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# $N = 4$ superconformal symmetry for the covariant quantum superstring

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**Abstract**

We extend our formulation of the covariant quantum superstring as a WZNW model with $N = 2$ superconformal symmetry to $N = 4$. The two anticommuting BRST charges in the $N = 4$ multiplet of charges are the usual BRST charge $Q_S$ and a charge $Q_V$ proposed by Dijkgraaf, Verlinde and Verlinde for topological models. Using our recent work on "gauging cosets", we then construct a further charge $Q_C$ which anticommutes with $Q_C + Q_V$ and which is intended for the definition of the physical spectrum.


## 1. Introduction and conclusions

The past four years a new approach to the covariant quantization of the superstring has been developed. The starting point is a BRST operator $Q_B = \oint \lambda^\alpha d_{z\alpha}$ in the left-moving sector of the superstring [1], depending on free spacetime coordinates $x^m, \theta^\alpha$ and their conjugate momenta $p(\theta)_{z\alpha}$ ($m = 0, \dots, 9$; $\alpha = 1, \dots, 16$), and commuting ghosts $\lambda^\alpha$. The constraints $d_{z\alpha} \approx 0$ define the conjugate momenta of $\theta^\alpha$, and this is the only information of the classical Green–Schwarz string that is kept [2,3]. The OPEs produce further currents $\Pi_{zm}$ and $\partial_z \theta^\alpha$, and these currents form a closed affine Lie algebra. Nilpotency of $Q_B$ can be achieved by imposing the pure spinor constraint $\lambda\gamma^m\lambda = 0$ [1], but in our approach [4] we have relaxed this constraint, and this produced new ghost pairs $(\xi^m, \beta_{zm})$ (anticommuting) and $(\chi_\alpha, \kappa_z^\alpha)$ (commuting), as well as a conjugate momentum $w_{z\alpha}$ for $\lambda^\alpha$ (we suppress from now on the index $z$ most of the time). We discovered in this approach that the superstring is a "gauged" WZNW model [5], based on a non-semisimple non-reductive superalgebra $\mathcal{A}$ which decomposes into coset generators $Q_\alpha$

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(associated with $-id_\alpha$ and $\lambda^\alpha, w_\alpha$) and Abelian subgroup generators, namely $P_m$ (associated with $\Pi_m$ and $\xi^m, \beta_m$) and fermionic central charges $K^\alpha$ (associated with $\partial\theta^\alpha$ and $\chi_\alpha, \kappa^\alpha$). It is non-reductive because $[Q_\alpha, P_m]$ is not proportional to $Q_\beta$, but rather to $K^\beta$. The matter currents $J_M = \{-id_\alpha, \Pi_m, \partial\theta^\alpha\}$ depend only on $x^m, \theta^\alpha$ and $p_\alpha$, and generate $\mathcal{A}$. From the ghost fields one can construct currents which also form a representation of $\mathcal{A}$ but without double poles. The gauging leads to a second set of matter currents $J_M^h$ depending on new variables $x_h^m, \theta_h^\alpha$ and $p_\alpha^h$ and also these $h$-currents generate the algebra $\mathcal{A}$ but with opposite central charges (opposite double poles). In terms of these currents a particular superconformal algebra was constructed, with BRST charge $j_{zB}^W$ containing the sum of the currents $J_M + J_M^h + \frac{1}{2}J_M^{gh}$, a stress tensor, a ghost current, and a spin 2 field $B_{zz}$ which contains the difference of the currents $J_M - J_M^h$. The central charge of this system vanishes. The spin 2 field $B_{zz}$ satisfies the relation $\oint j_{zB}^W(z)B_{zz}(w) = T_{zz}(w)$. In our earlier work we had tried to find an expression for it [4], but only after gauging the model did we finally succeed. The gauged WZNW model also improved our earlier work in another respect: no longer was it necessary to introduce further ghosts and antighosts by hand, but only the minimal set of ghosts $c^M = \{\lambda^\alpha, \xi^m, \chi_\alpha\}$ is present, and still BRST nilpotency and vanishing of the central charge is achieved.

The next step concerned the definition of physical states $|\mathrm{phys}\rangle$. It became clear to us that in addition to the usual condition $Q_B^W|\mathrm{phys}\rangle = 0$ we needed further conditions, whose role was to remove the dependence of the cohomology on the extra coordinates $x^{m,h}, \theta^{\alpha,h}$ and $p_\alpha^h$. In addition, we expected to need a condition of the form $B_0|\mathrm{phys}\rangle = 0$ where $B_0$ is the zero mode of $B_{zz}(z)$. Also we knew from the work of [6] that in purely topological models there exists a second BRST charge $Q_V$, which has the more familiar Virasoro form $Q_V = \oint c^z T_{zz} + \cdots$. Indications that our approach has topological aspects were already encountered in our first paper on the subject [4].

Two BRST charges suggest the presence of an $N = 4$ algebra and that is the subject of this Letter. It is desirable to first discuss the motivations that have led to the present work, before commenting on the steps needed to obtain our results.

(1) All the known models of string theory on flat Minkowskian space can be embedded into an $N = 4$ superconformal algebra. This suggests to investigate whether this also applies to the present formulation.

(2) The construction of the pure spinor formulation [1] is based on a BRST charge and pure (first class) spinor constraints. This means that the observables (BRST cohomology) of the theory are constructed on a functional space with additional constraints. This construction is known as homological perturbation theory and it can be reformulated as an unconstrained system with the help of more than one BRST charge [3,7]. The reformulation thus obtained has several advantages: it provides an unconstrained functional space with manifestly geometrical properties such as supersymmetry and Lorentz covariance. However, since string theory is a conformal field theory, it is necessary to extend the construction of the BRST charges to a complete set of generators forming a closed algebra. We shall construct this algebra; it is an $N = 4$ superconformal algebra and this gives us a well established context to study the correlation function of the theory.

