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ORIGINAL ARTICLE

Characterization of the frictional losses and heat transfer of oscillatory viscous flow through wire-mesh regenerators



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Abstract In this paper, new relations for calculating heat transfer and pressure drop characteristics of oscillatory flow through wire-mesh screen regenerator such as Darcy permeability, Forchheimer's inertial coefficient, and heat transfer area per unit volume, as a function of the wire diameter are presented. According to the derived relations, thinner wires have higher pressure drop and higher heat transfer rate. The relations are applicable for all regenerative cryocoolers. Embedding the new relations into a numerical model, three Stirling-type orifice pulse tube cryocoolers with three regenerators different in length and diameter but same volume in a variety of wire diameters, have been modeled. The results achieved by the model reveal that the local heat transfer coefficient decreases with increase of the wire diameter and the length-to-diameter ratio. In addition, it was shown that the mean absolute gas–solid wire temperature difference is a linear function of wire diameter in the range investigated. The results show that for larger length-to-diameter ratios, Forchheimer's effect will dominate frictional losses, and the variations of the frictional losses are proportional to the inverse of the wire diameter. Wire diameter has been optimized to maximize the coefficient of performance of the cryocooler. Shorter regenerators have thinner optimum wires. © 2015 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

The key role of the regenerator in the regenerative cryocoolers such as pulse tube and Stirling cryocoolers is well recognized,

and improving the performance of various types of regenerators is of great importance. The regenerator in a regenerative cryocoolers has a micro-porous metallic structure which is typically the largest source of power loss in cryocoolers. Axial heat conduction, imperfect gas–solid heat transfer, and frictional losses are the main origins of irreversibility. The results presented in the works done by Radebaugh [1], Flakes and Razani [2], Cha et al. [3], demonstrated that the numerical models can be used to simulate cryocoolers as an acceptable design and analysis tool. However, the accuracy

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Nomenclature

A	cross sectional area
c_F	Forchheimer's inertial coefficient
C_P	specific heat at constant pressure
C_V	specific heat at constant volume
C_d	flow coefficient of orifice
COP	coefficient of performance
d_h	hydraulic diameter
d_w	wire diameter
d_{Reg}	regenerator diameter
f	friction factor
h	heat transfer coefficient
k	thermal conductivity
K	Darcy permeability
L_{Reg}	regenerator length
l	mesh distance
\dot{m}_{or}	mass flow rate through orifice
N	number of packed screens
n	number of packed screens per unit length
p	pressure
V	volume
V_{Com}	compressor total volume
Pr	Prandtl number

R	ideal gas constant
Re_h	hydraulic Reynolds number
t	time
T	temperature
u	velocity
V_d	dead volume of compressor
V_S	swept volume of compressor
x	longitude coordinate

Greek symbols

α	heat transfer area per unit volume
β	opening area ratio of the screen
γ	specific heat ratio
ε	porosity
Γ	dimensionless parameter
λ	matrix conductivity factor
μ	viscosity
ρ	density
ω	angular frequency of operation
σ	pitch

of these numerical predictions depends on the accuracy of the closure relations they used. The dependence of the hydrodynamic parameters (such as Forchheimer's inertial coefficient and Darcy permeability) and heat transfer parameters (such as heat transfer area) on geometry is thus among the most important closure relations. An efficient regenerator must have: (1) a large heat transfer area to result in high gas–solid heat transfer rate to make it tend to gas–solid thermal equilibrium; (2) a large thermal inertia to decrease temperature oscillations; and (3) small pressure drop to decrease power consumption. Meeting all these goals is of great importance, and optimization and compromise are often needed.

In a regenerator as a porous medium, larger solid particles result in larger heat transfer area and hence, better thermal performance of the regenerator. Consequently, the enthalpy flow from the hot end to the cold end of the regenerator as a loss mechanism will decrease and cooling power will be increased. But, increase of size of solid particles causes a higher pressure drop through the regenerator, so that the mechanical performance of regenerator will be disrupted. A part of compressor work is consumed to overcome the resistance forces of the packing material of the regenerator. Thus, for a regenerator with specified dimensions, increase of size of particles, increases both cooling power and compressor power. Apparently, there is an optimum size that maximizes COP of the cryocooler. Some of the common solid matrixes for regenerators are wire-mesh screens, perforated disks, spherical powders, and foam metal. Most of regenerators are constructed by wire-mesh screens (Fig. 1). Wire-mesh screens are categorized into two types: plain and twill.

