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The spin dependent odderon in the diquark model

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ABSTRACT

In this short note, we report a di-quark model calculation for the spin dependent odderon and demonstrate that the asymmetrical color source distribution in the transverse plane of a transversely polarized hadron plays an essential role in yielding the spin dependent odderon. This calculation confirms the earlier finding that the spin dependent odderon is closely related to the parton orbital angular momentum.

The large size of the observed transverse single spin asymmetries (SSAs) in high energy scattering experiments [1] has stimulated a lot of theoretical developments as it allows us to address some most fundamental aspects of QCD and gain more insight into the hadron structure as well. The various mechanisms beyond the naive parton model [2] have been proposed to explain the large SSAs [3–8]. The common feature of these mechanisms is that an imaginary phase required for the non-vanishing SSAs is generated by taking into account an additional gluon exchange between the active parton and the remanent part of the transversely polarized hadron. For a small angle scattering in the high energy limit, the spin independent cross section is dominated by the two t-channel gluon exchange at leading order, which can be viewed as a pomeron exchange. The three gluon exchange responsible for the spin dependent cross section is then naturally re-interpreted as an odderon exchange [9–12], which is a C-odd object. The contributions to SSAs from a such tri-gluon exchange have also been extensively studied in the context of the collinear twist-3 framework [13–18].

Apart from the conventional perturbative QCD description [19–22], an odderon exchange also can be formulated in the dipole approach [23] and in the Color Glass Condensate (CGC) framework [24,25]. According to the saturation model calculation [26], the odderon exchange is absent when an unpolarized target has uniform color source (valence quark) distribution. This observation motivated one of us to introduce the spin dependent odderon [27] by noticing the fact that the valence quark distribution is strongly distorted in the transverse plane of the transversely polarized target [28]. In a more recent work [29], it has been found that such spin dependent odderon is the only source of SSAs at small x, as the three dipole type T-odd gluon TMD in a transversely polarized target dominating small x dynamics are determined by the spin dependent odderon. We also note that the same subject has been studied in an earlier literature [30].

The objective of this short note is to demonstrate the relation between the spin dependent odderon and the distorted impact dependent parton distribution in a more transparent way. For this aim, we compute the SSA for a quark scattering off an onium that consists of one quark and one scalar di-quark. As we focus on the high energy limit, we formulate our calculation in a standard k_T factorization approach [31,32] (also often referred to as the high energy factorization), in which the cross section can be presented in terms of an impact factor involving the convolution with the wave function of the incoming hadron. In this work, we use the Brodsky–Hwang–Ma–Schmidt (BHMS) di-quark model [7,33] to describe the transversely polarized projectile. Since the BHMS model evidently incorporates the parton orbital angular momentum effect that leads to an asymmetric impact dependent parton distribution inside a transversely polarized hadron [28], we use it to determine the light cone wave function of the onium. The current work can be viewed as the one more effort to address the topical issue: the interplay of spin physics and saturation physics [34].

We start by fixing the relevant kinematical variables for the process under consideration. In the di-quark model, the partonic subprocess is expressed as the following,

$$N(P, S_\perp) + Q(p) \to q(k) + S(r) + Q(p'), \tag{1}$$

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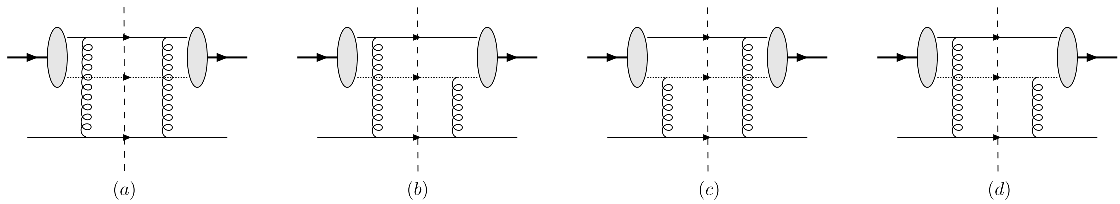

Fig. 1. Pomeron exchange between quark and onium. The solid line represent quark, while the doted line stands for the scalar di-quark.

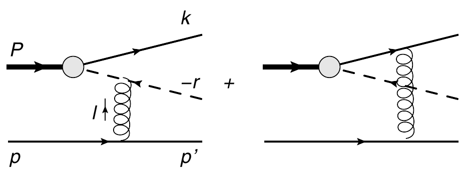

Fig. 2. Feynman diagrams contributing to $\mathcal{T}^{(1)}$.

where the polarized nucleon N is described within the quark–scalar diquark BHMS model, see the Chap. 4 of Ref. [33]. Q, q and S represent the incoming quark target, the produced quark and the scalar di-quark, respectively.

