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Propagation of Rayleigh waves in anisotropic layer overlying a semi-infinite sandy medium

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Abstract The present investigation deals with the propagation of Rayleigh waves in anisotropic layer overlying a sandy medium. Anisotropic material is in the nature of most general case i.e. of triclinic crystal and sandy medium is of alluvial soil type. The solutions for layer and half-space are obtained analytically. The displacement components in x and z directions are obtained for both the media. The dispersion relation is obtained subjected to certain boundary conditions. The special cases are considered. The numerical results are presented in the form of wave number and phase velocity $(k - c)$ analytical curves.

1. Introduction

The study of wave propagation in elastic media with different boundaries is of great importance to seismologists as well as to geophysicists because the knowledge in this field helps them to understand and predict the seismic behavior at different margin of earth. Elastic properties are generally anisotropic or inhomogeneous in sedimentary layers. For sediments deposited in horizontal layers or for rocks compressed by the increasing weight of later sediments, all properties would be expected to show the symmetry about the vertical, since all horizontal directions are equivalent. Rasolofosaon and Zinszner [1] analyzed comparison between permeability anisotropy and elasticity anisotropy of reservoir rocks. They developed new experimental and theoretical tools for the measurement and the characterization of arbitrary elasticity tensors and permeability tensors in rocks. In addition, they have given a complete set of the 21 elastic coefficients for various types of reservoir rocks. The problem of elastic waves propagating on the free surface of a semi-infinite elastic body is a well-covered research topic within the context of classical linear elasticity. The surface waves are subjected, along the direction of propagation, to the attenuation of horizontal and vertical displacements due to mechanical and geometrical phenomena of dissipation, and due to effects of dispersion (Richart et al. [2]). The dispersion is the variation of the wave velocity in relation to the frequency of vibration (Ewing et al. [3]). The surface waves, in an elastic half-space, have a velocity of propagation independent by the frequency and have only one mode shape of vibration. Instead in layered soil the velocity is dependent by the frequency and the propagation is due to the different

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modes of vibration. Chadwick [4,5] presented very comprehensive theory for the elastic wave propagation in a transversely isotropic material and proved that three waves can propagate in the material. He also discussed the surface waves in that material.

Rayleigh waves propagation in layered heterogeneous media has been studied in details by Wilson [6], Biot [7], Newlands [8] and Stonely [9]. Dutta [10] illustrated Rayleigh waves propagation in a two-layer anisotropic media whereas propagation of Rayleigh waves in an elastic half space of orthotropic material has been discussed by Abd-Alla [11]. Vishwakarma and Gupta [12] have discussed the Rayleigh waves in a layer over a sandy half space as one of the cases under the effect of rigid boundary. Tomar and Kaur [13] have considered the SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space. Vinh and Ogden [14] have analyzed the formulas for the speed of Rayleigh waves in orthotropic compressible elastic materials are obtained in explicit form by using the theory of cubic equations. Singh and Kumar [15] have studied the problem of propagation of Rayleigh waves due to a finite rigid barrier in a shallow ocean.

In the present investigation, an attempt has been made to study the behavior of Rayleigh waves when upper boundary plane is considered as free surface. Anisotropic material is in the nature of most general case i.e. of triclinic crystal and sandy medium is of alluvial soil type. The solutions for layer and half-space are obtained analytically. The displacement components in x and z directions are obtained for both the media. The dispersion relation is obtained subjected to certain boundary conditions. The special cases are reduced for (i) when the layer is taken as orthotropic material, (ii) when the layer is of isotropic material, (iii) when the depth is zero i.e. only sandy half-space is considered and (iv) when the depth is zero and $\eta = 1$ i.e. isotropic half-space. The numerical results are presented in the form of wave number and phase velocity $(k - c)$ analytical curves.

2. Formulation of the problem

We have considered an anisotropic elastic layer of finite thickness h lying over a semi-infinite sandy medium. The interface of these two media is considered at $z = 0$ whereas the free surface is at $z = -h$. Here, z axis is directed vertically downward and x axis is assumed in the direction of the propagation of wave with velocity c. The geometrical configuration is depicted in Fig. 1. For Rayleigh waves the displacement do not depend on y and if (u, v, w) be the displacement at any point $P(x, y, z)$ into the medium then $v = 0$ and u, w are function of x, z and t.