A one-dimensional theoretical analysis of the regenerator of a typical pulse tube refrigerator has been carried out by Roach et al. [4] based on assuming simple sinusoidal oscillation. In their study, by using the numerical solution of the derived

equation along with boundary conditions, optimum regenerator length to diameter ratio (L/D) for two different regenerators with 150-mesh screens and 250-mesh screens has been achieved. Qiu et al. [5] investigated theoretically and experimentally the effect of mesh size of woven wire screens of a three-layer regenerator on the performance of the single-stage G–M (Gifford–McMahon) type pulse tube cryocooler and obtained a lowest no-load refrigeration temperature of 11.1 K with an input power of 6 kW.

In this paper, new relations are derived for characterization of a wire mesh screen regenerator by using geometrical and empirical relations. Then, all parts of a pulse tube cryocooler have been modeled numerically, and optimization of the regenerator has been performed based on the derived relations.

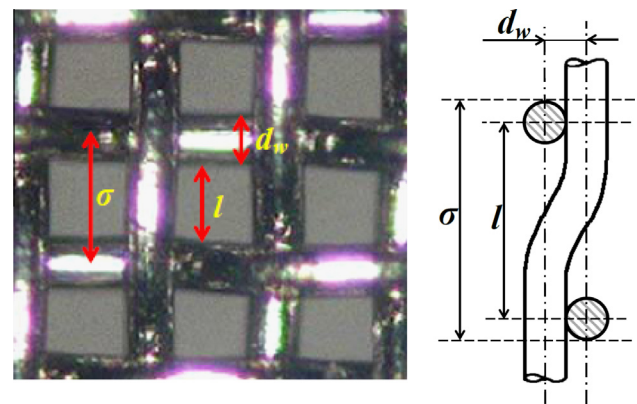


Figure 1 Photo of wire-mesh screen and the schematic of its lateral view.

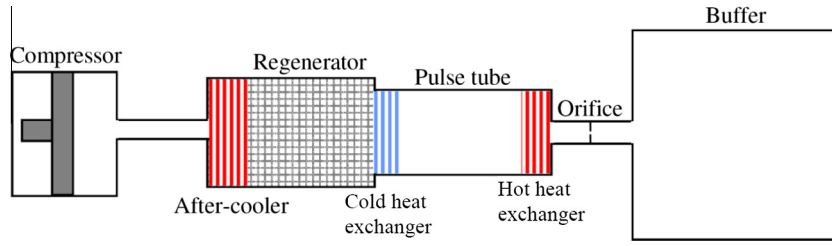


Figure 2 Schematic of Stirling-type orifice pulse tube cryocooler.

2. Physical domain

A schematic of the Stirling-type pulse tube cryocooler is illustrated in Fig. 2. The horizontal coordinate x is adjusted at the piston face. The cooler contains a piston compressor, heat exchangers, a regenerator, a pulse tube, an orifice, and a buffer (reservoir).

3. Numerical model

3.1. Governing equations

For simplicity, the following assumptions are made:

1. Flow is one-dimensional.
2. Working fluid is an ideal gas.
3. Solid material of porous media is incompressible.
4. The heat exchangers have constant wall temperature.
5. Body forces are negligible.
6. Properties of the solid and the gas are temperature dependent.

With the above assumptions, governing equations will be [6]:

Continuity equation:

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{1}{A} \frac{\partial}{\partial x}(\rho\varepsilon Au) = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial}{\partial t}(\rho\varepsilon u) + \frac{1}{A} \frac{\partial}{\partial x}(\rho\varepsilon Au^2) = -\varepsilon \frac{\partial P}{\partial x} - \varepsilon \left(\underbrace{\frac{\mu}{K} \varepsilon u}_{\text{Darcy term}} + \underbrace{\frac{c_F \rho}{\sqrt{K}} \varepsilon^2 u |u|}_{\text{Forchheimer's term}} \right) \quad (2)$$