(3) In [5] we showed that the pure spinor formulation arizes if one quantizes WZNW models. In particular, we showed how to select a physical space when the constraints are represented by the generators of a coset instead of the usual construction of the BRST charge based on the generators of a subgroup [5]. This unavoidably leads to first class constraints on the ghost fields which can be treated in the context of homological perturbation theory as discussed above. In addition, WZNW models are conformal field and $N = 1, 2$ and $N = 4$ superconformal symmetry is pivotal to derive some important result such as non-renormalization theorems, finiteness, computation of correlation functions, computation of elliptic genera, and partition functions. So, besides applications to string theory, the motivations to extend the results obtained in [5] to $N = 4$ algebra is to reproduce the known results of gauged WZNW models in the new framework of "gauging" the coset of the underlying gauge algebra.

(4) An $N = 4$ structure has some important implications: it implies that there is a natural picture changing operation, and it allows the construction of the measure for higher genus computations.

(5) In the context of the pure spinor formulation, the role of the Virasoro constraints, which have been essential for all the string models (bosonic, RNS, WZNW, String field theory) is obscure. The present formulation sheds some light on the problem. In fact, the construction clearly shows that one needs to couple the theory to a topological gravity multiplet bringing in the ghosts for diffeomorphisms. A careful gauging of the present formalism would lead to the present structure of BRST operators.

In the rest of the introduction, we explain the several steps needed to obtain the desired result. Starting form the WZNW model, we found that the superconformal algebra is not an $N = 2$ algebra, but rather a Kazama algebra [8]; such an algebra has extra higher spin currents (namely two spin 3 currents). However, it is known that one can add a gravitational topological quartet (which we call the Koszul quartet[1] $K'$ below) and modify the Kazama currents such that the sums of the currents of the combined system form a genuine $N = 2$ algebra [9,10]. In particular the BRST charge of this combined system is the sum of the separate charges, $Q_S^W + Q_S^{K'}$, but the spin 2 current $B^W$ of WZNW model is modified into $\tilde{B}^W$ by adding terms depending on the fields of $K'$.

The fact that such a Koszul quartet is a gravitational topological quartet was welcome news, because it enables us to introduce worldsheet diffeomorphisms into our work. It is known from the work of Dijkgraaf–Verlinde–Verlinde [6], that there exist two BRST charges in topological models: a charge $Q_S^K$ for the Koszul quartet and a charge $Q_V$ which is related to diffeomorphisms and which has the form

$$Q_V = \oint c\left(T^W + \frac{1}{2}T^K\right) + \gamma\left(\tilde{B}^W + \frac{1}{2}B^K\right) + \cdots.$$

Here $T^W$ is the stress tensor of the matter topological system which in our case corresponds to the sum of WZNW the $K$ system, and $\tilde{B}^W$ is the modified spin 2 field mentioned above. The two charges $Q_V$ and $Q_S^K$ anticommute.

However, as noticed recently [11], in order that $Q_V$ and $Q_S^W + Q_S^{K'}$ anticommute, the Koszul quartet needed to turn the Kazama algebra into $N = 2$ algebra cannot be the same as the Koszul quartet needed to construct $Q_V$. Thus there are two Koszul quartets, which we already denoted above by $K'$ and $K$. The quartet $K'$ modifies the current $B^W$ of the WZNW model, while $K$ enters in the construction of $Q_V$. At this point we have the following BRST charges: $Q_S^W + Q_S^{K'}$, $Q_S^K$ and $Q_V$. The first one is a spacetime object, while the latter two are worldsheet objects. They are all nilpotent and anticommute with each other.

Although we had now constructed three BRST charges, none of them contained the information that the theory originally contained the pure spinor constraints. So the problem of finding an additional BRST charge $Q_C$ remained. We decided to start a study of general Lie algebras and constraints of the kind encountered in the superstring [12]. In this study we divided generators into the commuting set of Cartan generators, and coset generators. The superstring is an example, with $Q_\alpha$ the coset generators, and $(P_m, K^\alpha)$ the Abelian subalgebra. We then “gauged the coset generators”. By this provocative statement we meant that we imposed constraints on the ghosts associated to the coset generators (corresponding to the pure spinor constraints [1]), and then relaxed these constraints in such a way that the cohomology remained unchanged. In the process we found the second BRST charge $Q_C$, but one has to introduce a doubling of the subgroup ghosts as well as an another copy of the subgroup ghosts which vanishing ghost number. In our case these new fields are denoted by $(\xi'_m, \beta'^{m}_{z}, \chi'_\alpha, \kappa'^{\alpha}_{z})$ and $(\phi_m, \bar{\phi}^m_z, \phi_\alpha, \bar{\phi}^\alpha_z)$. There is a separate BRST charge for the coset fields which we denote by $Q_S^{\mathrm{co}}$ and a contribution of the coset fields to $Q_V$ which we denote by $Q_V^{\mathrm{co}}$.

Following the procedure of [12] the BRST charge $Q_C^W$ for the WZNW model with $K$ and $K'$ quartets and coset fields was recently constructed in [11], but it was found not to anticommute with the total charge $\mathbf{Q}_S + \mathbf{Q}_V$ where $\mathbf{Q}_S = Q_S^W + Q_S^{K'} + Q_S^K + Q_S^{\mathrm{co}}$ and $\mathbf{Q}_V = Q_V + Q_V^{\mathrm{co}}$. We construct below a charge $Q_C$ which does anticommute with $\mathbf{Q}_S + \mathbf{Q}_V$. Our construction is based on the observation that all currents so far have been constructed without

[1] This quartet consists of the ghosts $(c'^z, b'_{zz}, \gamma'^z, \beta'_{zz})$ with conformal spins $(-1, 2, -1, 2)$ and the ghost charges $(1, -1, 2, -2)$. Later we introduce a second quartet $K = (c^z, b_{zz}, \gamma^z, \beta_{zz})$ with same quantum numbers in order to construct the topological BRST charge $Q_V$.