We parameterize momenta of particles using Sudakov's light-cone vectors p_1 and p_2 ($p_1^2 = 0 = p_2^2$, $2p_1 \cdot p_2 = s$). The momenta of incoming nucleon P with mass M and transverse polarization vector $S_\perp$, the incoming massless quark target p, the produced quark k with mass m_q and the helicity $\lambda_k = \pm$ and the scalar di-quark r with mass m_s have the forms

$$
\begin{aligned}
P &= p_1 + \frac{M^2}{s} p_2, \qquad p = p_2\,, \\
k &= z\,p_1 - \frac{(zl_\perp + v_\perp)^2 - m_q^2}{sz}\, p_2 + zl_\perp + v_\perp\,, \\
r &= \bar{z}\,p_1 - \frac{(\bar{z}l_\perp - v_\perp)^2 - m_s^2}{s\bar{z}}\, p_2 + \bar{z}l_\perp - v_\perp\,,
\end{aligned}
\tag{2}
$$

where $v_\perp$ is the relative transverse momentum between the produced quark and diquark, $\bar{z} = 1 - z$ and the momentum transfer in the t-channel $\Delta = p - p'$ is in high-energy kinematics mostly the transverse vector $l_\perp$, with suppressed like $\mathcal{O}(1/s)$ component along p_2 Sudakov vector.

It is instructive to first review how to compute the spin independent cross section of the onium–quark scattering with a single gluon exchange as shown in Fig. 1. Using the Feynman rules given in the Appendix I and applying the eikonal approximation to both quark and diquark lines, it is straightforward to derive the $\mathcal{T}^{(1)}$ matrix for the scattering process with single gluon exchange in momentum space, corresponding to the diagrams in the Fig. 2,

$$
\begin{aligned}
\mathcal{T}^{(1)} &= 2s\lambda_s g^2 t^a_{c_k c_r} t^a_{c_{p'} c_p} \\
&\quad \left[\frac{z(1-z)\bar{u}(z, zl_\perp + v_\perp, \lambda_k) u(P, S_\perp)}{(zl_\perp + v_\perp)^2 - \tilde{M}^2} \right. \\
&\quad \left. - \frac{z(1-z)\bar{u}(z, v_\perp - (1-z)l_\perp, \lambda_k) u(P, S_\perp)}{(v_\perp - (1-z)l_\perp)^2 - \tilde{M}^2} \right] \frac{\delta_{\lambda_p \lambda_{p'}}}{l_\perp^2}\,,
\end{aligned}
\tag{3}
$$

where λ_p and $\lambda_{p'}$ are helicities of the scattered quark. Note that $\tilde{M}^2 = \bar{z}m_q^2 + zm_s^2 - z\bar{z}M^2$ is positive. For simplicity of notation, we show as an argument of quark spinor $\bar{u}(k, \lambda_k)$ only the momentum component along Sudakov vector p_1 and the transverse component. The $\mathcal{T}^{(1)}$ scattering amplitude expressed as a convolution in the impact parameter $x_\perp$ (conjugate to the transverse momentum $v_\perp$) is given by,

$$
\begin{aligned}
\mathcal{T}^{(1)} &= 2sg^2 t^a_{c_k c_r} t^a_{c_{p'} c_p} \\
&\quad \int d^2x_\perp \Psi(x_\perp, z) e^{ix_\perp \cdot (zl_\perp + v_\perp)} \left(1 - e^{-ix_\perp \cdot l_\perp}\right) \frac{\delta_{\lambda_p \lambda_{p'}}}{l_\perp^2}\,,
\end{aligned}
\tag{4}
$$

with

$$
\Psi(x_\perp, z) = \lambda_s \int \frac{d^2v_\perp}{(2\pi)^2} e^{-ix_\perp \cdot v_\perp} \frac{z(1-z)\bar{u}(z, v_\perp, \lambda_k) u(P, S_\perp)}{v_\perp^2 - \tilde{M}^2}\,,
\tag{5}
$$

which is the nucleon wave function in impact (coordinate) representation. We proceed to compute the product of spinors in Eq. (5) using the spinor technique of Ref. [35]. The result expressed in terms of two orthogonal vectors of a basis in the transverse plane,

$$
\tilde{e}_1^\mu = S_\perp^\mu\,, \qquad \tilde{e}_2^\mu = \epsilon_\perp^{S_\perp \mu}\,, \qquad \tilde{e}_1^2 = -1 = \tilde{e}_2^2\,, \quad \tilde{e}_1 \cdot \tilde{e}_2 = 0\,,
\tag{6}
$$

takes the form

$$
\begin{aligned}
&\bar{u}(z, v_\perp, \lambda_k) u(P, \lambda_P, S_\perp) \\
&\quad = \delta_{\lambda_k +} \frac{1}{\sqrt{2z}} \left[Mz + m_q - (\tilde{e}_1 + i\tilde{e}_2) \cdot v_\perp \right] \\
&\qquad + \delta_{\lambda_k -} \frac{1}{\sqrt{2z}} \left[Mz + m_q + (\tilde{e}_1 - i\tilde{e}_2) \cdot v_\perp \right]\,,
\end{aligned}
\tag{7}
$$

