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## sQGP as hCFT

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## ABSTRACT

We examine the proposal to make quantitative comparisons between the strongly coupled quark–gluon plasma and holographic descriptions of conformal field theory. In this Letter, we calculate corrections to certain transport coefficients appearing in second-order hydrodynamics from higher curvature terms to the dual gravity theory. We also clarify how these results might be consistently applied in comparisons with the sQGP.

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## 1. Introduction

Recent experimental results from the Relativistic Heavy Ion Collider (RHIC) have revealed a new phase of nuclear matter, known as the strongly coupled quark–gluon plasma (sQGP) [1]. At the same time, the AdS/CFT correspondence has matured into a powerful tool to study thermal and hydrodynamic properties of strongly coupled gauge theories [2]. Even though QCD is not (yet) a gauge theory which has a controllable description in the framework of a rigorous gauge theory/string theory correspondence, some of its properties just above the deconfinement phase transition are remarkably similar to those found for plasmas in holographic conformal field theory (hCFT) [3,4]. This hints that certain aspects of the physics may be universal and so may be accessible in a general AdS/CFT framework. The canonical model is most notably the  $\mathcal{N} = 4$   $SU(N)$  supersymmetric Yang–Mills (SYM) theory in the planar limit and for large 't Hooft coupling [5]. The holographic dual for this and a large class of superconformal gauge theories is simply Einstein gravity in  $AdS_5$  [6]. By adding higher curvature interactions to the dual gravity theory, one expands the range of physical characteristics of the gauge theory and in [7] it was suggested that this may provide a phenomenological approach for quantitative comparisons between hCFT plasmas and the sQGP. An interesting step in this direction was made in [8].

In this Letter, we further examine the proposal for such quantitative comparisons. First in Section 2, we briefly describe the framework of the dual gravity theory with higher curvature corrections and also present various results for thermal and hydrodynamic properties of the corresponding conformal plasma. While much of these results are collected from the previous literature,

the  $R^2$  and  $R^3$  corrections to the relaxation time  $\tau_{\Pi}$  and second-order transport coefficient  $\lambda_1$  are new. As the calculations are straightforward, we only present results here. In Section 3, we translate the results from a description in terms of the couplings in the dual gravity theory to one in terms of physical parameters which directly characterize the underlying CFT. Further, we discuss how these results can be consistently applied in comparing the results of the holographic calculations to data for the sQGP. In particular, we distinguish scenarios with and without exactly marginal couplings.

## 2. Holographic conformal hydrodynamics

Following [7], we consider the holographic description of a strongly coupled CFT with the following higher curvature corrections:

$$I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} \left[ \frac{12}{L^2} + R + L^2 \alpha_1 C_{abcd} C^{abcd} + L^4 \alpha_2 C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab} + L^6 \alpha_3 W(C) \right], \quad (2.1)$$

where  $C_{abcd}$  is a five-dimensional Weyl tensor.  $W(C) \sim C^4$  is a particular quartic contraction of  $C_{abcd}$  which naturally arises in type IIB supergravity [6]. We emphasize that we assume that  $\alpha_n \ll 1$  which allows us to treat the higher curvature couplings perturbatively. In the following, we work to linear order in  $\alpha_{2,3}$  while keeping corrections to second order in  $\alpha_1$ . We discuss the rationale for this approach in Section 3.

In general, in constructing the gravitational action (2.1), one might have added many more higher curvature interactions, however, within the present perturbative framework, most of these can be removed by field redefinitions without affecting the final physical results, as explained in detail in [7]. Hence our perturbative

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results are completely general for a gravitational action with interactions quadratic and cubic in curvatures.<sup>1</sup> In fact, in complete generality, there are five independent interactions that would appear at order  $C^4$  [10]. Hence our analysis is specialized at this order by focusing on the particular supergravity term in (2.1).

A regular black brane solution describes the equilibrium thermodynamics of the CFT plasma. It is straightforward to incorporate the higher curvature corrections to the Einstein equations [9,11,12] and to order in  $\mathcal{O}(\alpha_1^2, \alpha_2, \alpha_3)$ , the equilibrium pressure is given by

$$P = \frac{\pi^4}{2} \frac{L^3}{\ell_p^3} T^4 (1 + 18\alpha_1 + 24\alpha_1^2 + 24\alpha_2 + 15\alpha_3). \quad (2.2)$$

This expression can also be related to the energy density  $\varepsilon$  or entropy density  $s$ , using standard relations that apply for any CFT, i.e.,  $\varepsilon = 3P$  and  $\varepsilon = \frac{3}{4}Ts$  (in the absence of a chemical potential).