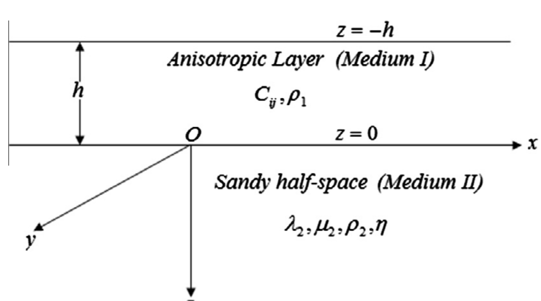

Figure 1 Geometry of the problem.

3. Basic equations and solution

3.1. Solution in the layer

The dynamical equations of motion for propagation of Rayleigh waves are given by

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2}, \tag{1}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = \rho_1 \frac{\partial^2 w_1}{\partial t^2}, \tag{2}$$

where ρ_1 is the density of the material of the layer, u_1 and w_1 are the displacement component in the layer along x and z direction respectively.

The stress–strain relations for anisotropic layer are taken as

$$\tau_{xx} = C_{11}e_{xx} + C_{12}e_{yy} + C_{13}e_{zz} + C_{14}e_{yz} + C_{15}e_{xz} + C_{16}e_{xy}, \tag{3a}$$

$$\tau_{zz} = C_{13}e_{xx} + C_{23}e_{yy} + C_{33}e_{zz} + C_{34}e_{yz} + C_{35}e_{xz} + C_{36}e_{xy} \tag{3b}$$

and

$$\tau_{xz} = C_{15}e_{xx} + C_{25}e_{yy} + C_{35}e_{zz} + C_{45}e_{yz} + C_{55}e_{xz} + C_{56}e_{xy}. \tag{3c}$$

Now, the equation of motion for propagation of Rayleigh waves in anisotropic medium by using (1), (2) and (3), we have

$$C_{11}\frac{\partial^2 u_1}{\partial x^2} + C_{15}\frac{\partial^2 w_1}{\partial x^2} + C_{55}\frac{\partial^2 u_1}{\partial z^2} + C_{35}\frac{\partial^2 w_1}{\partial z^2} + 2C_{15}\frac{\partial^2 u_1}{\partial x \partial z} + (C_{13} + C_{55})\frac{\partial^2 w_1}{\partial x \partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \tag{4}$$

and

$$C_{15}\frac{\partial^2 u_1}{\partial x^2} + C_{55}\frac{\partial^2 w_1}{\partial x^2} + C_{35}\frac{\partial^2 u_1}{\partial z^2} + C_{33}\frac{\partial^2 w_1}{\partial z^2} + (C_{13} + C_{55})\frac{\partial^2 u_1}{\partial x \partial z} + 2C_{35}\frac{\partial^2 w_1}{\partial x \partial z} = \rho_1 \frac{\partial^2 w_1}{\partial t^2}. \tag{5}$$

Assuming the solution of above equations as $u_1(x, z, t) = U_1(z)e^{ik(x-ct)}$ and $w_1(x, z, t) = W_1(z)e^{ik(x-ct)}$ and substituting in (4) and (5), we have

$$\left[C_{55}D^2 + 2ikC_{15}D + (\rho_1 k^2 c^2 - c_{11}k^2)\right]U_1 + \left[C_{35}D^2 + ik(C_{13} + C_{55})D - c_{15}k^2\right]W_1 = 0, \tag{6}$$

$$\left[C_{35}D^2 + ik(C_{13} + C_{55})D - c_{15}k^2\right]U_1 + \left[C_{33}D^2 + 2ikC_{35}D + (\rho_1 k^2 c^2 - c_{55}k^2)\right]W_1 = 0, \tag{7}$$

where k is wave number and c is phase velocity.

Following the orthodox method of solving simultaneous linear equations with constant coefficients, we write $U_1(z) = Ae^{-ksz}$, $W_1(z) = Be^{-ksz}$ and using in (6) and (7), we have

$$[C_{55}s^2 - 2iC_{15}s + (\rho_1 c^2 - C_{11})]A + [C_{35}s^2 - i(C_{13} + C_{55})s - C_{15}]B = 0 \tag{8}$$

and

$$[C_{35}s^2 - i(C_{13} + C_{55})s - C_{15}]A + [C_{33}s^2 - 2iC_{35}s + (\rho_1 c^2 - C_{55})]B = 0. \quad (9)$$

In order to obtain nontrivial solution of (8) and (9), we have

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0, \quad (10)$$

where a_0, a_1, a_2, a_3 and a_4 have been defined in Appendix A.