bosonization, so that the zero modes $\oint \eta_z$ and $\oint \eta'_z$ due to bosonization of the superghosts of the two Koszul quartets $K$ and $K'$ trivially anticommute with all the other currents. We propose to take the zero mode $\oint \eta'_z$ and to make a similarity transformation with the whole BRST charge $\mathbf{Q}_S$ as follows

$$\mathbf{Q}_C^W = e^{-R} \oint \eta'_z e^R, \quad \text{where } R = \left\{ \mathbf{Q}_S, \oint \xi' X_z^W \right\}.$$

Here $\xi'$ is the partner of $\eta'_z$ and $X_z^W$ is defined by $[\mathbf{Q}_S, \oint X_z^W] = Q_C^W$ with $Q_C^W$ the charge given in [12]. Of course, $\mathbf{Q}_S$ itself remains unchanged under this similarity transformation and $\mathbf{Q}_C^W$ is of the form $\oint \eta'_z + Q_C^W + \cdots$ and is independent of $K$. The extra terms denoted by $\cdots$ follow straightforwardly the double- and higher-order commutators, and are needed in order that $\mathbf{Q}_C^W$ anticommute with $\mathbf{Q}_S$.

Having constructed the extra charge $\mathbf{Q}_C^W$ which we expect to be needed to define the correct physical spectrum, we return to the issue of an $N = 4$ superconformal algebra. A small $N = 4$ superconformal algebra needs a triplet of $SU(2)$ currents, which for a twisted model (the case we are considering) have spins $(0, 1, 2)$ and ghost numbers $(2, 0, -2)$ [13].

We use the free fields of the $K$ quartet to construct the Wakimoto representation of these $SU(2)$ currents [14]. There are now at least two ways to proceed: use $\mathbf{Q}_S$ and $\mathbf{Q}_V$, or use $Q_S^W + Q_S^{K'} + Q_S^{\mathrm{co}}$ and $\mathbf{Q}_C^W$ to construct another $N = 4$ algebra. In this Letter we perform the first construction. It may clarify if we summarize the various charges in a diagram, and indicate the various $N = 2$ and $N = 4$ subalgebras which could conceivably be constructed. Those whose existence is only conjectured are indicated by a question mark. From this picture another conjecture emerges: the various $N = 4$ algebras are all subalgebras of an enveloping $N = 8$ superconformal algebra.

$$
\left.\begin{array}{l}
\left.\begin{array}{r|l}
 \text{Without coset fields} & \text{With coset fields} \\
\begin{matrix}\text{SPACETIME}\\ N=4?\end{matrix}\left\{\begin{matrix}\oint\eta'_z\\ \left.Q_S^W+Q_S^{K'}\right\}N=2\end{matrix}\right. & \left.\begin{matrix}\mathbf{Q}_C^W\\ Q_S^W+Q_S^{K'}+Q_S^{\mathrm{co}}\end{matrix}\right\}N=4? \\
\hline
\begin{matrix}\text{WORLDSHEET}\\ N=4\end{matrix}\left\{\begin{matrix}\left.Q_S^K+Q_V\right\}N=2\\ \oint\eta_z\end{matrix}\right. & \left.\begin{matrix}Q_S^K+Q_V+Q_V^{\mathrm{co}}\\ \mathbf{Q}_C^{\mathrm{top}}\end{matrix}\right\}N=4?
\end{array}\right\}N=4
\end{array}\right\}N=8?
$$

Mutually anticommuting BRST charges of $N = 2, 4$ subalgebras

In the spacetime sector we begin with the BRST charge $Q_S^W$ of the WZNW model [5] (see the left upper part of the diagram). The BRST charge $Q_S^W + Q_S^{K'}$ belongs to an $N = 2$ algebra [5]. The BRST charge[2] $\oint \eta'_z$ anticommutes $Q_S^W + Q_S^{K'}$ and these two charges might be part of a $N = 4$ algebra. The coset fields are needed to construct $Q_C^W$ according to [12] and hence one finds the BRST charge $Q_S^{\mathrm{co}}$ for the coset fields in the right upper part of the diagram. Comparing the left- and right-hand side of the diagram, we conjecture that the BRST charges $\mathbf{Q}_C^W$ which we discussed above and $Q_S^W + Q_S^{K'} + Q_S^{\mathrm{co}}$ are part of another $N = 4$ algebra.

In the worldsheet sector we find the BRST charge $Q_S^K + Q_V$ which is part an $N = 2$ algebra, as discussed in [6], see the lower left part of the diagram. The zero mode $\oint \eta_z$ forms another anticommuting BRST charge, and together these two BRST charges form an $N = 4$ superconformal algebra as shown by Berkovits and Vafa [13]. We can repeat our procedure of the spacetime sector and make a similarity transformation on $\oint \eta_z$ with the BRST charge of the worldsheet sector to obtain $\mathbf{Q}_C^{\mathrm{top}}$, see the lower right part of the diagram. The formula reads

$$\mathbf{Q}_C^{\mathrm{top}} = e^{-R^{\mathrm{top}}} \oint \eta_z e^{R^{\mathrm{top}}}, \quad \text{where } R^{\mathrm{top}} = \left\{ Q_S^K + \mathbf{Q}_V, \oint \xi X_z^{\mathrm{top}} \right\},$$

[2] The bosonization formulas for $K$ are $\gamma^z = \eta_z e^{-\varphi}$ and $\beta_{zz} = \partial\xi e^{\varphi}$ with $\varphi(z)\varphi(w) \sim -\ln(z - w)$. Similarly for $K'$.