Energy equation of the gas:

$$\begin{aligned} \frac{\partial}{\partial t} \left[\rho\varepsilon \left(C_V T + \frac{u^2}{2} \right) \right] + \frac{1}{A} \frac{\partial}{\partial x} \left[\rho\varepsilon Au \left(C_P T + \frac{u^2}{2} \right) \right] \\ = \frac{1}{A} \frac{\partial}{\partial x} \left(k\varepsilon A \frac{\partial T}{\partial x} \right) + h\alpha(T_s - T) \end{aligned} \quad (3)$$

State equation of the gas:

$$P = \rho RT \quad (4)$$

Energy equation of the solid:

$$\frac{\partial}{\partial t} [\rho_s(1-\varepsilon)C_s T_s] = \frac{1}{A} \frac{\partial}{\partial x} \left(\lambda k_s(1-\varepsilon)A \frac{\partial T_s}{\partial x} \right) + h\alpha(T - T_s) \quad (5)$$

Also, the momentum equation for the orifice is as follows [7]:

$$\dot{m}_{or} = C_d \sqrt{2 \frac{\gamma}{\gamma-1} P_{up} \rho_{up} \left[\left(\frac{P_{down}}{P_{up}} \right)^{2/\gamma} - \left(\frac{P_{down}}{P_{up}} \right)^{(\gamma+1)/\gamma} \right]} \quad (6)$$

3.2. Initial and boundary conditions

Reciprocation of the compressor's piston has been implemented as left boundary condition. Compressor volume changes with time as the following relation:

$$V_{com} = V_d + 0.5V_s(1 - \sin(\omega t)) \quad (7)$$

No penetration (zero velocity) condition must be satisfied at two ends of the cooler.

Initial conditions: In the present problem, mechanical behavior of the system rapidly reaches to periodic steady state. But, because of large heat capacity of solid matrix, thermal behavior takes a long time to reach to periodic steady state. Hence, to reduce the CPU time, we guess the final temperature distribution as initial condition. A linear temperature profile assumed for regenerator and pulse tube parts and a constant temperature guessed for heat exchangers. Other parts that are adjacent to ambient have ambient temperature. Initial velocity is zero in whole domain and initial pressure distribution is uniform and equals mean pressure of the system.

3.3. Solution method

Finite volume approach is implemented for discretization of differential Eqs. (1)–(5). The second order backward method is used for time derivatives except for the first step calculations in which first order backward (Euler implicit) method is used. The second order upwind is used for convective (enthalpy flow) terms as well as momentum flow. For boundary nodes the first order upwind is used. Thermal conductive terms are approximated by central difference scheme. Other details about the numerical method and solution procedure were explained in the previously published paper of Boroujerdi et al. [8].

4. Characteristics of the regenerator

4.1. Pressure drop characteristics

In this section, crucial pressure drop characteristics of the regenerator are related to its geometrical parameters.

In the present numerical model, resistance force of solid (wire-mesh screen or tube wall) and the pressure gradient are related as

$$\frac{\partial p}{\partial x} \equiv - \left(\frac{\mu}{K} \varepsilon u + \frac{c_F \rho}{\sqrt{K}} \varepsilon^2 u |u| \right) \quad (8)$$

On the other hand, some researchers (especially in experimental works) such as Nam and Jeong [9], Zhang et al. [10], and Banjare et al. [11] used the following form to describe the resistance force based on correlation of Miyabe et al. [12] which presents the steady flow pressure drop over the wire-mesh screen:

$$f_F = \frac{n \varepsilon d_h^2}{4 l \beta} \frac{33.6}{Re_h} + 0.337 \frac{n \varepsilon^2 d_h^2}{4 \beta^2} \quad (9)$$

Hydraulic Reynolds number is defined as

$$Re_h = \frac{\rho |u| d_h}{\mu} \quad (10)$$

The following empirical equation was implemented by Nam and Jeong [9], Zhang et al. [10], and Banjare et al. [11] to evaluate the pressure gradient in the porous medium of the regenerator:

$$\frac{\partial p}{\partial x} = -f_F \frac{2}{d_h} \rho u |u| \quad (11)$$

By comparing Eqs. (8), (9), and (11), Darcy permeability and Forchheimer's inertial coefficient, can be obtained as

