Massless monopole clouds and electric–magnetic duality

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Abstract

We discuss the Montonen–Olive electric–magnetic duality for the BPS massless monopole clouds in $\mathcal{N} = 4$ supersymmetric Yang–Mills theory with non-Abelian unbroken gauge symmetries. We argue that these low energy non-Abelian clouds can be identified as the duals of the infrared bremsstrahlung radiation of the non-Abelian massless particles. After we break the $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 1$ by adding a superpotential, or to $\mathcal{N} = 0$ by further adding soft breaking terms, these non-Abelian clouds will generally condense and screen the non-Abelian charges of the massive monopole probes. The effective mass of these dual non-Abelian states is likely to persist as we lower the energy to the QCD scale, if all the non-Abelian Higgs particles are massive. This can be regarded as a manifestation of the non-Abelian dual Meissner effect above the QCD scale, and we expect it to continuously connect with the confinement as we lower the supersymmetry breaking scale to the QCD scale.

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1. Introduction

The $\mathcal{N} = 4$ supersymmetric Yang–Mills theory is conjectured to have a remarkable electric–magnetic duality [1–4]. A special form of this conjecture suggests that the electric theory is dual to the magnetic theory with a dual group and an inverse coupling constant.

This conjecture originated in the study of an $SU(2)$ theory spontaneously broken to $U(1)$ [1,2], where there is only one type of fundamental (anti-)monopole. The supersymmetric multiplet based on this monopole is dual to the massive gauge supermultiplet.

If the rank $r$ of the gauge group is higher than one, when the non-Abelian gauge symmetry is maximally broken to $U(1)^r$, the monopole configurations can be treated as superpositions of fundamental monopoles associated with simple roots [5]; while for elementary particles, each root of the dual group corresponds to a massive gauge supermultiplet. Studies of supersymmetric sigma models on the monopole moduli spaces [6] show that these supersymmetric fundamen-
We expect it to continuously go to the non-Abelian dual Meissner effect in QCD confinement when we lower the supersymmetry breaking scale to the QCD scale.

2. Massless monopole clouds and bremsstrahlung radiation

We use the same $SO(5)$ example. The gauge symmetry is now partially broken to $SU(2) \times U(1)$ by a Higgs expectation value $h$ orthogonal to the root $\gamma$ or $\gamma^*$ [19]. Correspondingly, the $\gamma$ monopoles or $\gamma^*$ elementary particles become massless. A spherically symmetric BPS magnetic monopole solution is found in [7]. It describes a massive monopole, embedded in the $SU(2)$ subgroup defined by the root $\beta$, surrounded by a non-Abelian cloud. There is a modulus $a$ characterizing the size of the cloud. We will be interested only in the non-Abelian fields which do not exponentially decay outside the massive monopole core $m_w^{-1}$:

$$A^a_{(i \gamma)} = \epsilon_{ain} \tilde{r}_m G(r), \quad \phi^a_\gamma = \hat{r}_a G(r).$$

where the subscripts $\gamma$ mean that the fields correspond to the triplet $SU(2)$ generators $t_a(\gamma)$ ($a = 1, 2, 3$) associated with the root $\gamma$, and

$$G(r) = \frac{1}{e r (1 + r/a)}.$$  

If the cloud size $a$ is infinite, we only have the massive monopole, carrying both Abelian and non-Abelian charges. If $a$ is finite, the cloud shields the non-Abelian charge of the massive one, so that the non-Abelian fields fall as $a/r^2$ outside of the radius $a$, as we can see from (1) and (2).

The metric for this massless monopole cloud can be obtained [8] by taking the zero reduced mass limit of the maximally broken case:

$$ds^2 = \frac{g^2}{8\pi} \left( \frac{da^2}{a} + a \sigma_1^2 + a \sigma_2^2 + a \sigma_3^2 \right).$$

where $g = 4\pi/e$ is the magnetic coupling and $\sigma_i$ ($i = 1, 2, 3$) are the one-forms describing the unbroken $SU(2)$. For this metric, the harmonic (anti-)self-dual form is not normalizable. So the massless monopole cloud is not bound. It has been a puzzle [8] why
this configuration, which is dual to the $\alpha^*$ gauge multiplet in the $Sp(4)$ theory, does not have a normalizable threshold bound state as in the maximally broken case.