and $X_z^{\text{top}} = c'b$. The construction of $X_z^{\text{top}}$ follows along the lines of [12]: one starts with a BRST operator which is nilpotent up to term which are a Lie derivative along $\gamma$ [15]. One requires that these terms vanish, which is the analog of the constraints involving $d_\alpha$, and then one relaxes this constraint [16]. In this context $\gamma$ plays the role of $\lambda^\alpha$, $c$ plays the role of $(\xi^m, \chi_\alpha)$, while $c'$ corresponds to $(\xi'^m, \chi'_\alpha)$, and $\gamma'$ corresponds to $(\varphi^m, \varphi_\alpha)$. Note that "gauging" of the coset of the topological quartet $K$ brings in the quartet $K'$ of the spacetime sector. This is the more fundamental reason that one needs two quartets. Perhaps again $Q_S^K + \mathbf{Q}_V$ and $\mathbf{Q}_C^{\text{top}}$ are part of an $N = 4$ algebra (see the lower right part of the diagram).

Finally, we come to the contents of this Letter. We show that $\mathbf{Q}_S$ and $\mathbf{Q}_V$ do indeed belong to an $N = 4$ superconformal algebra. We construct this algebra in steps. In Section 2 we construct an $N = 4$ algebra for the quartet $K$ with the Wakimoto triplet, in Section 3 we add the coset fields, and in Section 4 we add the WZNW model coupled to the quartet $K'$. The final result is given by Eq. (45).

It is also easy to construct a charge $Q_C$ which anticommute with $\mathbf{Q}_S + \mathbf{Q}_V$, namely $Q_C = e^{-R} \oint (r\eta + s\eta')e^R$ with arbitrary $r, s$, and $R = \{\mathbf{Q}_S + \mathbf{Q}_V, \oint X\}$. One choice for $X$ is $X = (\xi' X_z^W + \xi X_z^{\text{top}})$. In order that physics after the similarity transformation is different from physics before, we expect that a suitable filtration (grading condition [4])[3] is needed.

Despite several important results of the pure spinor formalism [17] obtained by N. Berkovits and the Stony Brook group, a deeper understanding of the formalism and its geometrical origins are still lacking. Several issues such as the relation with the kappa symmetry of Green–Schwarz string theory, the Virasoro constraints (and therefore the diffeomorphism invariance), and the role of picture changing operators in a path-integral construction have to be explored and the present Letter might shed some light on these aspects. We mention that the details how to combine the left-moving sector with the right-moving sector have recently been worked out [11].

## 2. The $N = 4$ gravitational Koszul quartet

The quantization of the superstring as a particular WZNW model based on a non-semisimple Lie algebra has led us in [5] to a twisted $N = 2$ superconformal field theory. Following [6] we introduced a gravitational $N = 2$ Koszul quartet which can be considered as the twisted version of the familiar spin $(2, -1, 3/2, -1/2)$ ghost quartet $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$ of the $N = 1$ RNS spinning string. The introduction of this quartet achieved two goals:

(i) it allowed the construction of a second BRST charge $Q_V$ as in topological models, and
(ii) it coupled our spacetime-supersymmetric superstring to worldsheet gravity.

The two BRST charges $Q_S^W$ and $Q_S^K$ are present in any topological model, so they cannot be used to eliminate the dependence on $x_h$ and $\theta_h$. We need another anticommuting operator, like an antighost, to eliminate this dependence. Moreover, if one has two BRST charges, it seems likely that one is dealing with an $N = 4$ model.

An $N = 2$ model with two spin-one BRST charges suggest that it is part of a twisted $N = 4$ model, which should consist of two spin-one BRST currents $G^+(z)$ and $\tilde{G}^+(z)$, two spin 2 $B$-fields $G^-(z)$ and $\tilde{G}^-(z)$, a stress tensor $T_{zz}$ with vanishing anomaly, and further $SU(2)$ currents. In a twisted $N = 4$ model the $SU(2)$ currents have spin $0, 1, 2$, rather than spin 1 [13]. We thus need a spin $(0, 1, 2)$ triplet of currents which separately form a closed algebra.

At this point we may recall that the well-known Wakimoto representation [14] of currents constructed from ghost fields satisfies these properties. One is thus led to study the original $N = 2$ gravitational Koszul quartet together with the Wakimoto triplet of currents, and try to extend this model to an $N = 4$ model.

[3] See [12] for a geometrical interpretation of the grading.

This quartet $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$ has spins $(2, -1, 2, -1)$ and ghost charges $(-1, 1, -2, 2)$. The ghosts $b_{zz}$ and $c^z$ are anticommuting with propagator $c(z)b(w) \sim (z-w)^{-1}$, while $\gamma^z$ and $\beta_{zz}$ are commuting and satisfy the OPE $\gamma(z)\beta(z) \sim (z-w)^{-1}$. The currents of this $N = 2$ model are given by

$$T_{zz} = -2b_{zz}\partial_z c^z - \partial_z b_{zz} c^z - 2\beta_{zz}\partial_z\gamma^z - \partial_z\beta_{zz}\gamma^z, \tag{1}$$

$$j_z^B = -b_{zz}\gamma^z, \qquad J_z = -b_{zz}c^z - 2\beta_{zz}\gamma^z, \tag{2}$$

$$B_{zz} = 2\beta_{zz}\partial_z c^z + c^z\partial_z\beta_{zz} + \mu b_{zz}. \tag{3}$$

The stress tensor is simply the sum of the stress tensors of two spin $(2, -1)$ doublets, and the factor 2 in the ghost current yields the ghost charges $(2, -2)$ for $\gamma^z$ and $\beta_{zz}$. The $B$ field $B_{zz}$ has spin 2 and ghost number $-1$, and the parameter $\mu$ is a free parameter (to be fixed to $\mu = 1$ later). The spin-1 BRST current $j_z^B$ and the spin-2 field $B_{zz}$ are the twists of the two spin $3/2$ currents of an untwisted $N = 2$ multiplet. From now on we shall drop the super- and subscripts $z$ when no confusion is possible.