Let $s_j (j = 1, \ldots, 4)$ be the roots of (10) and the ratio of the displacement component U_{1j}, W_{1j} from (8) corresponding to $s = s_j$ is

$$\frac{W_{1j}}{U_{1j}} = \frac{B_j}{A_j} = \frac{-[C_{55}s_j^2 - 2iC_{15}s_j + (\rho_1 c^2 - C_{11})]}{[C_{35}s_j^2 - i(C_{13} + C_{55})s_j - C_{15}]} = m_j. \quad (11)$$

Thus the solution of (4) and (5) can be written as

$$u_1 = \left(A_1 e^{-ks_1 z} + A_2 e^{-ks_2 z} + A_3 e^{-ks_3 z} + A_4 e^{-ks_4 z}\right) e^{ik(x-ct)} \quad (12)$$

and

$$w_1 = (m_1 A_1 e^{-ks_1 z} + m_2 A_2 e^{-ks_2 z} + m_3 A_3 e^{-ks_3 z} + m_4 A_4 e^{-ks_4 z}) e^{ik(x-ct)}. \quad (13)$$

3.2. Solution in half space

The dynamical equations of motion for propagation of Rayleigh waves are given by

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \rho_2 \frac{\partial^2 u_2}{\partial t^2}, \quad (14)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = \rho_2 \frac{\partial^2 w_2}{\partial t^2}, \quad (15)$$

where ρ_2 is the density of the sandy medium, u_2 and w_2 are the displacement component in the layer along x and z direction respectively.

For sandy medium, the stress displacements relations are

$$\tau_{xx} = \eta\left[(\lambda_2 + 2\mu_2)\frac{\partial u_2}{\partial x} + \lambda_2 \frac{\partial w_2}{\partial z}\right],$$
$$\tau_{zz} = \eta\left[(\lambda_2 + 2\mu_2)\frac{\partial w_2}{\partial z} + \lambda_2 \frac{\partial u_2}{\partial x}\right] \text{ and } \tau_{xz} = \eta\mu_2\left(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}\right). \quad (16)$$

where λ_2, μ_2 are Lame constants and η is sandiness parameter given by $\frac{E}{\mu_2} = 2\eta(1 + v_2)$ (Weiskopf [16]), where E and v_2 are the Young modulus and Poisson's ratio respectively.

Putting (16) into (14) and (15), the two equations become

$$\eta(\lambda_2 + 2\mu_2)\frac{\partial^2 u_2}{\partial x^2} + \eta\mu_2 \frac{\partial^2 u_2}{\partial z^2} + \eta(\lambda_2 + \mu_2)\frac{\partial^2 w_2}{\partial x \partial z} = \rho_2 \frac{\partial^2 u_2}{\partial t^2} \quad (17)$$

and

$$\eta\mu_2 \frac{\partial^2 w_2}{\partial x^2} + \eta(\lambda_2 + 2\mu_2)\frac{\partial^2 w_2}{\partial z^2} + \eta(\lambda_2 + \mu_2)\frac{\partial^2 u_2}{\partial x \partial z} = \rho_2 \frac{\partial^2 w_2}{\partial t^2}. \quad (18)$$

For time harmonic wave propagating in positive x-direction, we have

$$u_2 = [Ce^{-kpz} + De^{kpz}] e^{ik(x-ct)}, \quad (19)$$

$$w_2 = [Ee^{-kpz} + Fe^{kpz}] e^{ik(x-ct)}, \quad (20)$$

where p is the parameter to be determined, c is the phase velocity and k is the wave number.

Now putting (19) and (20) into (17) and (18), the following four equations are obtained:

$$\begin{aligned}
&[\rho_2 c^2 - \eta(\lambda_2 + 2\mu_2) + \eta\mu_2 p^2]C - i\eta(\lambda_2 + \mu_2)pE = 0 \\
&[\rho_2 c^2 - \eta(\lambda_2 + 2\mu_2) + \eta\mu_2 p^2]D + i\eta(\lambda_2 + \mu_2)pF = 0 \\
&[\rho_2 c^2 - \eta\mu_2 + \eta(\lambda_2 + 2\mu_2)p^2]E - i\eta(\lambda_2 + \mu_2)pC = 0 \\
&[\rho_2 c^2 - \eta\mu_2 + \eta(\lambda_2 + 2\mu_2)p^2]F + i\eta(\lambda_2 + \mu_2)pD = 0
\end{aligned} \quad (21)$$