where $\epsilon_\perp^{\nu\mu} = \frac{2}{s} \epsilon^{p_1 p_2 \nu\mu}$. After carrying out the integration over $v_\perp$, one can rewrite the wave function Eq. (5) as,

$$
\begin{aligned}
\Psi(x_\perp, z) &= \delta_{\lambda_k +} \frac{(-\lambda_s \sqrt{z\bar{z}})}{2\pi\sqrt{2}} \left[\left(Mz + m_q\right) K_0(\tilde{M}|x_\perp|) \right. \\
&\qquad \left. - \left(\tilde{e}_1 + i\tilde{e}_2\right) \cdot x_\perp \frac{i\tilde{M}}{|x_\perp|} K_1(\tilde{M}|x_\perp|) \right] \\
&\quad + \delta_{\lambda_k -} \frac{(-\lambda_s \sqrt{z\bar{z}})}{2\pi\sqrt{2}} \left[\left(Mz + m_q\right) K_0(\tilde{M}|x_\perp|) \right. \\
&\qquad \left. + \left(\tilde{e}_1 - i\tilde{e}_2\right) \cdot x_\perp \frac{i\tilde{M}}{|x_\perp|} K_1(\tilde{M}|x_\perp|) \right]\,,
\end{aligned}
\tag{8}
$$

where $|x_\perp| = \sqrt{-x_\perp^2}$. This leads to the following expression for the wave function squared,

$$
\begin{aligned}
&\sum_{\lambda_k = \pm} |\Psi(x_\perp, z)|^2 \\
&\quad = \lambda_s^2 \frac{z\bar{z}^2}{(2\pi)^2} \left[\left(Mz + m_q\right)^2 K_0^2(\tilde{M}|x_\perp|) + \tilde{M}^2 K_1^2(\tilde{M}|x_\perp|) \right. \\
&\qquad \left. + 2\left(Mz + m_q\right) \tilde{e}_2 \cdot x_\perp K_0(\tilde{M}|x_\perp|) \frac{\tilde{M}}{|x_\perp|} K_1(\tilde{M}|x_\perp|) \right]\,.
\end{aligned}
\tag{9}
$$

With the above results one can compute the cross section which reads,

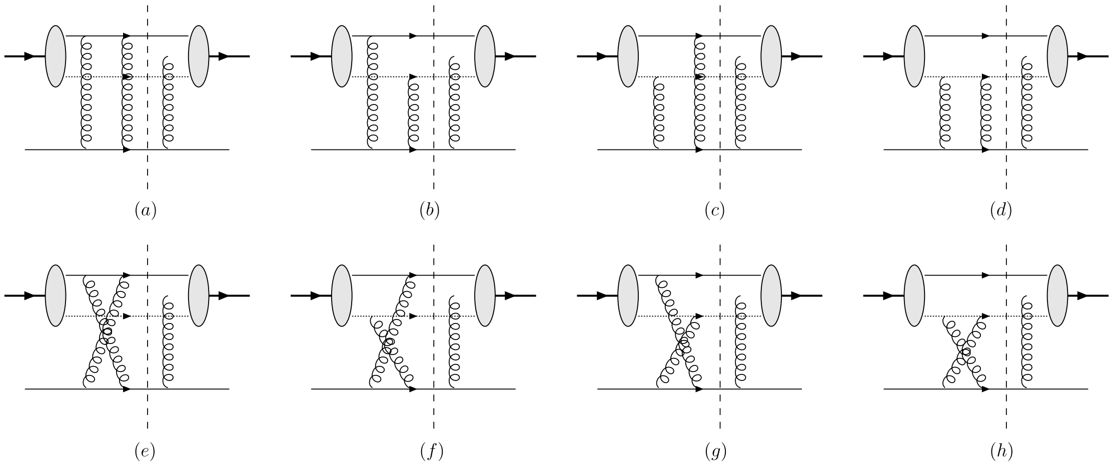

Fig. 3. Odderon exchange between quark and onium. The conjugate diagrams are not shown here.