$$K = \frac{2 l \beta}{33.6 n} \quad (12)$$

$$c_F = \frac{0.337 n \sqrt{K}}{2 \beta^2} \quad (13)$$

Now, geometrical parameters of Eqs. (12) and (13) must be expressed in terms of two main parameters of wire diameter and pitch. In packing wire-mesh screens, the actual porosity can vary by several percent based on the degree of nesting. The half thickness of one screen is very close to the wire diameter suggesting that the screens were not nesting (Harvey [6]). Ideally, the length of screen is twice the wire diameter. Pitch, opening area ratio and porosity are as follows [9]

$$\sigma = l + d_w \quad (14)$$

$$\beta = \left(\frac{l}{\sigma} \right)^2 \quad (15)$$

$$\varepsilon = 1 - \frac{\pi d_w}{4 \sigma} \quad (16)$$

Also, the number of packed screens per unit length for the case of perfect stacking of square-mesh screens in which the weaving causes no inclination of the wires and screen layer not separated, can be calculated as

$$n = \frac{N}{L_{Reg}} = \frac{N}{N L_{Screen}} = \frac{1}{2 d_w} \quad (17)$$

We have defined dimensionless parameter Γ as the ratio of the wire diameter to the pitch, so we have

$$\Gamma = \frac{d_w}{\sigma}, \quad 0 < \Gamma < 1 \quad (18)$$

$$\varepsilon = 1 - \frac{\pi}{4} \Gamma \quad (19)$$

By substituting l from Eq. (14), β from Eq. (15), n from Eq. (16), σ from Eq. (17) in Eqs. (12) and (13), Darcy permeability, and Forchheimer's inertial coefficient can be expressed as follows:

$$K = \frac{4}{33.6} \frac{d_w^2}{\Gamma} \frac{(1 - \Gamma)^3}{\Gamma} \quad (20)$$

$$c_F = \frac{0.337}{2 \sqrt{33.6}} \frac{1}{\sqrt{\Gamma(1 - \Gamma)^5}} \quad (21)$$

4.2. Heat transfer characteristics

Heat transfer coefficient of the gas–solid thermal interaction in the regenerator is given by Bin-Nun and Manidakos [13]:

$$h = \frac{k [1 + 0.99 (Re_h Pr)^{0.66}] \varepsilon^{1.79}}{d_h} \quad (22)$$

Heat transfer area per unit volume is the ratio of the total perimeter of wires to total volume of the regenerator:

$$\alpha = \frac{\pi d_w L_w}{V_{Reg}} \quad (23)$$

So L_w is the sum of length of wires inside regenerator.

By using definition of the porosity we have

$$1 - \varepsilon = \frac{V_{Solid}}{V_{Reg}} = \frac{L_w \pi d_w^2 / 4}{V_{Reg}} \quad (24)$$

Combining Eqs. (19), (23), and (24) gives the heat transfer area per unit volume as a function of Γ and wire diameter:

$$\alpha = \frac{\pi \Gamma}{d_w} \quad (25)$$

The Eqs. (20), (21), and (25) provide a better understanding of effects of geometrical parameters on the characteristics of the regenerator. The porosity of most wire-mesh screen regenerators used in pulse tube cryocoolers has a value between 0.65 and 0.7. If porosity is specified and constant, then from Eq. (19) Γ will be known. Thus, Darcy permeability, Forchheimer's inertial coefficient, and heat transfer area per unit volume will be only a function of the wire diameter.

It is interesting to ask how the performance of a pulse tube cryocooler depends on the wire diameter and length-to diameter ratio of the regenerator. For a specified geometry (length and diameter) of the regenerator, there should be an optimum wire diameter, since when porosity is constant, according to the Eqs. (20), (21), and (25), heat transfer area per unit volume and inertial resistance term are proportional to the inverse of the wire diameter, and Darcy term is proportional to the inverse of the square of the wire diameter, so increase of the wire diameter, increases pressure drop as unfavorable phenomenon, and thermal performance of the regenerator as a favorable phenomenon.

$$\frac{1}{K} \propto \frac{1}{d_w^2}, \quad \frac{c_F}{\sqrt{K}} \propto \frac{1}{d_w}, \quad \alpha \propto \frac{1}{d_w} \quad (26)$$

Furthermore, since the axial conduction heat leak will become very large at short lengths and the pressure drop will become very large at long lengths, it can be concluded that for each wire diameter there is an optimum length-to-diameter ratio.