Eliminating C, D, E and F from four equations in (21), we get a bi-quadratic equation with reference to p, (a dimensionless parameter) as follows:

$$\eta^2 \frac{\beta_2^2}{\alpha_2^2} p^4 + \left[\eta^2\left(1 - \frac{\beta_2^2}{\alpha_2^2}\right)^2 + \eta\left(\frac{c^2}{\alpha_2^2} - \eta\right) + \eta\frac{\beta_2^2}{\alpha_2^2}\left(\frac{c^2}{\beta_2^2} - \eta\right)\right] p^2 + \frac{\beta_2^2}{\alpha_2^2}\left(\frac{c^2}{\alpha_2^2} - \eta\right)\left(\frac{c^2}{\beta_2^2} - \eta\right) = 0, \quad (22)$$

where $\alpha_2^2 = \frac{(\lambda_2 + 2\mu_2)}{\rho_2}$ and $\beta_2^2 = \frac{\mu_2}{\rho_2}$.

Let $\pm p_1$, $\pm p_2$ are the roots of Eq. (22). Then from (19) and (20) the displacements in the sandy layer are given by

$$u_2 = [C_1 e^{-kp_1 z} + C_2 e^{-kp_2 z} + D_1 e^{kp_1 z} + D_2 e^{kp_2 z}] e^{ik(x-ct)}, \quad (23)$$

$$w_2 = [n_1 C_1 e^{-kp_1 z} + n_2 C_2 e^{-kp_2 z} - n_1 D_1 e^{kp_1 z} - n_2 D_2 e^{kp_2 z}] e^{ik(x-ct)}, \quad (24)$$

where $E_j = n_j C_j$ and $F_j = - n_j D_j$, in which

$$n_j = \frac{\frac{c^2}{\alpha_2^2} - \eta + \eta\frac{\beta_2^2}{\alpha_2^2} p_j^2}{i\eta\left(1 - \frac{\beta_2^2}{\alpha_2^2}\right) p_j} (j = 1, 2). \quad (25)$$

The approximate solution to Eqs. (23) and (24) for half-space is given by

$$u_2 = [C_1 e^{-kp_1 z} + C_2 e^{-kp_2 z}] e^{ik(x-ct)} \quad (26)$$

and

$$w_2 = [n_1 C_1 e^{-kp_1 z} + n_2 C_2 e^{-kp_2 z}] e^{ik(x-ct)}. \quad (27)$$

4. Boundary conditions

(1) At the interface, $z = 0$, the continuity of the displacement along x direction requires that $u_1 = u_2$ and $w_1 = w_2$.
(2) At the interface, $z = 0$, the continuity of the stress requires that $(\tau_{xz})_1 = (\tau_{xz})_2$ and $(\tau_{zz})_1 = (\tau_{zz})_2$, where τ_{xz} and τ_{zz} are the stress component. Subscript '1' is taken for the layer and '2' for the half space.
(3) At the upper boundary plane (Free Surface) i.e. $z = -h$, the stresses vanish i.e. $(\tau_{xz})_1 = 0$ and $(\tau_{zz})_1 = 0$.

Using the boundary conditions first, second, third and Eqs. (12), (13), (26) and (27) respectively, we have

$$A_1 + A_2 + A_3 + A_4 - C_1 - C_2 = 0 \quad (28)$$

$$m_1 A_1 + m_2 A_2 + m_3 A_3 + m_4 A_4 - n_1 C_1 - n_2 C_2 = 0 \quad (29)$$

$$K_1A_1+K_2A_2+K_3A_3+K_4A_4-\eta\mu_2(in_1-p_1)C_1-\eta\mu_2(in_2-p_2)C_2=0 \quad (30)$$

$$K_5A_1+K_6A_2+K_7A_3+K_8A_4-\eta\{i\lambda_2-(\lambda_2+2\mu_2)p_1n_1\}\\ C_1-\eta\{i\lambda_2-(\lambda_2+2\mu_2)p_2n_2\}C_2=0 \quad (31)$$