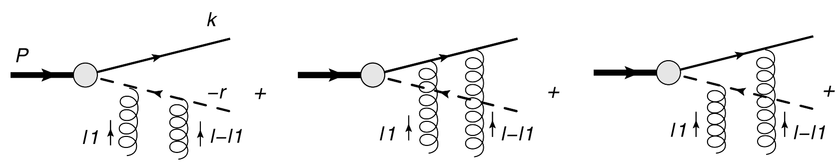

Fig. 4. Three Feynman diagrams with two gluons contributing to the transition vertex of proton into quark and diquark. The remaining 3 diagrams are obtained by the interchange of gluonic lines $l_1 \leftrightarrow l - l_1$ in the above diagrams.

$$\frac{d\sigma}{d^2 l_\perp} = \frac{C_F}{2\pi}\frac{\alpha_s^2}{(l_\perp^2)^2}\int \frac{dz}{z\bar{z}} d^2x_\perp \sum_{\lambda_k=\pm} |\Psi(x_\perp, z)|^2 (1 - e^{-ix_\perp l_\perp})(1 - e^{ix_\perp l_\perp}) \,. \tag{10}$$

If one integrates over the azimuthal angle of $x_\perp$ in Eq. (10), the spin dependent term in Eq. (9) drops out. Therefore, to derive the spin dependent cross section, one has to take into account one additional gluon exchange as shown in Fig. 3. Following the similar procedure as in calculation of $\mathcal{T}^{(1)}$, we compute the $\mathcal{T}^{(2)}$ scattering amplitude for the two gluon exchange in the symmetric 8_S color octet state. In the k_T-factorization approach, the $\mathcal{T}^{(2)}$ amplitude is expressed as a factorized convolution in transverse momenta of t-channel gluons

$$\mathcal{T}^{(2)} = \frac{1}{i\,2!}\left(\frac{2}{s}\right)^2 \frac{s}{2}\int \frac{d^2 l_{1\perp}}{(2\pi)^4}\frac{(-i)^2}{l_{1\perp}^2 (l_\perp - l_{1\perp})^2} \left[\int d\beta_1\, \mathcal{S}^{(2)}(P \to q\, s)_{\mu\nu} p_2^\mu p_2^\nu\right] \left[\int d\alpha_1\, \mathcal{S}^{(2)}(p \to p')_{\mu'\nu'} p_1^{\mu'} p_1^{\nu'}\right]. \tag{11}$$

It involves $\mathcal{S}^{(2)}(P \to q\,s)$-matrix for transition of a proton into quark and diquark with two gluons in 8_S state, having longitudinal polarizations $\sim p_2^\mu$, which is integrated over β_1 Sudakov component of l_1 momentum. Similarly, $\mathcal{S}^{(2)}(p \to p')_{\mu'\nu'} p_1^{\mu'} p_1^{\nu'}$ is the $\mathcal{S}$-matrix for transition of target quark p into outgoing quark p'. It is in turn integrated over α_1 Sudakov component of l_1 momentum. The combinatorial factor $1/(2!)$ assures that the $\mathcal{S}^{(2)}(P \to q\,s)$-matrix element is represented as a sum of six Feynman diagrams shown in Fig. 4. The expression for $\mathcal{T}^{(2)}$ has the form,

$$\mathcal{T}^{(2)} = -\frac{isg^4(N_c^2-4)t^a_{c_k c_r} t^a_{c_{p'} c_p}\delta_{\lambda_p\lambda_{p'}}}{2^4\pi^2 N_c} \int \frac{d^2 l_{1\perp}}{l_{1\perp}^2 (l_\perp - l_{1\perp})^2} d^2x_\perp\, \Psi(x_\perp, z)\, e^{ix_\perp(v_\perp + z l_\perp)} (1 - e^{-ix_\perp l_{1\perp}})(1 - e^{ix_\perp (l_{1\perp} - l_\perp)}) \,. \tag{12}$$

The corresponding cross section reads,

$$\frac{d\sigma}{d^2 l_\perp} = \frac{i\alpha_s^3 C_F (N_c^2 - 4)}{2^3\pi^2 N_c} \int \frac{dz\, d^2x_\perp}{z\bar{z}} \sum_{\lambda_k=\pm} \frac{|\Psi(x_\perp, z)|^2\, d^2 l_{1\perp}}{l_{1\perp}^2 (l_\perp - l_{1\perp})^2 l_\perp^2} (1 - e^{-ix_\perp l_\perp})(1 - e^{ix_\perp l_{1\perp}})(1 - e^{-ix_\perp(l_{1\perp} - l_\perp)}) + c.c.\,, \tag{13}$$

with $|\Psi(x_\perp, z)|^2$ given by Eq. (9). We proceed to integrate out $l_{1\perp}$ by the Feynman parameter method and obtain the expression ($\bar{a} = 1 - a$),

$$\int \frac{d^2 l_{1\perp}}{(2\pi)^2}\frac{1}{l_{1\perp}^2}\frac{1}{(l_\perp - l_{1\perp})^2}\left(1 - e^{il_{1\perp}\cdot x_\perp}\right)\left(1 - e^{i(l_\perp - l_{1\perp})\cdot x_\perp}\right) = -\frac{1}{4\pi l_\perp^2}\int_0^1 \frac{da}{a\bar{a}}\Big\{1 + e^{il_\perp\cdot x_\perp} - \sqrt{a\bar{a}x_\perp^2 l_\perp^2}\, K_1(\sqrt{a\bar{a}x_\perp^2 l_\perp^2}\,) \left(e^{ial_\perp\cdot x_\perp} + e^{i\bar{a}l_\perp\cdot x_\perp}\right)\Big\}, \tag{14}$$