The Wakimoto representation is given by

$$J^{++} = -bc\gamma + \frac{3}{2}\partial\gamma - \beta\gamma\gamma, \tag{4}$$

$$J_3 = -bc - 2\beta\gamma, \tag{5}$$

$$J^{--} = \beta. \tag{6}$$

The superscripts denote the ghost number. The ghost current is identified with $J_3$. These currents satisfy the following OPE

$$J_3(z)J^{\pm\pm}(w) \sim \pm 2\frac{J^{\pm\pm}(w)}{z-w}, \qquad J_3(z)J_3(w) \sim \frac{-3}{(z-w)^2}, \tag{7}$$

$$J^{++}(z)J^{--}(w) \sim \frac{-3/2}{(z-w)^2} + \frac{J_3(w)}{z-w}, \qquad T(z)J_3(w) \sim \frac{3}{(z-w)^3} + \frac{J_3(z)}{(z-w)^2}. \tag{8}$$

Closure of the algebra fixes all coefficients in the currents. We could rescale these currents such that the terms with double poles in $J^{++}J^{++}$ and $J_3J_3$ become equal, but the formulas are simpler by keeping the present normalization.

We now present the $N = 4$ extension of the $N = 2$ Koszul model. This result has been obtained before in [18] with $\mu = 0$, but we keep $\mu$ arbitrary. The stress tensor and $SU(2)$ triplet are unchanged, while we have the following anticommuting currents

$$G^+ = -b\gamma \ \overset{J^{++}}{\longleftarrow}\overset{J^{--}}{\longrightarrow}\ \tilde{G}^- = -b, \tag{9}$$

$$G^- = 2\beta\partial c + c\partial\beta + \mu b \ \underset{J^{--}}{\longleftarrow}\underset{J^{++}}{\longrightarrow}\ \tilde{G}^+ = -\frac{3}{2}\partial^2 c + bc\partial c + 2\partial c\beta\gamma + c\partial\beta\gamma + \mu b\gamma. \tag{10}$$

The currents $G^\pm$ are equal to the BRST current and the $B$ field of the $N = 2$ model. As the notation indicates the currents $J^{++}$ and $J^{--}$ map the currents $G^+$ and $\tilde{G}^-$ into each other, and also $G^-$ and $\tilde{G}^+$ are mapped into each other by $J^{++}$ and $J^{--}$

$$J^{++}(z)G^+(w) \sim 0, \qquad J^{--}(z)G^-(w) \sim 0, \tag{11}$$

$$J^{++}(z)\tilde{G}^+(w) \sim 0, \qquad J^{--}(z)\tilde{G}^-(w) \sim 0, \tag{12}$$

$$J^{++}(z)G^-(w) \sim \frac{-\tilde{G}^+(w)}{z-w}, \qquad J^{--}(z)\tilde{G}^+(w) \sim \frac{-G^-(w)}{z-w}, \tag{13}$$

$$J^{++}(z)\tilde{G}^-(w) \sim \frac{-G^+(w)}{z-w}, \qquad J^{--}(z)G^+(w) \sim \frac{-\tilde{G}^-(w)}{z-w}. \tag{14}$$

Only the calculation of $J^{++}(z)\tilde{G}^+(w)$ is involved.

The superscripts of these currents denote their ghost number

$$J_3(z)G^{\pm}(w) \sim \pm\frac{G^{\pm}(w)}{z-w}, \qquad J_3(z)\tilde{G}^{\pm}(w) \sim \pm\frac{\tilde{G}^{\pm}(w)}{z-w}. \tag{15}$$

The conformal spin of $G^+$ and $G^-$ is 1 and 2, respectively [5], while it is straightforward to verify that $\tilde{G}^{\pm}(w)$ have the same conformal spin as $G^{\pm}$

$$T(z)\tilde{G}^+(w) \sim \frac{\tilde{G}^+(w)}{(z-w)^2} + \frac{\partial\tilde{G}^+(w)}{z-w}. \tag{16}$$

The crucial test is whether the OPEs of two fermionic currents close. They do indeed close. We find the following OPEs

$$G^+(z)\tilde{G}^+(w) \sim \frac{2J^{++}(w)}{(z-w)^2} + \frac{\partial J^{++}(w)}{z-w}, \tag{17}$$

$$G^-(z)\tilde{G}^-(w) \sim \frac{2J^{--}(w)}{(z-w)^2} + \frac{\partial J^{--}(w)}{z-w}, \tag{18}$$

$$G^+(z)G^-(w) \sim \frac{-3}{(z-w)^3} + \frac{J_3(w)}{(z-w)^2} + \frac{T_{zz}(w)}{z-w}, \tag{19}$$

$$\tilde{G}^+(z)\tilde{G}^-(w) \sim \frac{3}{(z-w)^3} + \frac{-J_3(w)}{(z-w)^2} + \frac{-T_{zz}(w)}{z-w}. \tag{20}$$

For our work it is important that the two BRST $\oint G^+$ and $\oint \tilde{G}^+$ charges are nilpotent and anticommute. This is indeed the case

$$G^+(z)\tilde{G}^-(w) \sim 0, \qquad G^+(z)G^+(w) \sim 0, \qquad \tilde{G}^-(z)\tilde{G}^-(w) \sim 0, \tag{21}$$

$$G^-(z)\tilde{G}^+(w) \sim 0, \qquad G^-(z)G^-(w) \sim 0, \qquad \tilde{G}^+(z)\tilde{G}^+(w) \sim 0. \tag{22}$$

For $\tilde{G}^+(z)\tilde{G}^+(w)$ we directly checked that the terms with $\mu$ cancel, but the vanishing of this OPE follows already from (13) and (23).