$$K_1A_1e^{ks_1h}+K_2A_2e^{ks_2h}+K_3A_3e^{ks_3h}+K_4A_4e^{ks_4h}=0 \quad (32)$$

$$K_5A_1e^{ks_1h}+K_6A_2e^{ks_2h}+K_7A_3e^{ks_3h}+K_8A_4e^{ks_4h}=0 \quad (33)$$

Eliminating A_1, A_2, A_3, A_4, C_1 and C_2 from (28)–(33), we have

$$\begin{vmatrix} 1 & 1 & 1 & 1 & -1 & -1 \\ m_1 & m_2 & m_3 & m_4 & -n_1 & -n_2 \\ K_1 & K_2 & K_3 & K_4 & -\eta\mu_2(in_1-p_1) & -\eta\mu_2(in_2-p_2) \\ K_5 & K_6 & K_7 & K_8 & -\eta\{i\lambda_2-(\lambda_2+2\mu_2)p_1n_1\} & -\eta\{i\lambda_2-(\lambda_2+2\mu_2)p_2n_2\} \\ K_1e^{ks_1h} & K_2e^{ks_2h} & K_3e^{ks_3h} & K_4e^{ks_4h} & 0 & 0 \\ K_5e^{ks_1h} & K_6e^{ks_2h} & K_7e^{ks_3h} & K_8e^{ks_4h} & 0 & 0 \end{vmatrix}=0 \quad (34)$$

Eq. (34) gives the dispersion relation of Rayleigh waves in anisotropic layer lying over sandy medium.

5. Special cases

Case I: When we consider $C_{15}=C_{35}=0$, then Eq. (34) reduces to dispersion relation of Rayleigh waves in orthotropic layer lying over sandy half space.

Case II: When we take $C_{11}=C_{33}=\lambda_1+2\mu_1, C_{13}=\lambda_1$, $C_{55}=\mu_1, C_{15}=C_{35}=0$, then Eq. (34) reduces to dispersion relation of Rayleigh waves in isotropic layer lying over sandy half space.

Case III: When $h=0$, then Eq. (34) reduces to

$$\begin{vmatrix} (in_1-p_1) & (in_2-p_2) \\ \{i\lambda_2-(\lambda_2+2\mu_2)p_1n_1\} & \{i\lambda_2-(\lambda_2+2\mu_2)p_2n_2\} \end{vmatrix}=0 \quad (35)$$

Eq. (35) gives the dispersion relation of Rayleigh waves in sandy half space.

Case IV: When $h=0$ and $\eta=1$, the Eq. (34) reduces to

$$\begin{vmatrix} (in_1'-p_1') & (in_2'-p_2') \\ \{i\lambda_2-(\lambda_2+2\mu_2)p_1'n_1'\} & \{i\lambda_2-(\lambda_2+2\mu_2)p_2'n_2'\} \end{vmatrix}=0 \quad (36)$$

where p_1' and p_2' are roots of

$$\frac{\beta_2^2}{\alpha_2^2}p^4+\left[\left(1-\frac{\beta_2^2}{\alpha_2^2}\right)^2+\left(\frac{c^2}{\alpha_2^2}-1\right)+\frac{\beta_2^2}{\alpha_2^2}\left(\frac{c^2}{\beta_2^2}-1\right)\right]p^2 +\frac{\beta_2^2}{\alpha_2^2}\left(\frac{c^2}{\alpha_2^2}-1\right)\left(\frac{c^2}{\beta_2^2}-1\right)=0$$

and n_1' and n_2' are given by

$$n_j'=\frac{\frac{c^2}{\alpha_2^2}-1+\frac{\beta_2^2}{\alpha_2^2}p_j'^2}{i\left(1-\frac{\beta_2^2}{\alpha_2^2}\right)p_j'}\,(j=1,2).$$

Eq. (36) gives the dispersion relation of Rayleigh waves in isotropic half space.

6. Numerical results and discussion

We have taken data for inhomogeneous anisotropic medium from Rasolofosaon and Zinszner [1].

$C_{11}=106.8$ GPa, $C_{22}=99.00$ GPa, $C_{33}=54.57$ GPa, $C_{12}=27.10$ GPa

$C_{13}=9.68$ GPa, $C_{14}=-0.03$ GPa, $C_{15}=0.28$ GPa, $C_{16}=0.12$ GPa

$C_{23}=18.22$ GPa, $C_{24}=1.49$ GPa, $C_{25}=0.13$ GPa, $C_{26}=-0.58$ GPa

$C_{34}=2.44$ GPa, $C_{35}=-1.69$ GPa, $C_{36}=-0.75$ GPa, $C_{44}=25.97$ GPa

$C_{45}=1.98$ GPa, $C_{46}=0.43$ GPa, $C_{55}=25.05$ GPa, $C_{66}=37.82$ GPa

$C_{56}=1.44$ GPa and $\rho_1=2727\ \text{kg/m}^3$.

For sandy half space, the data are taken as

$\lambda_2=2.46$ GPa, $\mu_2=5.66$ GPa, and $\rho_2=7800\ \text{kg/m}^3$.