The shear viscosity  $\eta$  and the relaxation time  $\tau_\Pi$  of the CFT plasma can be extracted from the two-point boundary stress-energy correlation functions in the black brane background [2,13], while the second-order coefficient  $\lambda_1$  can be extracted by studying the holographic dual of the boost-invariant expansion of plasma [14]. The leading order results for  $\{\eta, \tau_\Pi, \lambda_1\}$  were obtained in [13,15,16] and the  $\mathcal{O}(\alpha_3)$  corrections were studied in [17–19]. The  $\mathcal{O}(\alpha_1)$  correction to the shear viscosity were first considered in [12] and these results were extended to include  $\mathcal{O}(\alpha_1^2, \alpha_2)$  corrections in [9]. It is straightforward to repeat the computations of [19] with the effective action (2.1) to determine corresponding  $\mathcal{O}(\alpha_1^2, \alpha_2)$  corrections to the second-order transport coefficients  $\tau_\Pi$  and  $\lambda_1$ .<sup>2</sup> We do not include any details of the analysis here but only present the final results. Hence, to order  $\mathcal{O}(\alpha_1^2, \alpha_2, \alpha_3)$ ,

$$\begin{aligned} \frac{\eta}{s} &= \frac{1}{4\pi} (1 - 8\alpha_1 + 112\alpha_1^2 - 384\alpha_2 + 120\alpha_3), \\ \tau_\Pi T &= \frac{1}{2\pi} \left( 2 - \ln 2 - 11\alpha_1 - 125\alpha_1^2 - 104\alpha_2 + \frac{375}{2}\alpha_3 \right), \\ \frac{\lambda_1 T}{\eta} &= \frac{1}{2\pi} (1 - 2\alpha_1 - 146\alpha_1^2 - 32\alpha_2 + 215\alpha_3). \end{aligned} \quad (2.3)$$

As discussed in the following section, the above results can also be expressed in terms of physical parameters of the dual CFT. Towards this end, we compute the two central charges in the CFT dual to (2.1) using the holographic trace anomaly [21]:

$$a = \pi^2 \frac{L^3}{\ell_p^3}, \quad c = \pi^2 \frac{L^3}{\ell_p^3} (1 + 8\alpha_1). \quad (2.4)$$

The  $R^2$  and  $R^3$  contributions to the central charges were considered in [22] and [9], respectively. However, in contrast with, e.g., (2.3) where we have the leading terms in an (infinite) expansion, we emphasize that these results (2.4) have no higher order corrections with the effective action (2.1). The fact that (2.4) is exact occurs because we have parameterized the higher curvature corrections only in terms of the Weyl tensor, which vanishes in the AdS<sub>5</sub> background.<sup>3</sup>

### 3. Discussion

Working perturbatively in the gravitational couplings in the effective action (2.1), we have the results for a number of interesting

<sup>1</sup> While the analysis of [9] introduces two  $R^3$  interactions, a certain combination of these terms vanishes by a Schouten identity after allowing for field redefinitions.

<sup>2</sup> In the context of Gauss-Bonnet gravity, numerical calculations of  $\alpha_1$ -modifications to  $\tau_\Pi$  were made in [20].

<sup>3</sup> We thank Aninda Sinha for discussions on this point.

properties of strongly coupled plasmas in the dual conformal field theory. Of course, these expressions for the pressure (2.2) and the transport coefficients (2.3) are given in terms of the gravitational couplings  $\alpha_n$ , as well as the dimensionless ratio  $L/\ell_p$ . As such, these results must also be specified as arising from our particular presentation of the effective action (2.1). In general, field redefinitions allow us to modify the form of the effective action but they will also change the precise form of these expressions, at the order that we have presented the results in the previous section. However, as we now discuss, this ambiguity can be avoided by parameterizing the results in terms of physical parameters of the underlying CFT.