Another important parameter that affects the performance of the regenerator is the matrix conductivity factor (λ). It is defined as the ratio of actual heat conduction of solid matrix to heat conduction of pure solid with the same cross sectional area $\left((1 - \varepsilon)\pi d_{Reg}^2/4 \right)$. There are thermal contact resistances between screens. For a regenerator with specified length and porosity, less wire diameter results in a greater number of packed screens and thermal contact resistances so that matrix conductivity factor increases with the increase of the wire diameter.

5. Results and discussion

Three Stirling-type orifice pulse tube cryocoolers with three regenerators different in length and diameter (short: $L/D = 1.8$, mean: $L/D = 4.375$, and long: $L/D = 8.0$) but with the same volume, have been modeled numerically. For each length-to-diameter ratio, wire-mesh diameter has been varied from 25 μm to 60 μm . Helium gas with a filling pressure 1.5 MPa is used as the working fluid. Compressor swept and dead volumes are 15 cm^3 and 4 cm^3 respectively and its operating frequency is 20 Hz. Ambient temperature, temperature of after-cooler and the temperature of the hot heat exchanger are 300 K, and the cooling temperature is 80 K. The regenerator has a volume of 14.07 cm^3 . The pulse tube has a diameter of 8 mm and a length of 70 mm. The orifice hole diameter is 0.70 mm. The volume of the buffer is 250 cm^3 .

After the solution reaches the periodic steady state, we focused on the results of the last cycle to investigate the behavior of the system. The local heat transfer coefficient has been averaged over one cycle. Fig. 3 shows the cycle-averaged gas–solid wire heat transfer coefficient along the length of

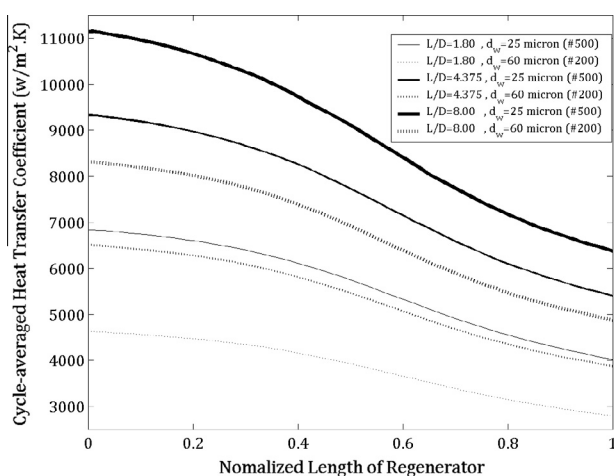


Figure 3 Cycle-averaged heat transfer coefficient along the regenerator for different regenerator L/D and different wire diameters.

the regenerator under the different wire diameters and, the different regenerator length to diameter ratios (L/D). According to this figure the local heat transfer coefficient gradually decreases along the length of the regenerator from the hot end to the cold end. From the hot end to the cold end of regenerator, thermal conductivity of gas decreases because of the lower temperature. In addition, because of lower density and smaller mass flow rate in the cold region, velocity is reduced toward the cold end. By the mentioned reasons, heat transfer coefficient decreases from the hot to the cold end.

According to Eqs. (22) and (25), by increasing the wire diameter, heat transfer coefficient decreases as well as heat transfer area and consequently, heat transfer and thermal performance of regenerator decrease. Furthermore, from Fig. 3, it is obvious that the heat transfer coefficient increases by an increase in length-to-diameter ratio. It can be attributed to this fact that with decreasing regenerator diameter, the gas velocity and Reynolds number increase due to the flow cross sectional area reduction. Moreover, from Fig. 3, for longer regenerators, the heat transfer coefficient difference between two sides of regenerator will be higher. Also, it is worth-mentioning that, there is a good consistency between our result and those obtained by Qiu et al. [5].

In the compression half-cycle, gas inside the regenerator has a higher temperature than solid so that heat flows from gas to wire-mesh, and vice versa for expansion half-cycle.

Fig. 4 illustrates cycle-averaged gas–solid temperature difference in the middle of the short regenerator ($L/D = 4.375$). It is clear that the gas–solid temperature difference changes linearly with wire diameter in the investigated range of wire diameter from 25 μm to 60 μm . However, it has an asymptote at the wire diameter of zero, because with an extremely fine mesh, heat transfer area tends to infinity, and then there is no temperature difference. In other words, gas and solid are in thermal equilibrium.

