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Semiclassical gravitational effects near a singular magnetic flux

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Abstract

We consider the backreaction of the vacuum polarization effect for a massive charged scalar field in the presence of a singular magnetic massless string on the background metric. Using semiclassical approach, we find the first-order (in $\hbar$ units) metric modifications and the corresponding gravitational potential and deficit angle. It is shown that, in certain region of values of coupling constant and magnetic flux, the gravitational potential and deficit angle can be positive as well as negative over all distances from the string and can even change its sign. Unlike the case of massless scalar field, the gravitational corrections were found to have short-range behavior.

1. Introduction

Gauge theories with spontaneous symmetry breaking predict the emergence of cosmic objects with topology defect in the early Universe. Such objects can possibly survive at the present day (see the review by Vilenkin [1] and references therein). In topology defect points the spontaneous symmetry breaking principle, giving the mass for fields, is no more valid. So physical fields need some boundary conditions, that cause the vacuum polarization and appearance of non-zero vacuum expectation value of the energy–momentum tensor of quantum fields like in Casimir effect [2]. Non-zero vacuum expectation value of the energy–momentum tensor in one's turn serves as a source of gravitation [3,4] and can take part in cosmological models of the Universe taking into account vacuum quantum effects.

One of the topology defect manifestations, whose existence is not in contradiction with observable data [5], is cosmic strings which are particularly interested both as possible "seed" for galaxy formation [6,7] and as possible gravitational lens [8]. Space–time metric of the cosmic strings in empty Universe in the linearized approach were

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found by Vilenkin [8] and exactly in [9,10]. In spite of large linear mass density of the string μ ($\sim 10^{22}$ g/cm) the space–time metric is not highly curved near the string and for a static, cylindrically symmetric cosmic string is conical and hence flat[1]

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1-4\mu)^2 r^2\, d\varphi^2 \tag{1.1}$$

with deficit angle $\triangle\varphi = 8\pi\mu$. The effects of the quantum-mechanical scattering of a test particle on a string were estimated in [11–13]. The vacuum polarization effect of quantum fields in the string background is considerably large near the string (see, for example, [14]). So it can significantly modify space–time metric in the vicinity of the string. The backreaction of the vacuum energy–momentum tensor on space–time metric was first investigated by Hiscock [15] in linear perturbations within the semiclassical approach. If the cosmic string carries a magnetic flux, the vacuum polarization has also contribution from the Bohm–Aharonov interactions [16,17]. In this case, the vacuum expectation value of the energy–momentum tensor of quantum fields was derived both for massless [18] and massive field [19–21], but backreaction of the vacuum polarization was analyzed in detail only for massless field [22].

As known, the energy–momentum tensor in the case of material field of zero mass is equivalent to the case of massive field at small distances. But, as one can see from [20], in the case of massless material field the physical peculiarities of the tensor components behavior at finite distances are lost. In particular, the short-range exponentially decreasing of the tensor components is absent, and, more interesting, the tensor components of the massless field in principle lose possibility of changing its sign under moving away from the string. Hence, the case of the massless field is a first order approximation of the general massive case. To illustrate above, we refer to [20], where the massive scalar field was detailed considered and the case of zero mass is a simple constant on the figures for dimensionless tensor component ($r^4 t^{00}, r^4 t^{rr}, r^6 t^{\varphi\varphi}, r^4 t^{33}$) instead of evidently complicated structure. In this respect, it seems to be of interest to carefully consider the backreaction in the more physical realistic and the common case of massive quantum field to see possible new features arising from the massiveness.

In this Letter, we use the analytically obtained result [20] to explicitly investigate in the linear approximation the backreaction of the massive field on the space–time metric and physical consequences of one.

In Section 2 we generalize the linear approximation method [15] to the case of arbitrary field in the background of cosmic string. Using results of Section 2 and [20], in Section 3 we analytically find expressions for the modified metric, Newtonian gravitation potential and deficit angle, that are analyzed in Sections 4, 5 in detail. Discussions of obtained results can be found in concluding remarks, some mathematical aspects are placed in Appendix A.