As alluded to above, two useful parameters which characterize any four-dimensional CFT are the central charges,  $a$  and  $c$ . Hence given the results of the holographic trace anomaly (2.4), it is convenient to replace:

$$\frac{L^3}{\ell_p^3} = \frac{a}{\pi^2}, \quad \alpha_1 = \frac{1}{8} \frac{c-a}{a} \equiv \frac{\delta}{8}. \quad (3.1)$$

Again, we emphasize that these expressions are exact and do not receive further perturbative corrections with our effective action (2.1).<sup>4</sup>

As the corresponding interaction is cubic in the Weyl tensor,  $\alpha_2$  naturally plays a role in defining the three-point function of the stress tensor in the dual CFT. In general, this three-point function depends on three independent constants [23]. In fact, the central charges,  $a$  and  $c$ , each corresponds to a certain linear combination of these parameters. Recently, it was also shown in [24] that these constants in the three-point function also define two new parameters with a clear physical significance in the CFT. They considered an “experiment” in which the energy flux was measured at null infinity after a local disturbance was created the insertion of the stress tensor  $\mathcal{O} = T^{ij}\epsilon_{ij}$ . The energy flux escaping at null infinity in the direction indicated by the unit vector  $\vec{n}$  is then [24]

$$\langle \mathcal{E}(\vec{n}) \rangle_{\mathcal{O}} = \frac{E}{4\pi} \left[ 1 + t_2 \left( \frac{\epsilon_{ij}^* \epsilon_{i\ell} n_\ell n_j}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{2}{15} \right) \right], \quad (3.2)$$

where  $E$  is the total energy of the state. The two constants,  $t_2$  and  $t_4$ , can be used to characterize the underlying CFT.

However, recall that the three-point function contains only three independent parameters which go into defining the four constants:  $a$ ,  $c$ ,  $t_2$  and  $t_4$ . Hence the latter are not all independent and rather satisfy the relation [25]

$$\frac{a}{c} = 1 - \frac{1}{6}t_2 + \frac{4}{45}t_4. \quad (3.3)$$

Hence we keep only  $t_4$  to characterize the CFT as it is most naturally connected to the cubic curvature interaction in the dual gravity action (2.1). To leading order, one finds

$$t_4 = 4320\alpha_2 + \mathcal{O}(\alpha_1\alpha_2, \alpha_3^2). \quad (3.4)$$

We should comment that this result was originally calculated in the absence of any quartic curvature interactions [24], however, there is no contribution linear in  $\alpha_3$  by essentially the same reasoning presented in discussing (2.4). The key point is that our higher curvature corrections in (2.1) are written in terms of the Weyl tensor, which vanishes in the AdS<sub>5</sub> background. Hence the quartic term  $W(C)$  cannot contribute the three-point function and  $t_4$  at linear order. Further  $t_4$  does not receive a contribution at

<sup>4</sup> Note that our present definition of  $\delta$  differs slightly from that in [7]. There we had  $\delta' = (c-a)/c \simeq 8\alpha_1 - 64\alpha_1^2 + \mathcal{O}(\alpha_1^3)$ . We return to this choice of parameters below.

order  $\alpha_1^2$ , as can be observed by noting that  $t_4$  vanishes identically when the gravitational dual contains only curvature-squared corrections [25,26]. Finally, we add that  $t_4$  has the interesting property that it vanishes when the underlying CFT is supersymmetric [24].

Turning to  $\alpha_3$ , one would have to find an analogous parameter that characterizes the CFT through the four-point function of the stress tensor. Unfortunately, the four-point function is much more difficult to analyze as it is less rigidly constrained by the symmetries of the theory (than the two- or three-point functions) and it depends on details of the spectrum of operators in the CFT and their couplings to the stress tensor. As a result the four-point function is less studied and we do not have a physical parameter to replace the gravitational coupling  $\alpha_3$ . However, we remind the reader that in many string constructions  $\alpha_3 \sim 1/\lambda^{3/2}$  where  $\lambda$  is the 't Hooft coupling in the dual superconformal gauge theory [6]. We also re-iterate here that at order  $C^4$  in the effective gravitational action, one could write down five independent contractions of the Weyl tensor. Hence in complete generality, there would be five independent gravitational couplings appearing at this order [10]. In our analysis, we have chosen one particular linear combination of interactions which arises naturally as the leading  $C^4$  term in constructions of type IIB superstring theory [6]. While we cannot be sure that the leading interaction at this order will have precisely the form of  $W(C)$  in (2.1), we can take our results as representative of the general case [10].