To answer that, we first look at the elementary particles in the weakly coupled electric theory of $Sp(4)$. Because the beta function vanishes in the $\mathcal{N} = 4$ supersymmetric gauge theory [20], the massless particles of the non-Abelian gauge multiplet $\gamma^*$ in this weakly coupled theory are not confined. Therefore, whenever a massive particle is coupled to these massless ones, it emits non-Abelian infrared bremsstrahlung radiation. For example, the massive $\alpha^*$ Higgs can become a massive $\beta^*$ Higgs by emitting an infrared gauge or Higgs boson associated with the root $\gamma^*$. Generalizing this, the massive gauge multiplets $\alpha^*$ and $\beta^*$ become indistinguishable through the emission and absorption of the massless gauge supermultiplet associated with $\gamma^*$.\footnote{It is interesting to compare this $SO(5)$ example to a single massive fundamental monopole in $SU(3) \to SU(2) \times U(1)$ theory, where the massless monopole cloud is absent. On the dual side, for a single massive elementary particle in this $SU(3)$ theory, the non-Abelian charge is unchanged (or gauge equivalent) after infrared radiation.}

These two descriptions for the monopoles and elementary particles are very different. The former describes a solitonic static field configuration, while the latter describes massless elementary particles that propagate in the speed of light. To see how they can be dual to each other, we need analyze the low energy supersymmetric quantum mechanics of the massless monopole clouds on the moduli space.

To see what happens, we need to find the spherically symmetric eigenstates of the Laplacian $\Delta = dd^* + d^*d$ corresponding to the metric (3) [21]. These non-normalizable scattering states can be described by sixteen harmonic differential forms which are the duals of the gauge supermultiplet $\gamma^*$. Up to constant factors, these are given by

0-form:

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi}),$$

(4)

1-forms:

$$\frac{1}{a} J_2(g\sqrt{Ea/2\pi}) da,$$

2-forms:

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi}) (da \wedge \sigma_1 + a\sigma_2 \wedge \sigma_3),$$

and cyclic,

(6)

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi}) (da \wedge \sigma_1 - a\sigma_2 \wedge \sigma_3),$$

and cyclic,

(7)

where $E$ is the arbitrarily small energy of the massless monopole cloud and $J$’s are Bessel functions. The $\sigma_i$’s and $da$ correspond to fermionic excitations. The 3-forms and 4-form are the Hodge duals of the 1-forms and 0-form, respectively. The $a$-dependence of these wave functions are all similar. For example, the 0-form wave function goes to a constant for $a \ll \frac{2g}{\pi E}$ and falls as $a^{-3/4} \cos(g\sqrt{Ea/2\pi} - 3\pi/4)$ for $a \gg \frac{2g}{\pi E}$.

However the moduli space approximation for the low energy solitons usually requires small velocities. For the case of the massless monopole cloud, this requires $|\dot{a}| < 1$. From the metric (3), this imposes the restriction $a < a_c = \frac{g^2}{4\pi E}$. Beyond this region the moduli space approximation fails and the cloud propagates as a wavefront at the speed of light [18]. So the wave function should be replaced by the spherical wave $\sim e^{iEa}/a$ as $a > a_c$, where $a$ becomes the position of the wavefront. As we turn to the weak magnetic coupling (small $g$) limit, the duality conjecture suggests that the monopoles and the elementary gauge particles exchange roles. Indeed, as we can see, the extent of the solitonic wave function $\alpha_c$ is much smaller then the wavelength ($\sim 1/E$) of the wavefront and, in addition, inside $a_c$ the wave function is nearly a constant. Thus the solitonic phase is negligible. (See Fig. 2.) The massless monopole always appears as infrared radiation and the elementary local field description takes over.