2. Perturbative approach

The exterior metric of a static, cylindrically symmetric cosmic string (with or without the magnetic flux Φ) is Eq. (1.1). This metric induces non-zero vacuum expectation values of the energy–momentum tensor $\langle T^\mu_\nu \rangle$ of a quantum field. We are interested in considering the backreaction of this energy–momentum tensor on a string's metric. To do this in a semiclassical approach, one has to solve the Einstein equations

$$G^\mu_\nu = 8\pi \langle T^\mu_\nu \rangle. \tag{2.1}$$

As we pointed out in introduction, such problem for the massless fields was first solved by Hiscock in [15]. In this section, we follow [15] to obtain the result for any type of fields.

The general static, cylindrically symmetric and invariant under Lorentz boosts along the z-axis metric has the form

$$ds^2 = e^{2\phi(r)}\left(-dt^2 + dz^2 + dr^2\right) + e^{2\psi(r)}\, d\varphi^2. \tag{2.2}$$

[1] Here and over all the paper we use Planck units: $G = \hbar = c = 1$ in which $\mu \sim 10^{-6}$.

Corresponding components of the Einstein tensor are

$$G_t^t = G_z^z = e^{-2\phi}\big(\phi'' + \psi'' + (\psi')^2\big), \tag{2.3}$$

$$G_r^r = e^{-2\phi}\big((\phi')^2 + 2\phi'\psi'\big), \tag{2.4}$$

$$G_\varphi^\varphi = e^{-2\phi}\big((\phi')^2 + 2\phi''\big), \tag{2.5}$$

where prime means the derivative by r.

If $G_\nu^\mu = 0$ in the exterior of a string, we have

$$\phi_0 = 0, \qquad \psi_0 = \ln(\alpha r). \tag{2.6}$$

To join interior and exterior solutions one have to put $\alpha = 1 - 4\mu$ (see [9] for details) and we recover (1.1).

Since $\langle T_\nu^\mu \rangle$ is small quantum correction, we can expand the solution of (2.1) about the background metric (2.6):

$$\phi = \phi_0 + \phi_1, \qquad \psi = \psi_0 + \psi_1, \tag{2.7}$$

where ψ_0 and ϕ_0 are from (2.6), and ϕ_1 and ψ_1 are supposed to be the first order of smallness, same as $\langle T_\nu^\mu \rangle$.

In the first order approximation, Eq. (2.1) takes form

$$\phi_1'' + \psi_1'' + \frac{2}{r}\psi_1' = 8\pi \langle T_t^t \rangle, \tag{2.8}$$

$$\frac{2}{r}\phi_1' = 8\pi \langle T_r^r \rangle, \tag{2.9}$$

$$2\phi_1'' = 8\pi \langle T_\varphi^\varphi \rangle, \tag{2.10}$$

and the exterior metric (2.2) modifies to

$$ds^2 = \big(1 + 2\phi_1(r)\big)\big[-dt^2 + dz^2 + dr^2\big] + (1-4\mu)^2 r^2 (1 + 2\psi_1)\, d\varphi^2. \tag{2.11}$$

Eqs. (2.9), (2.10) that define ϕ_1 function are adjusted if $\langle T_r^r \rangle + r(\langle T_r^r \rangle)' = \langle T_\varphi^\varphi \rangle$ which is just a r-component of the covariant conservation condition for the energy–momentum tensor ($\nabla_\mu \langle T_\nu^\mu \rangle = 0$). We propose it to be justified.

Using substitution $\psi_1' = \chi / r^2$, solution of Eqs. (2.8), (2.9) can be easily found:

$$\phi_1(r) = 4\pi \int_\infty^r dr' \cdot r' \langle T_r^r(r') \rangle, \tag{2.12}$$

$$\psi_1(r) = 4\pi \int_\infty^r \frac{dr'}{r'^2} \int_\infty^{r'} dr'' \cdot r''^2 \big[2\langle T_t^t(r'') \rangle - \langle T_\varphi^\varphi(r'') \rangle\big]. \tag{2.13}$$

Lower limits of integration defined so the $\phi_1(r)$ and $\psi_1(r)$ to be vanishing at infinity.[2] In other words, we neglect the homogeneous solution of (2.8)–(2.10) as having no relation to our effect.

[2] We expect the induced vacuum expectation values of energy–momentum tensor to be decreasing function of distance from the string, so it is natural to choose the constants of integration so the metric (2.11) is flat at infinity.