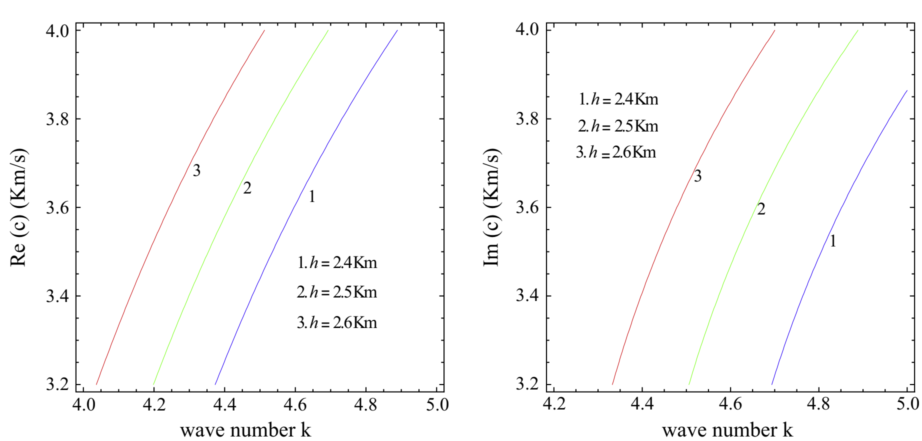

Figure 2 Variation of Re(c) and Im(c) with k for different value of h when $\eta=2.54$ and layer is of anisotropic material.

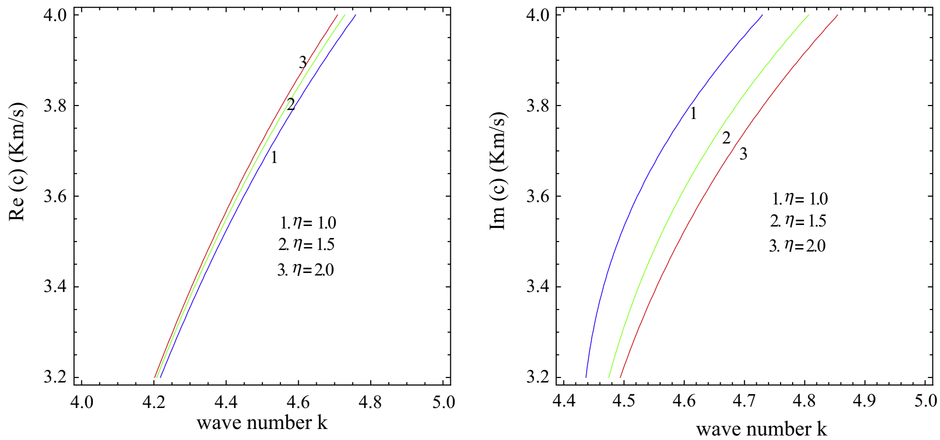

Figure 3 Variation of Re(c) and Im(c) with k for different value of η when h = 2.5 km and layer is of anisotropic material.

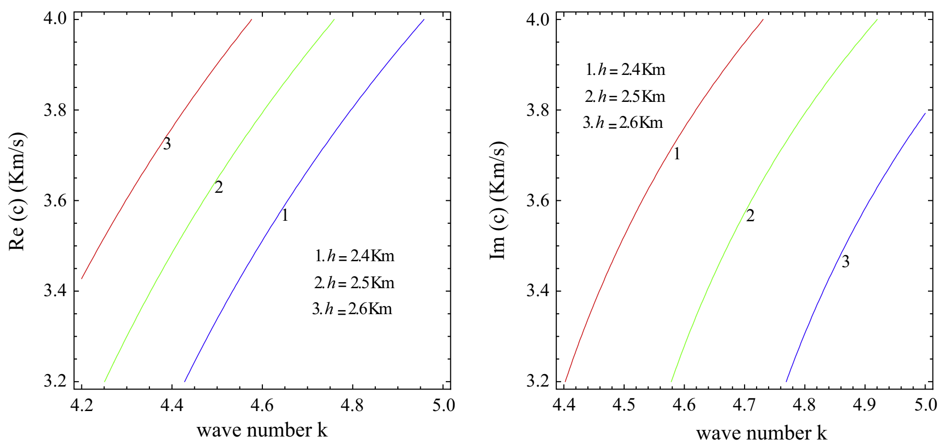

Figure 4 Variation of Re(c) and Im(c) with k for different value of h when η = 2.54 and layer is of orthotropic material.