The above discussion is in accordance with the classical dynamics discussed in [18], i.e., the prediction of the moduli space approximation from (3) that $a \sim t^2 E/g^2$ is good only for a time period of order $g^2/E$ during which the cloud speed $\dot{a} \sim 1$. According to the uncertainty principle, for $g < 1$, it is quantum mechanically unobservable.
Fig. 2. The cloud wave functions for the two different cases, $g \gg 1$ and $g \ll 1$. The three different length scales $a_c \approx g^2/E$, $\lambda_w \approx 1/E$ and $\lambda_s \approx 1/(g^2E)$ are indicated in these two different cases. The part within $a_c$ is the solitonic phase, whose wave function is given by Eqs. (4)–(7). Outside $a_c$ is the spherical wave. For $g \gg 1$, $a_c$ is the biggest length scale, while for $g \ll 1$, $a_c$ is the smallest.

So instead of being a problem, the absence of the massive and massless monopole bound states is in fact consistent with the duality. As we turn the coupling $g$ from strong to weak, the unbound cloud becomes the infrared non-Abelian radiation of elementary particles and the solitonic phase of the cloud disappears quantum mechanically.

In the above discussion, we always studied the weakly coupled theory in the elementary particle sector where particles are local excitations of fields, and the strongly coupled theory in the solitonic sector where solitons are non-local objects from the point of view of the elementary fields. Both descriptions can happen either in the electric or magnetic theory, depending on the couplings. We will continue to use this method throughout the Letter.

3 Since we are considering the case where all six Higgs vevs in the $\mathcal{N} = 4$ theory are proportional in the gauge space, a global $SO(6)$ rotation can make all but one of them zero. An important difference when we go from big $g$ to small $g$ is that, while the Higgs profile of the strongly coupled non-Abelian massless monopole clouds take non-zero values only in one Higgs direction, as we go to the weak coupling limit, we can see from the interaction terms in the field theory that these massless radiations can oscillate in all the Higgs directions. This fact will also be useful later.

3. Dynamics of non-BPS non-Abelian monopole clouds

Non-Abelian $\mathcal{N} = 1$ or $\mathcal{N} = 0$ supersymmetric gauge theories have the important property of confinement. Significant insights have been made by Seiberg and Witten in [22]. From the exact $\mathcal{N} = 2$ low energy theory, they explicitly show that a superpotential breaking the supersymmetry to $\mathcal{N} = 1$ causes the massless magnetic monopole field to condense. This confinement is described in a weakly coupled magnetic theory through the dual Meissner effect [23]. Related issues starting from $\mathcal{N} = 4$ have also been studied (see, e.g., [24] and references therein).

It is natural to ask what roles the non-Abelian clouds we have studied may play in this QCD confinement. To see this, we will focus on the energy region above the QCD scale $\Lambda_{\text{QCD}}$. Specifically, we start with a $\mathcal{N} = 4$ theory with a weak electric coupling at high energy. In this theory we have argued that, in the presence of certain massive monopoles, we can identify the low energy magnetic non-Abelian clouds as the dual infrared non-Abelian particles by exploring the duality conjecture. When we break the supersymmetry at low energy, we break the original electric–magnetic symmetry. But the dual states we identified should still exist and we will be interested in how they evolve as the supersymmetry is broken. As mentioned, we will focus mostly on the energy region above $\Lambda_{\text{QCD}}$, where the strongly coupled magnetic theory is described by non-BPS monopoles. Then we will discuss some implications for the low energy theory below $\Lambda_{\text{QCD}}$.

We explicitly break the supersymmetry to $\mathcal{N} = 1$ at low energy by adding a superpotential for the $\mathcal{N} = 1$ chiral multiplets. We expand the Higgs around those vacua where part of the non-Abelian symmetry is unbroken and use $\phi$ to represent the non-Abelian components of the deviations. Among all the terms in the expansion, we will study the quadratic terms

$$\frac{1}{2} m_\phi^2 \text{tr}(\phi^2)$$

as examples. This gives an $\mathcal{N} = 4$ supersymmetry scale $m_\phi$. As mentioned, the fact that $m_\phi > \Lambda_{\text{QCD}}$ is guaranteed as long as the electric coupling is weak at the supersymmetry breaking scale $m_\phi$. We
will be interested in the limit where the non-Abelian Higgs masses $m_\phi$ are much smaller than the massive gauge bosons $m_W$. We also want the $U(1)$ Higgs masses to be much smaller than the non-Abelian Higgs. By doing this, we effectively make the $U(1)$ parts remain BPS so we can concentrate on the non-BPS properties of the non-Abelian parts only. This is why we have neglected the $U(1)$ mass terms in (8).\(^4\)

To study the non-BPS monopoles, it is enough to add a superpotential in the direction of the non-zero Higgs. But for later purposes to connect with confinement, we will also add superpotentials for the other two chiral multiplets. This can be simply given by the mass terms with zero Higgs vev. It has no effect on the monopole properties we will discuss.