It is more convenient to introduce new radial coordinate ρ, which is the measure of proper distance from the string:

$$d\rho = \sqrt{1+2\phi_1(r)}\,dr \implies \rho \approx r + \int_\infty^r \phi_1(r')\,dr', \tag{2.14}$$

where we again chose the arbitrary constant so ρ and r to be equal at infinity. With the same accuracy up to linear terms

$$r \approx \rho - \int_\infty^\rho \phi_1(\rho')\,d\rho', \ \phi_1(r) \approx \phi_1(\rho) \quad \text{and} \quad \psi_1(r) \approx \psi_1(\rho). \tag{2.15}$$

Using (2.14) and (2.15) we can rewrite the induced metric (2.11) in the form

$$ds^2 = \big(1+2\phi_1(\rho)\big)\big[-dt^2+dz^2\big] + d\rho^2 + (1-4\mu)^2\rho^2\left(1+2\psi_1(\rho) - \frac{2}{\rho}\int_\infty^\rho \phi_1(\rho')\,d\rho'\right)d\varphi^2. \tag{2.16}$$

The condition of validity of this result is the smallness of first order perturbation comparing to one:

$$\big|\phi_1(\rho)\big| \ll 1, \qquad \big|\psi_1(\rho)\big| \ll 1. \tag{2.17}$$

Newtonian gravitational potential V is recovered from the g_{00} component of the metric as $g_{00} = -(1+2V)$, so it is $\phi_1(\rho)$ in our case. Gravitational force acting at the probe particle with unit mass is

$$f(\rho) = -\phi_1'(\rho) = -4\pi\rho\langle T_r^r(\rho)\rangle, \tag{2.18}$$

where we used (2.9). The length L of the circumference of constant ρ in (2.16) is

$$L = \rho(2\pi - \triangle\varphi),$$

where

$$\triangle\varphi = 2\pi\left(4\mu + (1-4\mu)\left[\frac{1}{\rho}\int_\infty^\rho \phi_1(\rho')\,d\rho' - \psi_1(\rho)\right]\right) \tag{2.19}$$

is a deficit angle.

3. Singular magnetic flux

In this section we will consider the particular case of a massive charged scalar field in the background of a singular massless ($\mu = 0$) and caring magnetic flux string. Vacuum expectation value of the induced energy–momentum tensor of a scalar field was computed in [20]:

$$\langle T_t^t\rangle = \langle T_z^z\rangle = -\frac{16\sin(F\pi)}{(4\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\big[(2F-1)\operatorname{arccosh}\upsilon\big]$$
$$\times\, \upsilon^{-3}\big\{\big[1+2(1-4\xi)\upsilon^2\big]K_2(2mr\upsilon) - 2(1-4\xi)mr\upsilon^3K_3(2mr\upsilon)\big\}, \tag{3.1}$$

$$\langle T_r^r\rangle = -\frac{16\sin(F\pi)}{(4\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\big[(2F-1)\operatorname{arccosh}\upsilon\big]\upsilon^{-3}\big(1-4\xi\upsilon^2\big)K_2(2mr\upsilon), \tag{3.2}$$

$$\langle T_\varphi^\varphi \rangle = -\frac{16\sin(F\pi)}{(4\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}} \cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr] \times \upsilon^{-3}\bigl(1-4\xi\upsilon^2\bigr)\bigl\{K_2(2mr\upsilon) - 2mr\upsilon K_3(2mr\upsilon)\bigr\}, \tag{3.3}$$

where m is the mass of a scalar field, F is the fractional part ($0 < F < 1$) of the string's magnetic flux Φ (in the units of quantum flux $2\pi\hbar/e$) and ξ is the coupling constant of the scalar field to the scalar curvature of the space–time (for details see [20]).

Using (3.2) and general relations (2.12), (2.18), one can obtain the following expressions for the gravitational force acting at a point particle of unit mass and for the gravitational potential:

$$f(\rho) = \frac{\sin(F\pi)}{\pi^2}\cdot\frac{m^2}{\rho}\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]\upsilon^{-3}\bigl(1-4\xi\upsilon^2\bigr)K_2(2m\rho\upsilon), \tag{3.4}$$

$$\phi_1(\rho) = \frac{\sin(F\pi)}{2\pi^2}\cdot\frac{m}{\rho}\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]\upsilon^{-4}\bigl(1-4\xi\upsilon^2\bigr)K_1(2m\rho\upsilon). \tag{3.5}$$