We conclude that we have constructed an $N=4$ extension of the gravitational $N=2$ Koszul quartet. We end this section with a few comments:

(1) The parameter $\mu$ of the term $\mu b$ in $G^-$ remains arbitrary; it is not fixed when one extends the $N=2$ Koszul model with a free $\mu$ to the $N=4$ Koszul model.
(2) Both $T$, $J_3$, $G^+$, $G^-$ and $T$, $J_3$, $\tilde{G}^+$, $\tilde{G}^-$ are $N=2$ multiplets. Since obviously for both the anomaly in $TJ_3$ is opposite to the anomaly in $J_3J_3$, both are topological $N=2$ multiplets. The anomaly in the stress tensor indeed vanishes.
(3) The OPEs of a twisted $N=4$ model are, for example, given in [13]. We obtain agrement with these OPEs if we rescale our current by factors $\pm i$.
(4) For $\mu=0$ this $N=4$ superconformal algebra has been derived before in [18], specifically equation (33).

## 3. An $N=4$ model for one Koszul quartet and coset fields

In this section we extend the construction to “coset fields”. These coset fields were first introduced in our paper [12], in order to construct a second BRST change for the superstring called $Q_C$. Subsequently these fields were added to our $N=2$ WZNW model for the superstring in [11]. The result of these articles is an $N=2$ conformal field theory containing two Koszul quartets, coset fields, and the fields of the WZNW model. In this section we construct an $N=4$ conformal field theory containing one Koszul quartet and the coset fields. This will pave the way to an $N=4$ formulation of the WZNW model.

The coset fields for the superstring consist of second set of ghosts $(\xi'_m, \beta_z'^m, \chi'_\alpha, \kappa_z'^\alpha)$, and a corresponding set of fields $(\varphi_m, \bar\varphi^m, \varphi_\alpha, \bar\varphi^\alpha)$. The fields $(\xi'_m, \beta'_m, \varphi_\alpha, \bar\varphi^\alpha)$ are anticommuting, while $(\chi'_\alpha, \kappa_z'^\alpha, \varphi_m, \bar\varphi^m)$ are commuting. The propagators are the standard ones

$$\xi'_m(z)\beta_z'^n(w) \sim \delta^n_m \frac{1}{z-w}, \qquad \chi'_\alpha(z)\kappa_z'^\beta(w) \sim \delta^\beta_\alpha \frac{1}{z-w}, \tag{23}$$

$$\varphi_m(z)\bar\varphi^n_z(w) \sim \delta^n_m \frac{1}{z-w}, \qquad \varphi_\alpha(z)\bar\varphi^\beta_z(w) \sim \delta^\beta_\alpha \frac{1}{z-w}. \tag{24}$$

From these fields one can construct an $N=2$ algebra.

Following [5,11,12] the stress tensor, ghost and $B$ currents are easily written down. For $T_{zz}$ we have the usual free field expression

$$\begin{aligned} T^{\mathrm{co}+K} = {} & -\beta'_{zm}\partial_z\xi'^m - \kappa_z'^\alpha\partial_z\chi'_\alpha - \bar\varphi_{zm}\partial_z\varphi^n - \bar\varphi^\alpha_z\partial_z\varphi_\alpha \\ & - 2b_{zz}\partial_z c^z - \partial_z b_{zz}c^z - 2\beta_{zz}\partial_z\gamma^z - \partial_z\beta_{zz}\gamma^z \quad \text{with } c_{TT}=0. \end{aligned} \tag{25}$$

The central charges of the $bc$ and $\beta\gamma$ system ($-26$ and 26) cancel each other, and also those of the coset fields cancel because the primed fields have opposite statistics from the $\varphi$ fields. The ghost current is the sum of the ghost currents of the two systems

$$J_z^{\mathrm{co}+K} = -\beta'_{zm}\xi'^m - \kappa_z'^\alpha\chi'_\alpha - b_{zz}c^z - 2\beta_{zz}\gamma^z \quad \text{with } c_{JJ} = -9. \tag{26}$$

Its anomaly is $c_{JJ}=-9$. (Twisting yields this anomaly in the $JJ$ OPE, while the conformal anomaly in $TT$ vanishes after twisting.) The BRST current is the sum of the two BRST currents of the coset and Koszul systems

$$j_{z,B}^{\mathrm{co}+K} = -\bar\varphi^m_z\xi'_m - \bar\varphi^\alpha_z\chi'_\alpha - b_{zz}\gamma^z. \tag{27}$$

Finally, the $B_{zz}$ field reads

$$B_{zz}^{\mathrm{co}+K} = \beta'_{zm}\partial_z\varphi^m + \kappa_z'^\alpha\partial_z\varphi_\alpha + 2\beta_{zz}\partial_z c^z + c^z\partial_z\beta_{zz} + \mu b_{zz}, \tag{28}$$

where we recall that $\mu$ is a free parameter.

The coset currents $T_{zz}^{\mathrm{co}}$, $J_z^{\mathrm{co}}$, $j_{z,B}^{\mathrm{co}}$ and $B_{zz}^{\mathrm{co}}$ form separately an $N=2$ superconformal algebra. In particular,

$$j_B^{\mathrm{co}}(z)B^{\mathrm{co}}(w) \sim \frac{-6}{(z-w)^3} + \frac{J^{\mathrm{co}}}{(z-w)^2} + \frac{T^{\mathrm{co}}}{z-w}, \tag{29}$$

$$J^{\mathrm{co}}(z)J^{\mathrm{co}}(w) \sim \frac{-6}{(z-w)^2}, \tag{30}$$

$$T^{\mathrm{co}}(z)J^{\mathrm{co}}(w) \sim \frac{6}{(z-w)^3} + \frac{J^{\mathrm{co}}(w)}{(z-w)^2} + \frac{\partial J^{\mathrm{co}}(w)}{z-w}. \tag{31}$$

However, in the extension to an $N=4$ system, couplings arise between the coset and the Koszul system, as we now show.