For any of the regenerator, if we assume a sinusoidal mass flow rate and a sinusoidal pressure wave, the mass flow rate can be separated as two parts. One is in phase with the pressure. Zhu and Matsubara [14] called it “progressive wave”. The other one is in -90° or 90° phase with the pressure. Zhu and Matsubara called it “standing wave”. The pressure oscillations and mass flow rate oscillations at the cold end of regenerator for three different length-to-diameter ratios are shown in Fig. 5. It can be concluded that pressure and mass flow oscillations are not sinusoidal and the phase shift for different L/D

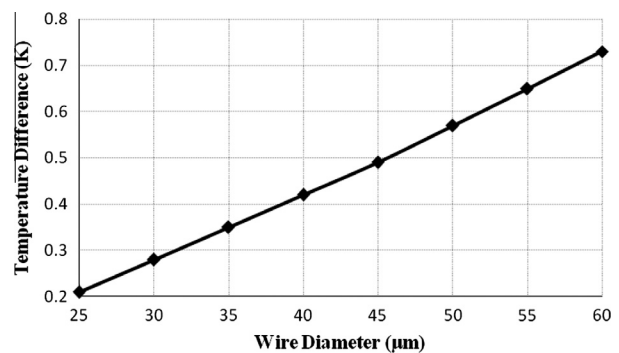


Figure 4 Mean cyclic gas–solid temperature difference at the middle of regenerator for $L/D = 4.375$.

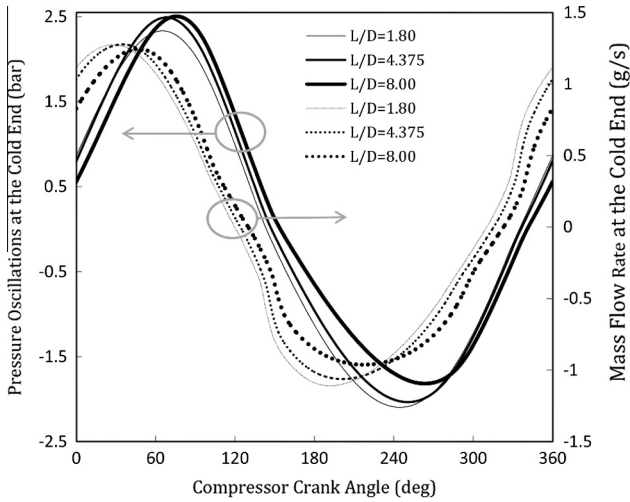


Figure 5 The pressure oscillations and mass flow rate oscillations versus compressor crank angle for three different length-to-diameter ratios.

values is about 30–38°. So that, there exists both standing wave and traveling wave. However, the standing wave has less amplitude than traveling wave. This is the condition of an orifice pulse tube cooler.

Frictional power dissipation which is the mean cyclic value of power dissipated by resistive forces (Darcy effect and Forchheimer's inertial effect) has been calculated as follows:

$$\dot{W}_{friction} = \frac{\omega}{2\pi} \oint \iiint_{Reg} \left(\frac{\mu}{K} \varepsilon u + \frac{c_F \rho}{\sqrt{K}} \varepsilon^2 u |u| \right) u dV dt \quad (27)$$

The cycle-averaged frictional power dissipation as a function of wire diameter for three ratios of regenerator length to diameter has been depicted in Fig. 6. Curve fitting of the three graphs of Fig. 6 gives

$$\begin{aligned} \dot{W}_{friction}(L/D = 1.8) &\propto \frac{1}{d_w^{1.53}}, & \dot{W}_{friction}(L/D = 4.375) \\ &\propto \frac{1}{d_w^{1.31}}, & \dot{W}_{friction}(L/D = 8.0) \propto \frac{1}{d_w} \end{aligned} \quad (28)$$

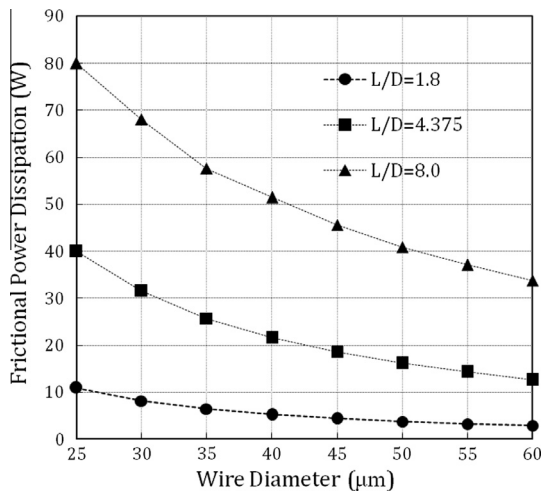


Figure 6 Mean cyclic frictional power dissipation as a function of wire diameter.

