/Expt	1/1.24	2/1.24	1/1.24	1/1.24	1/1.24	1/1.24	1/1.24	1/1.24
LR ^{UC} /LR ^{IND}	0.05**/0.03**	0.40**/0.14**	0.05**/0.03**	0.05**/0.03**	0.05**/0.03**	0.05**/0.03**	0.05**/0.03**	0.05**/0.03**
2.5%	-2.1700	-2.1022	-2.2863	-2.1729	-2.1859	-2.1563	-2.1817	-2.2651
Actl/Expt	1/1.55	2/1.55	1/1.55	1/1.55	1/1.55	1/1.55	1/1.55	1/1.55
LR ^{UC} /LR ^{IND}	0.22**/0.03**	0.12**/0.14**	0.22**/0.03**	0.22**/0.03**	0.22**/0.03**	0.22**/0.03**	0.22**/0.03**	0.22**/0.03**
5.0%	-1.7228	-1.6436	-1.8181	-1.7266	-1.7351	-1.7061	-1.7274	-1.8045
Actl/Expt	2/3.1	4/3.1	1/3.1	1/3.1	2/3.1	2/3.1	1/3.1	2/3.1
LR ^{UC} /LR ^{IND}	0.47**/0.14**	0.25**/0.56**	2.01**/0.03**	2.01**/0.03**	0.47**/0.14**	0.47**/0.14**	2.01**/0.03**	0.47**/0.14**

Note: *, ** denote that the null hypothesis cannot be rejected at 5% and 1%, respectively.

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