To obtain the extension to an $N=4$ system we need to extend the $U(1)$ ghost current to an $SU(2)$ current triplet with conformal spin $(0,1,2)$. The following is such a system

$$J^{++} = J_z^{\mathrm{co}}\gamma^z + \frac{9}{2}\partial_z\gamma^z - \gamma^z\gamma^z\beta_{zz} - \gamma^z b_{zz}c^z - c^z j_B^{\mathrm{co}}, \tag{32}$$

$$J_3 = -\beta'_{zm}\xi'^m - \kappa_z'^\alpha\chi'_\alpha - b_{zz}c^z - 2\beta_{zz}\gamma^z, \tag{33}$$

$$J^{--} = \beta_{zz}. \tag{34}$$

The ghost values of these currents are $(2, 0, -2)$, respectively,

$$J^{\pm\pm}(z)J_3(w) \sim \mp 2\frac{J^{\pm\pm}(w)}{z-w}, \tag{35}$$

$$J^{++}(z)J^{--}(w) \sim \frac{-9/2}{(z-w)^2} + \frac{J_3(w)}{z-w}, \tag{36}$$

$$J_3(z)J_3(w) \sim \frac{-9}{(z-w)^2}. \tag{37}$$

All coefficients in the $SU(2)$ current are fixed by requiring closure, in particular the coefficient of the total derivative $\frac{9}{2}\partial_z \gamma^z$.

We can now construct the currents $\tilde{G}^+$ and $\tilde{G}^-$ by acting with $J^{++}$ and $J^{--}$ on $j_{zB}^{\mathrm{co}+K} \equiv G_z^+$ and $B_{zz}^{\mathrm{co}+K} \equiv G_{zz}^-$. One finds easily

$$J^{--}(z)G^+(w) \sim \frac{-\tilde{G}^-(w)}{z-w} \;\Rightarrow\; \tilde{G}^- = -b. \tag{38}$$

The calculation of $\tilde{G}^+$ is more involved. We start from

$$\begin{aligned} &-\frac{\tilde{G}^+(w)}{z-w} \sim J^{++}(z)G^-(w) \\ &\quad = \left(J^{\mathrm{co}}\gamma - cj_B^{\mathrm{co}} + \frac{9}{2}\partial\gamma - \gamma\gamma\beta - \gamma bc\right)(z)\big(B^{\mathrm{co}} + 2\beta\partial c + c\partial\beta + \mu b\big)(w). \end{aligned} \tag{39}$$

We obtain

$$\tilde{G}^+ = cT^{\mathrm{co}} + \gamma B^{\mathrm{co}} - \partial\big(cJ^{\mathrm{co}}\big) - \mu j_B^{\mathrm{co}} - \frac{9}{2}\partial^2 c + bc\partial c + 2\gamma\beta\partial c + \gamma c\partial\beta + \mu\gamma b. \tag{40}$$

Triple and double poles nicely cancel here, confirming the coefficient $9/2$ of the term with $\partial\gamma$ in $J^{++}$. The crucial question is whether the simple structure of $\tilde{G}^+$ in the coset sector also holds in the Koszul sector. We find

$$bc\partial c + 2\gamma\beta\partial c + \gamma c\partial\beta = c\left(\frac{1}{2}T^K\right) + \gamma\left(\frac{1}{2}B^K\right) - \partial\left(c\frac{1}{2}J^K\right) + \frac{\mu}{2}j_B^K. \tag{41}$$

Hence, the total $\tilde{G}^+$ is indeed of a simple form

$$\tilde{G}^+ = c\left(T^{\mathrm{co}} + \frac{1}{2}T^K\right) + \gamma\left(B^{\mathrm{co}} + \frac{1}{2}B^K\right) - \partial\left(c\left(J^{\mathrm{co}} + \frac{1}{2}J^K\right)\right) - \mu\left(j_B^{\mathrm{co}} + \frac{1}{2}j_B^K\right) - \frac{9}{2}\partial^2 c. \tag{42}$$

Also $J^{++}$ can be written in this way

$$J^{++} = \gamma\left(J^{\mathrm{co}} + \frac{1}{2}J^K\right) - c\left(j_B^{\mathrm{co}} + \frac{1}{2}j_B^K\right) + \frac{9}{2}\partial\gamma. \tag{43}$$

## 4. The WZWN model coupled to two Koszul quartets and coset fields

In the previous section we saw how an $N=2$ "matter" system (the coset fields) could be coupled to a Koszul quartet such that an $N=4$ model resulted. We only needed the OPEs of the currents of the matter system. This reveals how to couple the WZWN model to these fields such that it becomes part of an $N=4$ model

(i) Use a first Koszul quartet denoted by $(b', c', \beta', \gamma')$ to construct a bona fide $N=2$ system for the WZWN model with currents $T^W, J^W, j^W, \tilde{B}^W$ [5]. This fixes the $\mu$ parameter of the first quartet to $\mu = 1$.

(ii) Couple this $N = 2$ system to a second Koszul quartet, denoted by $(b, c, \beta, \gamma)$, to obtain an $N = 4$ model in the same way as for the coset fields. The $\mu$ parameter of this Koszul quartet is arbitrary. Instead of coupling only to the second Koszul quartet we shall couple to the sum of the second Koszul multiplet and the coset fields. This combined system was discussed in the previous section and is what is needed below.