As depicted in Fig. 6, frictional power dissipation decreases as wire diameter increases. It is because of the fact that frictional power dissipation is a function of interface area of gas and wire-meshes. The larger contact area yields the more frictional power dissipation. From Eq. (25), as wire diameter increases, interface area decreases and consequently frictional power dissipation decreases.

Increase of L/D implies that L increases and/or D decreases (because the volume is constant). Then, from continuity, the velocity increases. So, growth of Forchheimer's inertial term ($c_F \rho \varepsilon^2 u |u| / \sqrt{K} \propto u^2 / d_w$) which is proportional to square of the velocity will be greater than Darcy term ($\mu \varepsilon u / K \propto u / d_w^2$) which is proportional to the velocity. It can be concluded that, for larger ratios of length to diameter (i.e. at higher velocities), Forchheimer's effect is the dominant frictional losses. Therefore, variations of the frictional losses are proportional to the inverse of the wire diameter (Eq. (28)). On the contrary, for smaller L/D , about unity, Darcy and Forchheimer's effects have equal contribution to dissipate the power.

Compressor power has been calculated by the following formula:

$$\dot{W}_{com} = \frac{\omega}{2\pi} \oint \left(P_{com} \frac{dV_{com}}{dt} \right) dt \quad (29)$$

Calculated values of the compressor power as a function of the wire diameter have been presented in Fig. 7. For three ratios of length to diameter, it is clear that for a long regenerator, the main part of compressor power is consumed to overcome the frictional losses in the regenerator. The variations of the compressor power versus wire diameter are due to the variations of frictional power loss, so the increments in Figs. 6 and 7 are approximately equal. The percentage of dissipated power of compressor for short regenerator varies from 16% (for wire diameter of 25 μm) to 5% (for wire diameter of 60 μm), and, for a long regenerator variation is between 49% (for wire diameter of 25 μm) and 30% (for wire diameter of 60 μm).

As the wire diameter decreases, gas–solid heat transfer, and consequently, thermal performance of the regenerator improve while the pressure loss increases. On the other hand, for a constant wire diameter, if the regenerator length is increased, conductive losses (gas, solid matrix, and wall conductions)

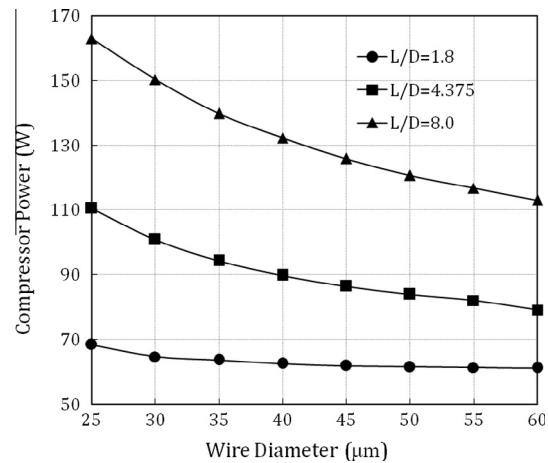


Figure 7 Compressor power as a function of wire diameter for three ratios of length to diameter.

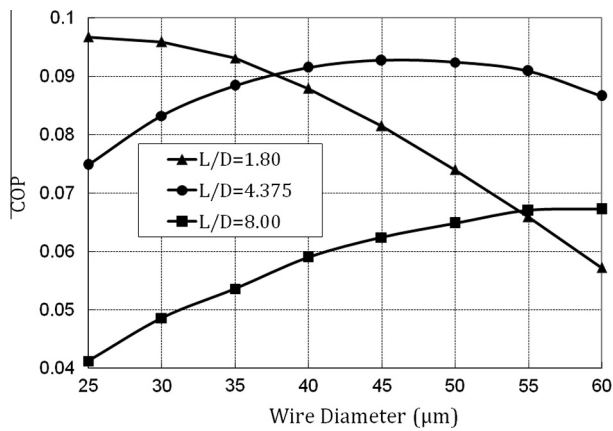


Figure 8 COP of the cooler as a function of wire diameter for three ratios of regenerator length to diameter.