We first study the example in $SO(5)$. The BPS fields are given in (1). When the non-BPS potential (8) is added, the non-Abelian Higgs field is exponentially cut off at a distance scale $m_\phi^{-1}$. Outside of the region $m_\phi^{-1}$ where the Brandt–Neri–Coleman (BNC) instability [25,26] applies, the gauge field decays to a magnetic-color neutral configuration, which corresponds to having a non-Abelian cloud inside $m_\phi^{-1}$. Since $m_\phi^{-1} \gg m_W^{-1}$, the BPS solution (1) is still a good approximation between $m_\phi^{-1}$ and $m_W^{-1}$. However, the cloud size is no longer a modulus. It is easy to see that, under the potential (8), it is classically energetically favored for the cloud to shrink. We can use the BPS solution to estimate this $a$-dependent potential. It is\(^5\)

$$\frac{g^2}{8\pi} m_\phi^2 a. \quad (10)$$

This should be a good approximation as the non-BPS potential is weak. The correction is given by factors of $m_\phi a$. The potential change within the core, $r < m_W^{-1}$, is negligible.

Using the metric (3) and this linear potential, we can study the quantum mechanics of this bounded non-Abelian cloud. This is non-supersymmetric, as the monopole breaks the $\mathcal{N} = 1$ supersymmetry. For the purpose of this Letter, we simply note that the ground state of the cloud has a mass gap of order $m_\phi$ and is concentrated in the region $\langle a \rangle \sim g^{-2} m_\phi^{-1} \ll m_\phi^{-1}$, since the factor $g^2$ can be absorbed in the $a$ in (3) and (10). Any multi-monopole configuration can be thought of as being a collection of these color singlets. Since we neglected the $U(1)$ Higgs mass, there are no net long-range forces between the monopoles when they are separated further than $m_\phi^{-1}$. Before discussing the physical interpretation of this result, we consider a case where the cloud encloses two massive monopoles.

We use the minimal symmetry breaking model of $SU(3)$ [9]. When the two massive monopoles are far apart, so that the non-Abelian Higgs has decayed exponentially, the relative orientation of their non-Abelian gauge charges is self-adjusted to minimize the energy [26]. The charges are then given by

$$\frac{1}{\sqrt{2}} \text{diag}(1, 0, -1), \quad \frac{1}{\sqrt{2}} \text{diag}(0, 1, -1), \quad (11)$$

respectively. Here, the first two entries of the matrices correspond to the unbroken $SU(2)$. Since only the non-Abelian part is non-BPS, these two monopoles are attracted by the Coulomb potential

$$\frac{g^2}{16\pi l^2} \quad (l > m_\phi^{-1}), \quad (12)$$

\(^4\) In certain models, the above mass relations can be achieved by adjusting the parameters in the potential. For instance, consider an $SU(2N)$ theory with superpotential

$$W(\Phi) = m \text{tr}(\Phi^2) + \lambda \text{tr}(\Phi^4) + \eta X \text{tr}(\Phi^2) - \mu^2, \quad (9)$$

where $m$, $\mu$, $\lambda$, and $\eta$ are real, and we have introduced a color singlet $X$ to have more adjustable parameters. At $X = -m/\eta$, this theory has a $\mathcal{N} = 1$ supersymmetric vacuum where the gauge symmetry is broken to $SU(N) \times U(1) \times SU(N)$. The mass relations can be satisfied by choosing $e \gg \sqrt{\lambda/\eta} \gg \eta$. Also consistent with this requirement, the dimensionless couplings in this superpotential have to be very small compared to the electric coupling $e$ at the supersymmetry breaking scale, so that above this scale the $\mathcal{N} = 4$ supersymmetry is restored. (Because the dimensionless parameters $\lambda$ and $\eta$ grow when we increase the energy, at much higher energy we again return to $\mathcal{N} = 1$. But this does not affect our argument as long as there is a region where $\mathcal{N} = 4$ supersymmetry is approximately held.) However to illustrate the properties of the non-Abelian clouds, we will use simpler groups such as the previously mentioned $SO(5)$. We will not try to construct the specific potential for each case, because the simple qualitative features which will be summarized after those examples are true for cases where these mass relations are satisfied.