Thus we obtain the following $N = 4$ superconformal currents for the WZWN model coupled to coset fields and two Koszul quartets

$$\begin{aligned}
&T = \left(T^W + T^{K'}\right) + T^{\mathrm{co}} + T^K \quad \text{with } c_{TT} = 0,\\
&J_3 = \left(J^W + J^{K'}\right) + J^{\mathrm{co}} + J^K \quad \text{with } c_{JJ} = -22 - 3 - 6 - 3 = -34,\\
&G^+ = j_B = \left(j_B^W + j_B^{K'}\right) + j_B^{\mathrm{co}} + j_B^K,\\
&G^- = B = \left(\tilde{B}^W + B^{K'}\right) + B^{\mathrm{co}} + B^K,\\
&J^{++} = \gamma\left(J^W + J^{K'} + J^{\mathrm{co}} + \frac{1}{2}J^K\right) - c\left(j_B^W + j_B^{K'} + j_B^{\mathrm{co}} + \frac{1}{2}j_B^K\right) + x\partial\gamma,\\
&\tilde{G}^+ = c\left(T^W + T^{K'} + T^{\mathrm{co}} + \frac{1}{2}T^K\right) + \gamma\left(\tilde{B}^W + B^{K'} + B^{\mathrm{co}} + \frac{1}{2}B^K\right)\\
&\qquad - \mu\left(j_B^W + j_B^{K'} + j_B^{\mathrm{co}} + \frac{1}{2}j_B^K\right) - \partial\left(c\left(J^W + J^{K'} + J^{\mathrm{co}} + \frac{1}{2}J^K\right)\right) + y\partial^2 c,\\
&J^{--} = \beta, \qquad \tilde{G}^- = -b.
\end{aligned} \tag{44}$$

The current $J^{++,K}$ contains a term $x\partial\gamma$ while the current $\tilde{G}^+$ contains a term $y\partial^2 c$. The same analysis as performed for the coset fields shows that also these currents satisfy an $N = 4$ superconformal algebra. The only parameters to be fixed are the values of $x$ and $y$. We fix $x$ by requiring that the double poles with $\gamma$ in the numerator cancel in the following OPE

$$J^{++}(z)J_3(w) \sim -2\frac{J^{++}(w)}{z-w} + \mathcal{O}\frac{1}{(z-w)^2}. \tag{45}$$

We find

$$\begin{aligned}
&\left[\gamma\left(J^W + J^{K'} + J^{\mathrm{co}} + \frac{1}{2}J^K\right) - c\left(j_B^W + j_B^{K'} + j_B^{\mathrm{co}} + \frac{1}{2}j_B^K\right) + x\partial\gamma\right](z)\left[J^W + J^{K'} + J^{\mathrm{co}} + J^K\right](w)\\
&\quad \sim \frac{2x\gamma(w)}{(z-w)^2} + \gamma(z)\frac{[-22 - 3 - 6 - (\frac{1}{2} + 4 - \frac{1}{2})]}{(z-w)^2} + \cdots.
\end{aligned} \tag{46}$$

This yields the value

$$x = 17. \tag{47}$$

Confirmation is obtained from

$$J_3(z)J_3(w) \sim \frac{-34}{(z-w)^2}, \qquad J^{++}(z)J^{--}(w) \sim \frac{-x}{(z-w)^2} + \frac{J_3(w)}{z-w} \tag{48}$$

which reproduces $x = 17$.

Finally we complete the construction of the $N = 4$ WZNW model by determining the value of $y$. We consider the OPE $J_3(z)\tilde{G}^+(w) \sim \tilde{G}^+(w)/z - w$ and require that all terms of the form $c(w)/(z-w)^3$ cancel. We find the following contributions

$$
\begin{aligned}
&(-bc)(z)\big(bc\partial c + y\partial^2 c\big)(w) \; - (2\beta\gamma)(z)(2\beta\gamma\partial c + \partial\beta\gamma c)(w) \\
&\quad + \big(J^W + J^{K'} + J^{\mathrm{co}}\big)(z)\big(c\big(T^W + T^{K'} + T^{\mathrm{co}}\big)\big)(w) - \big(J^W + J^{K'} + J^{\mathrm{co}}\big)(z)\partial\big(c\big(J^W + J^{K'} + J^{\mathrm{co}}\big)\big)(w) \\
&\qquad \sim \big[1 + 2y + 2 + (-22 - 3 - 6) - 2(-22 - 3 - 6)\big]c(w)/(z-w)^3.
\end{aligned} \tag{49}
$$

Thus

$$
y = -17. \tag{50}
$$

As a check we determine the term with $\partial^2 c$ in $\tilde{G}^+$ from $J^{++}(z)G^-(w) \sim -\tilde{G}^+(w)/(z-w)$. We find

$$
\begin{aligned}
&\begin{bmatrix} \gamma(J^W + J^{K'} + J^{\mathrm{co}}) + \frac{1}{2}\gamma(-bc - 2\beta\gamma) \\ -c(j_B^W + j_B^{K'} + j_B^{\mathrm{co}}) + \frac{1}{2}cb\gamma + x\partial\gamma \end{bmatrix}(z)\big[\tilde{B}^W + B^{K'} + B^{\mathrm{co}} + 2\beta\partial c + c\partial\beta + \mu b\big](w) \\
&\quad \sim (cb\gamma - \beta\gamma\gamma + x\partial\gamma)(z)(2\beta\partial c + c\partial\beta)(w) \\
&\qquad - c(z)\big[j_B^W(z)\tilde{B}^W(w) + j_B^{K'}(z)B^{K'}(w) + j_B^{\mathrm{co}}(z)B^{\mathrm{co}}(w)\big] + \cdots \\
&\quad \sim \frac{3c(z)}{(z-w)^3} - \frac{2xc(w)}{(z-w)^3} - \frac{2x\partial c(w)}{(z-w)^2} - \frac{c(z)}{(z-w)^3}[-22 - 3 - 6] + \cdots.
\end{aligned} \tag{51}
$$

The triple poles cancel for $x = 17$, confirming again the result for $x$. Then also the double poles cancel, while from the simple poles we find that $\tilde{G}^+$ contains a term $-17\partial^2 c$. This yields again $y = -17$.

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