Hence we characterize the CFT with the physical parameters  $\{a, \delta = (c - a)/a, t_4, \alpha_3\}$ . Then using (3.1) and (3.4), we can re-express  $\{P, \frac{\eta}{s}, \tau_{\Pi}, \lambda_1\}$  as:

$$\begin{aligned}
 P &= \frac{\pi^2}{2} a T^4 \left\{ 1 + \frac{9}{4} \delta + \frac{3}{8} \delta^2 + \frac{1}{180} t_4 + 15 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta^3, \delta t_4, t_4^2, \alpha_3^2) \right\}, \\
 \frac{\eta}{s} &= \frac{1}{4\pi} \left\{ 1 - \delta + \frac{7}{4} \delta^2 - \frac{4}{45} t_4 + 120 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta^3, \delta t_4, t_4^2, \alpha_3^2) \right\}, \\
 \tau_{\Pi} T &= \frac{1}{2\pi} \left\{ 2 - \ln 2 - \frac{11}{8} \delta - \frac{125}{64} \delta^2 - \frac{13}{540} t_4 + \frac{375}{2} \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta^3, \delta t_4, t_4^2, \alpha_3^2) \right\}, \\
 \frac{\lambda_1 T}{\eta} &= \frac{1}{2\pi} \left\{ 1 - \frac{1}{4} \delta - \frac{73}{32} \delta^2 - \frac{1}{135} t_4 + 215 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta^3, \delta t_4, t_4^2, \alpha_3^2) \right\}. \tag{3.5}
 \end{aligned}$$

We have explicitly noted that these expressions will receive higher order corrections to remind the reader that we are still working within a perturbative framework. In particular, consistency of the holographic calculations requires that  $a \gg 1$ ,  $\delta \ll 1$  and  $t_4 \ll 1$ , as well as  $\alpha_3 \ll 1$ .

In certain situations, it may be convenient to use  $\{c, \delta' \equiv (c - a)/c, t_4, \alpha_3\}$  to characterize the CFT instead. In this case, (3.1) is replaced with

$$\begin{aligned}
 \frac{L^3}{\ell_p^3} &= \frac{c}{\pi^2} (1 - \delta'), \\
 \alpha_1 &= \frac{1}{8} \frac{\delta'}{1 - \delta'} = \frac{1}{8} \delta' + \frac{1}{8} \delta'^2 + \mathcal{O}(\delta'^3). \tag{3.6}
 \end{aligned}$$

In terms of these physical parameters  $\{P, \frac{\eta}{s}, \tau_{\Pi}, \lambda_1\}$  become:

$$\begin{aligned}
 P &= \frac{\pi^2}{2} c T^4 \left\{ 1 + \frac{5}{4} \delta' + \frac{3}{8} \delta'^2 + \frac{1}{180} t_4 + 15 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta'^3, \delta' t_4, t_4^2, \alpha_3^2) \right\}, \\
 \frac{\eta}{s} &= \frac{1}{4\pi} \left\{ 1 - \delta' + \frac{3}{4} \delta'^2 - \frac{4}{45} t_4 + 120 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta'^3, \delta' t_4, t_4^2, \alpha_3^2) \right\}, \\
 \tau_{\Pi} T &= \frac{1}{2\pi} \left\{ 2 - \ln 2 - \frac{11}{8} \delta' - \frac{213}{64} \delta'^2 - \frac{13}{540} t_4 + \frac{375}{2} \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta'^3, \delta' t_4, t_4^2, \alpha_3^2) \right\}, \\
 \frac{\lambda_1 T}{\eta} &= \frac{1}{2\pi} \left\{ 1 - \frac{1}{4} \delta' - \frac{81}{32} \delta'^2 - \frac{1}{135} t_4 + 215 \alpha_3 \right. \\
 &\quad \left. + \mathcal{O}(\delta'^3, \delta' t_4, t_4^2, \alpha_3^2) \right\}. \tag{3.7}
 \end{aligned}$$