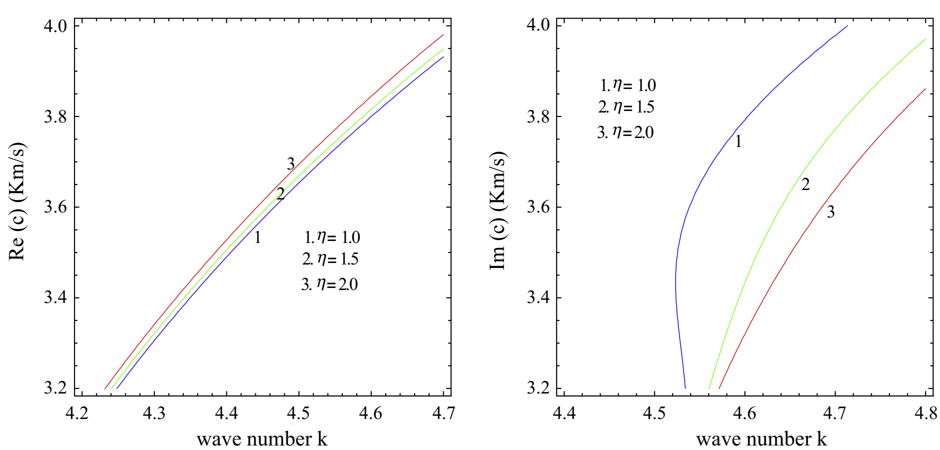

Figure 5 Variation of Re(c) and Im(c) with k for different value of η when h = 2.5 km and layer is of orthotropic material.

The dispersion relation (34) is a complex function. The Eq. (34) can be expressed in the form of Re(c) + iIm(c), where Re(c) and Im(c) are functions of k and constants of both media. The real part gives the dispersion of phase velocity and imaginary gives the damping effect. The graphs are plotted separately for both real and imaginary parts for phase velocity against wave number. In Fig. 2, the graphs are plotted for Re(c) and Im(c) against wave number k for different values of h and for fixed value of η = 2.54. It can be observed from the figure that both Re(c) and Im(c) increases as h increases with increasing wave number k. In Fig. 3, the graphs are plotted for Re(c) and Im(c) against wave number k for different values of η and for fixed value of h = 2.5 km. The figure reflects that as η increases Re(c) increases while Im(c) decreases. In both the figures, the Re(c) and Im(c) increases with increasing wave number k. In Fig. 4, the graphs are

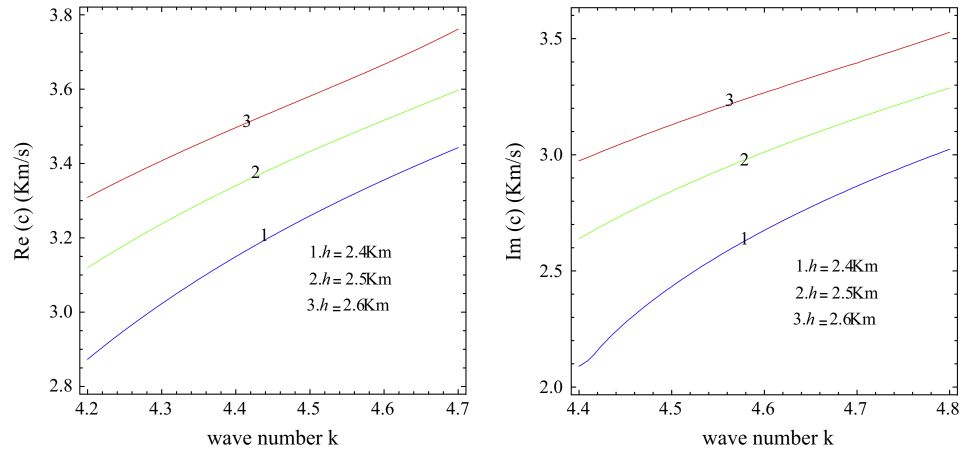

Figure 6 Variation of Re(c) and Im(c) with k for different value of h when η = 2.54 and layer is of isotropic material.