where K_1 is the modified Bessel function of the second kind. For a very small onium $x_\perp << 1/l_\perp$, one can make the following approximation,

$$\sqrt{a\bar{a}x_\perp^2 l_\perp^2}\, K_1(\sqrt{a\bar{a}x_\perp^2 l_\perp^2}\,) \approx 1\,, \tag{15}$$

and to perform integration over Feynman parameter a

$$\int_0^1 \frac{da}{a\bar{a}}\left\{1 + e^{il_\perp\cdot x_\perp} - e^{ial_\perp\cdot x_\perp} - e^{i\bar{a}l_\perp\cdot x_\perp}\right\}$$

$$\approx \frac{(x_\perp \cdot l_\perp)^2}{2}\,(1+e^{ix_\perp \cdot l_\perp})\,, \tag{16}$$

The cross section is simplified as,

$$\frac{d\sigma}{d^2 l_\perp} = \frac{\alpha_s^3 C_F\,(N_c^2-4)}{2^3 \pi N_c} \int \frac{dz\, d^2 x_\perp}{z\bar{z}} \sum_{\lambda_k=\pm} |\Psi(x_\perp, z)|^2 \frac{2(x_\perp \cdot l_\perp)^2}{l_\perp^4}\, \sin(x_\perp \cdot l_\perp)\,. \tag{17}$$

If $|\Psi(x_\perp, z)|^2$ were an azimuthal symmetric wave function, the integration over the angle of $x_\perp$ would lead to an vanishing cross section. However, it has been realized long time ago that the parton distribution in the transverse plane of a transversely polarized target is strongly distorted [28]. It can be clearly seen from the expressions Eq. (9) that this is indeed the case in the quark–scalar diquark model of a nucleon. The term in Eq. (9) involving $\tilde{e}_2 \cdot x_\perp$ gives after angular integration in Eq. (17)

$$\int_0^{2\pi} d\phi_{x_\perp}\, \tilde{e}_2 \cdot x_\perp (x_\perp \cdot l_\perp)^2 \sin(x_\perp \cdot l_\perp)$$

$$= \tilde{e}_2 \cdot l_\perp\, 2\pi \left[J_3(|x_\perp||l_\perp|) - \frac{3 J_2(|x_\perp||l_\perp|)}{|x_\perp||l_\perp|} \right] |x_\perp|^3 |l_\perp|\,. \tag{18}$$

With the help of this formula, one obtains the spin dependent cross section,

$$\frac{d\Delta\sigma}{d^2 l_\perp} = -\tilde{e}_2 \cdot l_\perp \frac{\lambda_s^2 \alpha_s^3 C_F\,(N_c^2-4)}{(2\pi)^2 |l_\perp|^2 N_c} \int_0^1 dz \frac{\bar{z}\left(Mz+m_q\right)}{4\tilde{M}^4}\,, \tag{19}$$

where we ignored the terms suppressed by the power of $|l_\perp|/M$. This approximation is justified for a small onium.

On the other hand, the spin dependent cross section for the quark initiated jet production in the backward region reads [27],

$$\frac{d^2\Delta\sigma}{d^2 l_\perp} = F_{x_g}(l_\perp^2) + \frac{1}{M} \epsilon_\perp^{ij} l_\perp^j S_\perp^i\, O_{1T,x_g}^{\perp}(l_\perp^2)\,. \tag{20}$$

Here F_{x_g} is the usual unintegrated gluon distribution, while $O_{1T}^{\perp}$ is the spin dependent odderon introduced in Ref. [27]. One thus can extract,

$$O_{1T,x_g}^{\perp}(l_\perp^2) = -\frac{\lambda_s^2 \alpha_s^3 C_F\,(N_c^2-4)}{(2\pi)^2 |l_\perp|^2 N_c} \int_0^1 dz \frac{\bar{z} M\left(Mz+m_q\right)}{4\tilde{M}^4}\,, \tag{21}$$

which is the main result of our short note. We refrain from performing the integration upon z as it is sufficient to clearly demonstrate the relation between the spin dependent odderon and the asymmetric color source distribution in the transverse plane of the polarized target at this step. Before summarizing the paper, few comments are in order. Though it is clear that the existence of the spin dependent odderon relies on the polarization dependent part of the wave function which is essentially the GPD E, one should note that the exact relations between the odderon and the GPD E are different in the MV model [27] and the di-quark model. In general, such relations are model dependent. Thus, it would be very interesting to work out a model independent relation between SSAs and the GPD E in the future.