Now with either parametrization, (3.5) or (3.7), these results might be used to make a quantitative comparison with the sQGP, as suggested in [6,7]. However, we should examine different scenarios that might naturally arise in hCFT where these results can be consistently applied for such a comparison. First, recall that, as discussed in [7], the gravitational couplings are typically suppressed by the ratio of the Planck scale to the AdS curvature scale with  $\alpha_n \sim (\ell_p/L)^{2n}$ . In this case, beyond having each  $\alpha_n \ll 1$ , there would be a hierarchy amongst the couplings with  $\alpha_{n+1}/\alpha_n \sim (\ell_p/L)^2 \ll 1$ .

In working with the gravitational action (2.1), we are limiting our attention to the behaviour of the stress-energy in the dual CFT and we are assuming that we can overlook the effects of any other operators on the properties of the plasma. It was explained in [7] that this approach is consistent up to first order in the expansion in  $(\ell_p/L)^2$ . However, additional considerations are required to go to higher orders when the spectrum of the CFT includes operators with dimension of  $\mathcal{O}(1)$  or exactly marginal operators, as we now describe.

Complications arise when the dual fields have linear couplings to higher curvature terms.<sup>5</sup> For example, we might consider a coupling of the form  $\phi C^2$  with some massive scalar field  $\phi$ . In this case, the leading order black hole solution implicitly includes  $\phi = 0$ . However, the scalar will acquire a nontrivial profile at higher orders when the effects of the higher curvature terms are included. That is, at higher orders, the dual operator acquires an expectation value in the CFT plasma. As the hydrodynamic properties of the plasma refer the physics at very long wavelengths, one might attempt to proceed by integrating out this massive scalar. To be explicit, imagine we have the scalar action

$$\mathcal{L} = -\frac{1}{2\ell_p^3} [(\nabla\phi)^2 - M^2\phi^2 + 2L^2\beta\phi C_{abcd}C^{abcd}], \tag{3.8}$$

where  $\beta \sim (\ell_p/L)^2$ , following the discussion in [7]. If we integrate out the scalar, the contribution to the action becomes

$$\begin{aligned}
 \mathcal{L} &= \frac{1}{2\ell_p^3} \left[ L^6 \frac{\beta^2}{M^2 L^2} (C_{abcd}C^{abcd})^2 \right. \\
 &\quad \left. + L^8 \frac{\beta^2}{M^4 L^4} (C_{abcd}C^{abcd}) \nabla^2 (C_{efgh}C^{efgh}) + \dots \right]. \tag{3.9}
 \end{aligned}$$

Now, when the scalar has a Planck scale mass, i.e.,  $M \sim 1/\ell_p$ , the couplings of these higher curvature terms are suppressed in accord

<sup>5</sup> Any couplings to the Einstein term can be eliminated by a conformal transformation. For example,  $\phi^n R$  is removed by redefining  $g_{ab} \rightarrow \phi^{-2n/3} g_{ab}$ .

with the expected hierarchy. Such a scalar would correspond to an operator with a very large dimension, which grows parametrically with the central charge. If instead, one considers a (scalar) operator with dimension of  $\mathcal{O}(1)$ , the dual field would have a mass  $M \sim 1/L$ , e.g., as might arise in the Kaluza–Klein reduction of a ten-dimensional string background. In this case, coupling of the new quartic curvature term is only suppressed by  $\beta^2 \sim (\ell_p/L)^4$  and so this term will correct the thermodynamic and the transport properties of the holographic plasma at the same order as  $\alpha_1^2$  and  $\alpha_2$ . In fact, all of the higher order terms in (3.9) have the same suppression and so can be expected to contribute at this same order. Essentially this demonstrates that integrating out this scalar is not an effective approach to incorporating the effects of the dual operator with  $\mathcal{O}(1)$ -dimension. Hence, if we want to work with a purely gravitational action beyond order  $(\ell_p/L)^2$ , we must impose the absence of  $\mathcal{O}(1)$ -dimension operators in the hCFT as a consistency requirement. Such a condition might naturally arise in non-supersymmetric large- $N$  gauge theories where a generic operator develops a large anomalous dimension.