\(^5\) Note we have a non-standard kinetic term for $a$ from Eq. (3).
where \( l \) is the monopole separation. Here a factor of \(-\frac{1}{4}\) is from the inner product of the non-Abelian part of (11), and the Abelian part is neglected because it is approximately BPS under our mass conditions mentioned before.\(^6\) When the two monopoles stay inside the range \( m_\phi^{-1} \), we can approximate the near-BPS fields outside of the massive cores by the superposition of two \( SU(2) \) monopoles at positions \( r_1 \) and \( r_2 \). This gives the Higgs fields at \( r \) as

\[
\text{diag}
\left(
\begin{array}{c}
\frac{1}{\sqrt{2er_1}} + \frac{1}{\sqrt{2er_2}}, \\
\frac{1}{\sqrt{2er_1}} - \frac{1}{\sqrt{2er_2}}, \\
\frac{1}{\sqrt{2er_1}} + \frac{1}{\sqrt{2er_2}}, \\
\frac{1}{\sqrt{2er_1}} - \frac{1}{\sqrt{2er_2}}
\end{array}
\right)
\]

(13)

if there were no non-Abelian cloud and

\[
\text{diag}
\left(
\begin{array}{c}
\frac{1}{\sqrt{2er_1}}, \\
\frac{1}{\sqrt{2er_1}}, \\
\frac{1}{\sqrt{2er_1}}, \\
\frac{1}{\sqrt{2er_1}}
\end{array}
\right)
\]

(14)

with a minimal size non-Abelian cloud, where \( \text{diag}(t_1, t_1, t_3) \) is the vacuum and \( r_i = |r - r_1| (i = 1, 2) \). In the latter case, the non-Abelian field is cancelled at a length scale bigger than the monopole separation \( l \). Therefore, under the potential (8), it is energetically favored to have a minimal size non-Abelian cloud surrounding the massive monopoles. However the non-Abelian Higgs field is still present within the separation scale \( l \). Integrating (8) over the spatial region up to \( m_\phi^{-1} \), we obtain an attractive potential\(^7\)

\[
\frac{g^2}{32\pi} m_\phi^3 l + O(g^2 m_\phi^3 l^2) \quad (l < m_\phi^{-1}).
\]

(15)

So, if the core size is ignored, the massive monopoles will coincide classically and have a non-Abelian cloud bound to them. This is similar to what we have seen in \( SO(5) \).

4. Non-Abelian monopole clouds and dual Meissner effect

The energy scale \( m_\phi \) and the linear property of the potentials (10) and (15) may receive corrections from the higher order terms neglected in (8). However, the following qualitative features do not depend on these terms and the specific examples. Within the \( \mathcal{N} = 4 \) supersymmetry length scale \( m_\phi^{-1} \) around the massive monopoles, the appearance of the non-BPS Higgs raises the energy above the vacuum due to the non-BPS potential; outside of this scale, we have the BNC instability; so, whenever the topology is allowed, the non-Abelian clouds will always contract to cancel the non-Abelian fields of the enclosed massive monopoles.

In our discussion, because the massive monopoles carry non-Abelian magnetic charges, they actually serve as probes so that we can study the properties of the dual non-Abelian states. Unlike the Coulomb-like phase in \( \mathcal{N} = 4 \) as we saw in Section 2, these dual states now have effective masses and the non-Abelian magnetic charges are screened. In other words, in this intermediate energy region where we describe the magnetic theory by solitons, breaking the supersymmetry by a superpotential (but maintaining the non-Abelian nature of the vacuum) in the weakly coupled electric theory causes the magnetic theory to be in an analogous dual Higgs phase. In the following, we will discuss the possibility of this phenomenon continuously going to the dual Meissner effect when we lower the energy scale \( m_\phi \) to that of the vacuum state (\( \Lambda_{\text{QCD}} \)), where the test massive solitonic monopole becomes the test elementary particle.