decrease, but mechanical losses (pressure drop) increase. For a specified regenerator length, there is a value for wire diameter that maximizes coefficient of performance (COP) of the cryocooler. Furthermore, for a specified wire diameter, there is an optimum regenerator length. COP is defined as ratio of heat absorbed by gas in cold heat exchanger (as cooling power) to the compressor work:

$$\text{COP} = \frac{\dot{Q}_{CHX}}{\dot{W}_{com}} \quad (30)$$

The optimum wire diameter for longer regenerators is larger than that of the shorter one. The calculated COP of the cooler has been shown in Fig. 8, as a function of the wire diameter for three ratios of regenerator length to diameter. The wire diameter of 25 μm is optimum value for $L/D = 1.8$, and the wire diameter of 60 μm for $L/D = 8.0$ is the optimum. The optimum wire diameter is 45 μm for $L/D = 4.375$. The maximum COP for short, mean, and long regenerator is 0.0967, 0.0927, and 0.0672, respectively. The longer regenerator has lower COP, because the consumed compressor power by friction is larger. When the regenerator is made with smaller length to diameter ratio, the COP is higher at the optimum wire diameter as shown in Fig. 8. Nevertheless, very small length to diameter ratio causes manufacturing problems especially for coupling the regenerator with the other parts of the cooler. The dependence of COP to L/D is less than the dependence of compressor power to L/D . The high velocity in larger values of L/D causes an increase of both heat transfer coefficient and frictional loss and consequently cooling power and compressor work increases.

6. Conclusion

New correlations for calculating heat transfer and pressure drop characteristics of oscillatory flow through wire-mesh screen regenerator such as Darcy permeability, Forchheimer's inertial coefficient, and heat transfer area per unit volume, as a function of the wire diameter of mesh screens are presented. The relations are applicable for all regenerative cryocoolers. Three Stirling-type orifice pulse tube cryocoolers with three regenerators different in length and diameter but

same volume in a variety of wire diameters, have been modeled and investigated. The main findings of the present work can be classified into two categories:

(1) Based on the relations derived in this paper:

- Darcy permeability is proportional to the square of the wire diameter and Darcy term ($\mu\epsilon u/K$) is proportional to the inverse of the square of the wire diameter.
- Forchheimer's inertial coefficient is independent of wire diameter and Forchheimer's term ($c_F \rho \epsilon^2 u / \sqrt{K}$) is proportional to the inverse of the wire diameter.
- Heat transfer area per unit volume is proportional to the inverse of the wire diameter.
- Consequently, an increase of the wire diameter, increases pressure drop as an unfavorable phenomenon, and increases thermal performance of the regenerator as a favorable phenomenon. Hence, there is a value for wire diameter that maximizes the coefficient of performance (COP) of the regenerative cryocooler.

(2) Based on the results achieved by the numerical model of the present work:

- Heat transfer coefficient gradually decreases along the length of the regenerator from the hot end to the cold end because of reduction of velocity and thermal conductivity of gas.
- By increasing the wire diameter, heat transfer coefficient decreases as well as heat transfer area.
- Heat transfer coefficient increases by an increase in length-to-diameter ratio because of higher gas velocity induced by smaller cross sectional area.
- Cycle-averaged gas–solid temperature difference changes linearly with wire diameter in the investigated range of wire diameter from 25 μm to 60 μm.
- Frictional power dissipation decreases as wire diameter increases.
- In higher velocities (larger ratios of length to diameter), Forchheimer's effect is the dominant frictional loss and is larger than Darcy effect.
- For a specified regenerator length, there is a value for wire diameter that maximizes coefficient of performance (COP) of the cryocooler, and, for a specified wire diameter, there is an optimum regenerator length to diameter ratio.
- The optimum wire diameter for longer regenerators is larger than that of the shorter one.
- The dependence of COP to L/D is less than the dependence of compressor power to L/D . The high velocity in larger values of L/D causes an increase of heat transfer coefficient and frictional loss, and consequently, both cooling power and compressor work increase.

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