In the case of an exactly marginal (scalar) operator, the situation is a bit more subtle. The dual scalar field  $\phi^M$  is precisely massless and so, in principle, it can have an arbitrary value in the AdS<sub>5</sub> vacuum. Further one should think that the coupling constants in the effective gravitational action have a(n unspecified) dependence on this scalar, i.e.,  $\alpha \rightarrow \alpha(\phi^M)$ . The key point then is that if  $\phi^M$  becomes very large, the couplings may not be suppressed as we had initially assumed above [7]. This scenario naturally arises in many supersymmetric realizations of the AdS/CFT correspondence in string theory where the dilaton, i.e., the string coupling, is dual to an exactly marginal operator. Further, in all examples of conformal gauge theories of which we are aware, the presence of an exactly marginal coupling implies supersymmetry. In turn, supersymmetry would imply that  $\alpha_2$  vanishes, as explained in [7], and hence the interactions quartic in the curvatures provide the next set of corrections in our perturbative expansion.

Hence we can identify two cases where our results can be consistently applied in a quantitative comparison with the sQGP:

- (i) The CFT does not have any operators (other than the stress-energy tensor) with  $\mathcal{O}(1)$ -dimension and in particular, has no exactly marginal operators. In this case, one can truncate the holographic theory to include only gravity, as in (2.1). Further, the expected hierarchy should hold amongst the gravitational couplings, so that  $\alpha_2 \sim \alpha_1^2 \gg \alpha_3$ . Hence it is consistent to work with (2.2) and (2.3) at order  $\mathcal{O}(\alpha_1^2, \alpha_2)$ , while dropping the  $\mathcal{O}(\alpha_3)$  contributions.
- (ii) The CFT has at least one exactly marginal (scalar) coupling, which we assume implies supersymmetry. As explained above,  $\alpha_2$  vanishes and we cannot necessarily assume  $\alpha_3 \ll \alpha_1$ , in this scenario. Hence such a case can be consistently described by working with our results at order  $\mathcal{O}(\alpha_1, \alpha_3)$ , while dropping the  $\alpha_2$  contributions.

While in either of these scenarios provides a framework in which data might be consistently fit with our holographic results (3.5), neither one seems a particularly good conjecture as to the type of CFT which might describe the sQGP. However, one can certainly imagine other interesting situations. For example, there may be CFT's where  $\beta$  vanishes or has some accidental suppression. This would reflect an unexpected suppression of the correlator of the dual operator and two stress tensors.<sup>6</sup> Hence one might

<sup>6</sup> This suppression might be achieved by some underlying symmetry, e.g.,  $\phi \rightarrow -\phi$ .

also explore less cautious comparisons without restricting to the two scenarios above [10].

What are the prospects of further developing the holographic model of sQGP? In the framework of holographic conformal models, a natural venue to pursue is the to go even higher order in all the couplings, ultimately to finite values of  $\{a, \delta, t_4, \dots\}$ , rather than restricting<sup>7</sup> to the 'Einstein gravity corner' where the higher curvature couplings are all small, i.e.,  $a \gg 1$ ,  $\delta \ll 1$  and  $t_4 \ll 1$ , as well as  $\alpha_3 \ll 1$ . The advantage of this approach is that conformal invariance severely constrains thermal and transport properties of the plasma, compared to the proliferation of the transport coefficients in non-conformal theories [28]. As we discussed above, the challenge here is that one must expand the dual gravitational theory to include additional fields which can account for the effects of the condensates of various  $\mathcal{O}(1)$ -dimension operators. Alternatively, one can stay in the linearized approximation of the large- $N$ , strong coupling supersymmetric CFT plasma (as in [7]), but include in addition (small) corrections due to breaking of the scale invariance. To leading order in the hydrodynamics approximation, this introduces a new viscous coefficient: the bulk viscosity. While the bulk viscosity is rather well studied in holographic models [29], much less is known about the second-order transport coefficients of non-conformal theories – see [30] for some initial investigations. Either way, the utility of further holographic models of sQGP, as well as hydrodynamic plasma simulations [3], rests upon ability to extract additional observables from RHIC and future LHC experiments.

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<sup>7</sup> Note that these physical parameters cannot take completely arbitrary values, as they are constrained by the consistency of the underlying CFT [20,24,26,27].

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