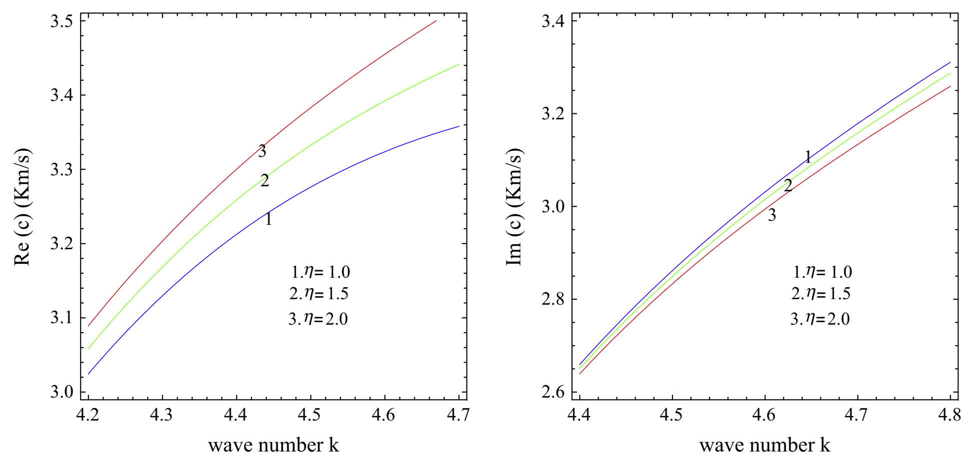

Figure 7 Variation of Re(c) and Im(c) with k for different value of η when h = 2.5 km and layer is of isotropic material.

plotted for Re(c) and Im(c) against wave number k for different values of h and for fixed value of η = 2.54 when the layer is considered of orthotropic materials. It can be observed from the figure that both Re(c) and Im(c) increases as h increases with increasing wave number k. In Fig. 5, the graphs are plotted for Re(c) and Im(c) against wave number k for different values of η and for fixed value of h = 2.5 km when the layer is considered of orthotropic materials. The figure reflects that as η increases Re(c) increases while Im(c) decreases. In Fig. 6, the graphs are plotted for Re(c) and Im(c) against wave number k for different values of h and for fixed value of η = 2.54 when the layer is considered of isotropic materials. It can be observed from the figure that both Re(c) and Im(c) increases as h increases with increasing wave number k.In Fig. 7, the graphs are plotted for Re(c) and Im(c) against wave number k for different values of η and for fixed value of h = 2.5 km when the layer is considered of isotropic materials. The figure reflects that as η increases Re(c) increases while Im(c) decreases.

7. Conclusions

The Rayleigh wave propagation in anisotropic layer lying over sandy half-space solid medium has been investigated. The dispersion relation is obtained analytically. The numerical results are discussed through figures by plotting graphs between phase velocity and wave number. It can be concluded from figures that as the thickness of layer increases magnitude of both real and imaginary phase velocity increase with wave number. Also, for increase of sandiness parameter the real phase velocity increases while imaginary decreases with wave number. The nature of effects of sandiness and depth is similar for different types of materials but the anisotropy of materials affects the phase velocity significantly. The velocity of seismic waves depends not only on the direction of wave propagation but also on the elastic properties and density of materials. The velocity of waves of seismic waves varies drastically in different materials and also at different depth. The sandiness of materials produces heterogeneity in the medium and has a great impact on the phase velocity. The heterogeneity and anisotropy plays a key role in the seismic wave propagation.

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Appendix A

$$a_0 = C_{33}C_{55} - C_{35}^2, \ a_1 = -2i(C_{15}C_{33} - C_{13}C_{35})$$

$$a_2 = (C_{33} + C_{55})\rho_0 c^2 - 4C_{15}C_{35} - C_{11}C_{33} + C_{13}^2 + 2C_{13}C_{15} + 2C_{15}C_{35}$$

$$a_3 = -2i\{(C_{15} + C_{35})\rho_0 c^2 - C_{11}C_{35} + C_{13}C_{15}\}$$

$$a_4 = \rho_0^2 c^4 - (C_{55} + C_{11})\rho_0 c^2 + C_{11}C_{55} - C_{15}^2$$

$$K_1 = iC_{15} - m_1 s_1 C_{35} + (im_1 - s_1)C_{55},$$
$$K_2 = iC_{15} - m_2 s_2 C_{35} + (im_2 - s_2)C_{55}$$

$$K_3 = iC_{15} - m_3 s_3 C_{35} + (im_3 - s_3)C_{55},$$
$$K_4 = iC_{15} - m_4 s_4 C_{35} + (im_4 - s_4)C_{55}$$

$$K_5 = iC_{13} - m_1 s_1 C_{33} + (im_1 - s_1)C_{35},$$
$$K_6 = iC_{13} - m_2 s_2 C_{33} + (im_2 - s_2)C_{35}$$

$$K_7 = iC_{13} - m_3 s_3 C_{33} + (im_3 - s_3)C_{35},$$
$$K_8 = iC_{13} - m_4 s_4 C_{33} + (im_4 - s_4)C_{35}$$

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