At this point, we would like to comment on the phenomenological implications of our work. Due to the C-odd nature of the odderon exchange, it doesn't contribution to the scattering between the onium and gluon. This implies that the SSAs caused by the spin dependent odderon disappear at mid-rapidity which is dominated by gluon. Therefore, we anticipate that the size of SSAs rises in the backward region of the transversely polarized target. This observation seems to be consistent with the measurement performed at RHIC [36].

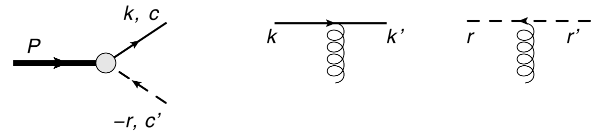

Fig. 5. Feynman rules in the diquark model.

To summarize, we calculate the SSA in the onium–quark scattering by taking into account an odderon exchange. It is shown that the asymmetric impact parameter dependent parton distribution inside a transversely polarized hadron computed from the diquark model is critical for having a non-vanishing odderon exchange. This calculation confirms that such a spin dependent odderon gives the potential access to the parton orbital angular momentum. This is also in agreement with the earlier observations that SSAs phenomenology is closely related to the parton orbital angular momentum [37,38].

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Appendix I

Based on Ref. [33], in the diquark model, the interaction between the nucleon, the quark, and the scalar diquark is described by the following Feynman rules for the nucleon–quark–diquark vertex, quark–gluon vertex, diquark–gluon vertex, and the diquark and quark propagators in Fig. 5, respectively,

$$i\lambda_s \bar{u}(k,\lambda_k) u(P, S_\perp)\delta^{cc'}, \quad -igt^a\gamma^\mu, \quad -igt^a(r+r')^\mu,$$
$$\frac{i}{r^2-m_s^2+i\epsilon}, \quad \frac{i(\hat{k}+m_q)}{k^2-m_q^2+i\epsilon}\,, \tag{22}$$

and the standard scalar diquark propagator, quark propagator and gluon propagator in the Feynman gauge,

$$\frac{i}{r^2-m_s^2+i\epsilon}, \quad \frac{i(\hat{k}+m_q)}{k^2-m_q^2+i\epsilon}\,, \quad \frac{-ig^{\mu\nu}\delta^{cc'}}{k^2+i\epsilon}\,, \tag{23}$$

where the subscripts c and c' are color indices in the adjoint representation and $\hat{k} = k_\mu \gamma^\mu$, t^a are $SU(N)$ gauge group generators in the fundamental representation.

Appendix II

In this appendix, we present an alternative way of determining the wave function in the diquark model. It is well known that the impact parameter dependent parton distribution can be parameterized as [28],

$$f_q(z, b_{\perp,q}) = \mathcal{H}_q(z, b_{\perp,q}^2) + \frac{1}{M}\epsilon_\perp^{ij} b_{\perp,q}^i S_\perp^j \frac{\partial \mathcal{E}_q(z, b_{\perp,q}^2)}{\partial b_{\perp,q}^2}\,, \tag{24}$$

$$f_s(z, b_{\perp,s}) = \mathcal{H}_s(z, b_{\perp,s}^2) + \frac{1}{M}\epsilon_\perp^{ij} b_{\perp,s}^i S_\perp^j \frac{\partial \mathcal{E}_s(z, b_{\perp,s}^2)}{\partial b_{\perp,s}^2}\,, \tag{25}$$

where $\mathcal{H}$ and $\mathcal{E}$ are the Fourier transform of the normal GPD H and E. The subscript q and s indicate the quark and the scalar diquark respectively. Using the relation,

$$b_{\perp,q} - b_{\perp,s} = x_\perp \ , \ \ zb_{\perp,q} + (1-z)b_{\perp,s} = 0 \, , \tag{26}$$

and the fact that light cone wave function and GPDs are normalized in the different way, one has,

$$\begin{aligned} &\int \frac{d^2x_\perp}{2\pi}\frac{dz}{2z(1-z)}|\Psi(x_\perp,z)|^2 \\ &= \int d^2[(1-z)x_\perp]dz\Big\{\mathcal{H}_q(z,(1-z)^2x_\perp^2) \\ &\quad + \frac{\epsilon_\perp^{ij}x_\perp^i S_\perp^j}{M(1-z)}\frac{\partial\mathcal{E}_q(z,(1-z)^2x_\perp^2)}{\partial x_\perp^2}\Big\} \, , \end{aligned} \tag{27}$$

where $\frac{d^2x_\perp}{2\pi}\frac{dz}{2z(1-z)}$ is the two particle phase space factor. The expression for the GPD E in momentum space derived in the diquark model has been given in Ref. [39]. By taking Fourier transform, one obtains,

$$\mathcal{E}_q(z,(1-z)^2x_\perp^2) = \frac{\lambda_s^2}{(2\pi)^3}(m_q + zM)MK_0^2(\tilde{M}|x_\perp|) \, . \tag{28}$$