To do this, we first note that, although the \( \mathcal{N} = 1 \) non-Abelian vacuum has the energy scale \( \Lambda_{\text{QCD}} \), we have only seen the non-Abelian clouds at \( m_\phi \) because we rely on the presence of massive non-Abelian monopoles. To look at these non-Abelian clouds at a lower energy scale \( \tilde{m}_\phi \) (\( m_\phi > \tilde{m}_\phi > \Lambda_{\text{QCD}} \)) with a corresponding bigger electric coupling \( \tilde{g} \) (according to the asymptotic freedom), we should change the setup by lowering the supersymmetry breaking scale to \( \tilde{m}_\phi \) and choose the \( \mathcal{N} = 4 \) theory above it to have the corresponding coupling \( \tilde{g} \). By the same argument we see that, after the supersymmetry breaking, the non-Abelian clouds of the \( \mathcal{N} = 1 \) theory with coupling \( \tilde{g} \)

\(^6\) In this \( SU(3) \) example, the assumption that the Abelian Higgs mass is small is important, because otherwise the Abelian gauge force has a factor of \( \frac{1}{2} \), which makes the overall color singlet configuration unstable.

\(^7\) For big \( g \), this kind of forces will affect the binding energies of the multiple monopole states considered in [27] where similar Higgs mass relations are taken.
are Higgsed and get a mass \( \sim m_\phi \). The same reasoning can go all the way to \( e \lesssim 1 \) (\( g \gtrsim 1 \)).

From \( e \sim g \sim 1 \) around \( \Lambda_{\text{QCD}} \), the solitonic description we used in the magnetic theory starts to deviate from being a good approximation. For the massive monopoles, the Compton wavelength begins to exceed the monopole core size. For the non-Abelian clouds, the potential becomes too shallow. Only one bound state can exist, with a mass gap \( g^2 m_\phi \), determined by the depth of the potential. This bound state has a wavelength of order \( g^{-3} m_\phi^{-1} \), which begins to exceed the range of the potential \( m_\phi^{-1} \) as \( g \lesssim 1 \). (Outside of \( m_\phi^{-1} \), we still have the BNC instability in the Higgs direction we are considering.) So as mentioned before, below \( g \sim 1 \) we should switch the roles of elementary particles and the solitons between the electric and magnetic theories. These analyses also suggest that the masses (which should be of order \( \Lambda_{\text{QCD}} \) from the last paragraph) of the dual non-Abelian fields are likely to vary continuously, rather than abruptly vanish, at \( g \sim 1 \).

This meets the expectation that the usual weakly coupled dual Higgs mechanism starts to take effect. Nielsen–Olesen electric flux tubes [28] appear as solitonic objects and this causes confinement of non-Abelian electric charge and electric fields. The quantum fluctuations of these tubes are of order \( g \) times the thickness of the flux tubes [28]. Here we comment that for big \( g \) above \( \Lambda_{\text{QCD}} \), these fluctuations are much bigger than the size of the electric flux. This is consistent with the fact that the electric fields are not confined above \( \Lambda_{\text{QCD}} \), despite of the analogous dual Higgs mechanism.

The coupling stops running soon after the magnetic perturbation theory starts to become valid, since all the non-Abelian magnetically charged particles obtain masses of order \( \Lambda_{\text{QCD}} \) through this dual Higgs mechanism.

Since so far all the non-BPS properties of the \( \mathcal{N} = 1 \) theory that we have used are shared by the non-supersymmetric theory, we can further break the \( \mathcal{N} = 1 \) supersymmetry by adding some soft breaking terms. For example, we can add a non-Abelian gaugino mass term with mass equal to the supersymmetry breaking scale \( m_\phi \) and get the same picture.

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**References**


Footnote 3: For the solitonic description of the magnetic theory at big \( g \), the massive monopoles can only probe one Higgs direction since the non-Abelian clouds are non-zero in only one of the Higgs fields. There it is enough that we have a superpotential for one chiral multiplet. But in order for this screening effect to continuously go to the case \( g \lesssim 1 \) where the non-Abelian Higgs can oscillate in all directions, the superpotential in the other two complex directions of the Higgs should also be present as we mentioned in footnote 3.