This leads to the spin dependent part of the wave function squared,

$$\begin{aligned} \Delta|\Psi(x_\perp,z)|^2 = &-\epsilon_\perp^{ij}x_\perp^i S_\perp^j\frac{2\lambda_s^2}{(2\pi)^2}(m_q+zM)(1-z)^2 z\frac{\tilde{M}}{|x_\perp|} \\ &K_0(\tilde{M}|x_\perp|)K_1(\tilde{M}|x_\perp|) \, , \end{aligned} \tag{29}$$

which is in complete agreement with Eq. (9) from the direct calculation. The same result is also found for the polarization independent piece of the wave function squared. The derivation presented here is essentially based on the observation that both the wave function squared and the GPDs in the position space have the clear probability interpretation.

References

[1] G. Bunce, et al., Phys. Rev. Lett. 36 (1976) 1113;
D.L. Adams, et al., E704 Collaboration, Phys. Lett. B 264 (1991) 462;
B.I. Abelev, et al., STAR Collaboration, Phys. Rev. Lett. 101 (2008) 222001;
S.S. Adler, et al., PHENIX Collaboration, Phys. Rev. Lett. 95 (2005) 202001;
I. Arsene, et al., BRAHMS Collaboration, Phys. Rev. Lett. 101 (2008) 042001.
[2] G.L. Kane, J. Pumplin, W. Repko, Phys. Rev. Lett. 41 (1978) 1689.
[3] D.W. Sivers, Phys. Rev. D 43 (1991) 261.
[4] J.C. Collins, Nucl. Phys. B 396 (1993) 161.
[5] A.V. Efremov, O.V. Teryaev, Sov. J. Nucl. Phys. 36 (1982) 140, Yad. Fiz. 36 (1982) 242;
A.V. Efremov, O.V. Teryaev, Phys. Lett. B 150 (1985) 383.
[6] J.-w. Qiu, G.F. Sterman, Phys. Rev. Lett. 67 (1991) 2264;
J.-w. Qiu, G.F. Sterman, Nucl. Phys. B 378 (1992) 52.
[7] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99, http://dx.doi.org/10.1016/S0370-2693(02)01320-5, arXiv:hep-ph/0201296.
[8] J.C. Collins, Phys. Lett. B 536 (2002) 43.
[9] M.G. Ryskin, Sov. J. Nucl. Phys. 46 (1987) 337, Yad. Fiz. 46 (1987) 611.
[10] N.H. Buttimore, B.Z. Kopeliovich, E. Leader, J. Soffer, T.L. Trueman, Phys. Rev. D 59 (1999) 114010.
[11] E. Leader, T.L. Trueman, Phys. Rev. D 61 (2000) 077504.
[12] A. Ahmedov, I.V. Akushevich, E.A. Kuraev, P.G. Ratcliffe, Eur. Phys. J. C 11 (1999) 703;
A. Ahmedov, E.N. Antonov, E. Bartos, E.A. Kuraev, E. Zemlyanaya, J. Phys. G 23 (2003) 521, J. Phys. G 29 (2003) 521.
[13] X.-D. Ji, Phys. Lett. B 289 (1992) 137.
[14] H. Beppu, Y. Koike, K. Tanaka, S. Yoshida, Phys. Rev. D 82 (2010) 054005.
[15] Z.-B. Kang, J.-W. Qiu, Phys. Rev. D 78 (2008) 034005.
[16] Z.-B. Kang, J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Rev. D 78 (2008) 114013.
[17] Y. Koike, S. Yoshida, Phys. Rev. D 84 (2011) 014026;
Y. Koike, S. Yoshida, Phys. Rev. D 85 (2012) 034030.
[18] A. Schafer, J. Zhou, arXiv:1308.4961 [hep-ph].
[19] J. Bartels, Nucl. Phys. B 175 (1980) 365;
J. Kwiecinski, M. Praszalowicz, Phys. Lett. B 94 (1980) 413.
[20] R. Engel, D.Y. Ivanov, R. Kirschner, L. Szymanowski, Eur. Phys. J. C 4 (1998) 93, http://dx.doi.org/10.1007/s100520050188, arXiv:hep-ph/9707362.
[21] J. Bartels, L.N. Lipatov, G.P. Vacca, Phys. Lett. B 477 (2000) 178.
[22] P. Hagler, B. Pire, L. Szymanowski, O.V. Teryaev, Eur. Phys. J. C 26 (2002) 261, http://dx.doi.org/10.1140/epjc/s2002-01054-9, arXiv:hep-ph/0207224;
B. Pire, F. Schwennsen, L. Szymanowski, S. Wallon, Phys. Rev. D 78 (2008) 094009;
A. Bzdak, L. Motyka, L. Szymanowski, J.-R. Cudell, Phys. Rev. D 75 (2007) 094023.
[23] Y.V. Kovchegov, L. Szymanowski, S. Wallon, Phys. Lett. B 586 (2004) 267.
[24] Y. Hatta, E. Iancu, K. Itakura, L. McLerran, Nucl. Phys. A 760 (2005) 172.
[25] S. Jeon, R. Venugopalan, Phys. Rev. D 70 (2004) 105012;
S. Jeon, R. Venugopalan, Phys. Rev. D 71 (2005) 125003.
[26] Y.V. Kovchegov, M.D. Sievert, Phys. Rev. D 86 (2012) 034028;
Y.V. Kovchegov, M.D. Sievert, Phys. Rev. D 86 (2012) 079906 (Erratum).
[27] J. Zhou, Phys. Rev. D 89 (7) (2014) 074050, http://dx.doi.org/10.1103/PhysRevD.89.074050, arXiv:1308.5912 [hep-ph].
[28] M. Burkardt, Phys. Rev. D 62 (2000) 071503;
M. Burkardt, Phys. Rev. D 66 (2002) 119903 (Erratum);
M. Burkardt, Int. J. Mod. Phys. A 18 (2003) 173.
[29] D. Boer, M.G. Echevarria, P. Mulders, J. Zhou, arXiv:1511.03485 [hep-ph].
[30] B.G. Zakharov, Sov. J. Nucl. Phys. 49 (1989) 860, Yad. Fiz. 49 (1989) 1386.
[31] S. Catani, M. Ciafaloni, F. Hautmann, Nucl. Phys. B 366 (1991) 135, http://dx.doi.org/10.1016/0550-3213(91)90055-3.
[32] J.C. Collins, R.K. Ellis, Nucl. Phys. B 360 (1991) 3, http://dx.doi.org/10.1016/0550-3213(91)90288-9.
[33] S.J. Brodsky, D.S. Hwang, B.Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311, http://dx.doi.org/10.1016/S0550-3213(00)00626-X, arXiv:hep-th/0003082.
[34] D. Boer, A. Dumitru, A. Hayashigaki, Phys. Rev. D 74 (2006) 074018;
F. Dominguez, B.W. Xiao, F. Yuan, Phys. Rev. Lett. 106 (2011) 022301;
F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan, Phys. Rev. D 83 (2011) 105005;
A. Metz, J. Zhou, Phys. Rev. D 84 (2011) 051503;
F. Dominguez, J.W. Qiu, B.W. Xiao, F. Yuan, Phys. Rev. D 85 (2012) 045003;
Z.B. Kang, F. Yuan, Phys. Rev. D 84 (2011) 034019;
Z.B. Kang, B.W. Xiao, Phys. Rev. D 87 (3) (2013) 034038;
Y.V. Kovchegov, M.D. Sievert, Phys. Rev. D 89 (5) (2014) 054035;
Y.V. Kovchegov, M.D. Sievert, Nucl. Phys. B 903 (2016) 164;
Y.V. Kovchegov, D. Pitonyak, M.D. Sievert, J. High Energy Phys. 1601 (2016) 072;
E. Akcakaya, A. Schäfer, J. Zhou, Phys. Rev. D 87 (5) (2013) 054010;
P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, J. High Energy Phys. 1509 (2015) 106;
A. Schäfer, J. Zhou, Phys. Rev. D 88 (7) (2013) 074012;
A. Schäfer, J. Zhou, Phys. Rev. D 90 (3) (2014) 034016;
A. Schäfer, J. Zhou, Phys. Rev. D 90 (9) (2014) 094012;
A. Schäfer, J. Zhou, arXiv:1308.4961 [hep-ph];
J. Zhou, Phys. Rev. D 92 (1) (2015) 014034;
J. Zhou, arXiv:1603.07426 [hep-ph];
Y. Hatta, B.W. Xiao, F. Yuan, arXiv:1601.01585 [hep-ph].
[35] R. Kleiss, Z. Phys. C 33 (1987) 433, http://dx.doi.org/10.1007/BF01552550.
[36] L.C. Bland, et al., AnDY Collaboration, Phys. Lett. B 750 (2015) 660, http://dx.doi.org/10.1016/j.physletb.2015.10.001, arXiv:1304.1454 [hep-ex].
[37] C. Boros, Z.T. Liang, T.C. Meng, Phys. Rev. Lett. 70 (1993) 1751, http://dx.doi.org/10.1103/PhysRevLett.70.1751.
[38] M. Burkardt, Phys. Rev. D 66 (2002) 114005, http://dx.doi.org/10.1103/PhysRevD.66.114005, arXiv:hep-ph/0209179.
[39] S. Meissner, A. Metz, K. Goeke, Phys. Rev. D 76 (2007) 034002, http://dx.doi.org/10.1103/PhysRevD.76.034002, arXiv:hep-ph/0703176.