The deficit angle (2.19) has the form (see Appendix A):

$$\triangle\varphi(\rho) = \frac{\sin(F\pi)}{\pi}\cdot\frac{m}{\rho}\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]\upsilon^{-4} \times\bigl[\bigl(3-2(1-2\xi)\upsilon^2\bigr)G(2m\rho\upsilon) + 4\bigl(1-(1-2\xi)\upsilon^2\bigr)K_1(2m\rho\upsilon)\bigr], \tag{3.6}$$

where

$$G(z) = \int_\infty^z K_0(z)\,dz = \frac{\pi}{2}\bigl(z\bigl[K_0(z)L_{-1}(z) + K_1(z)L_0(z)\bigr] - 1\bigr) \tag{3.7}$$

and $L_\nu(z)$ is the modified Struve function of order ν [23].

Asymptotics of (3.5) and (3.6) at small and large distances from the string could be easily computed using the asymptotical expressions for (3.1)–(3.3) given in [20]. For the gravitational potential one has

$$\phi_1(\rho) \sim \frac{F(1-F)\gamma(F,\xi)}{12\pi\rho^2}, \quad \rho \ll \frac{1}{m}, \tag{3.8}$$

$$\phi_1(\rho) \sim (1-4\xi)\frac{\sin F\pi}{8\pi}\cdot\frac{e^{-2m\rho}}{\rho^2}, \quad \rho \gg \frac{1}{m} \tag{3.9}$$

and for the deficit angle

$$\triangle\varphi(\rho) \sim \frac{2F(1-F)\delta(F,\xi)}{3\rho^2}, \quad \rho \ll \frac{1}{m}, \tag{3.10}$$

$$\triangle\varphi(\rho) \sim (4\xi-1)\frac{\sin \pi F}{4}\cdot\frac{e^{-2m\rho}}{\rho^2}, \quad \rho \gg \frac{1}{m}, \tag{3.11}$$

where we use

$$\gamma(F,\xi) = F(1-F) - 2(6\xi-1), \tag{3.12}$$

$$\delta(F,\xi) = F(1-F) + 6\xi - 1. \tag{3.13}$$

Finally, the general expression (2.16) for the metric in our case ($\mu = 0$) takes form:

$$ds^2 = \bigl(1 + 2\phi_1(\rho)\bigr)\bigl[-dt^2 + dz^2\bigr] + d\rho^2 + \rho^2\left(1 - \frac{\triangle\varphi(\rho)}{\pi}\right)d\varphi^2, \tag{3.14}$$

where $\phi_1(\rho)$ and $\triangle\varphi(\rho)$ are defined in Eqs. (3.5), (3.6). Analyzing Eqs. (3.5), (3.6) one can conclude that linear corrections to the metric has a symmetry $F \leftrightarrow 1 - F$ like as components of the energy–momentum tensor (3.1)–(3.3).

4. Gravitational potential

Consider the expression for the gravitational potential (3.5). The function integrated over υ is product of $K_1(2m\rho\upsilon)$ and $W(\upsilon)$, where

$$W(\upsilon) = \frac{\cosh[(2F-1)\operatorname{arccosh}\upsilon]}{\upsilon^4\sqrt{\upsilon^2 - 1}}\bigl(1 - 4\xi\upsilon^2\bigr).$$

In the region $\upsilon \in [1,\infty)$ $W(\upsilon)$ is negative if $\xi > 1/4$ and once change the sign at $\upsilon = \frac{1}{2\sqrt{\xi}}$ if $\xi < 1/4$. At the same time $K_1(2m\rho\upsilon)$ is decreasing positive function, which at $\upsilon \sim \frac{1}{m\rho}$ become less than one and exponentially goes to zero with increasing υ. Analyzing this product of functions, one can conclude that the integral in (3.5) is negative for all values of ρ if $\xi > 1/4$ and may change its sign at some value of ρ if $\xi < 1/4$ but only once.

To clarify gravitational potential behavior in the region $\xi < 1/4$ one need to analyse sign of the asymptotical expressions (3.8) and (3.9). For $\rho \gg 1/m$ one can immediately get

$$\phi_1(\rho) \to +0, \qquad \xi < \frac{1}{4} \quad \text{and} \quad \phi_1(\rho) \to -0, \qquad \xi > \frac{1}{4}. \tag{4.1}$$

For knowing sign of $\phi_1(\rho)$ asymptotic at small distances ($\rho \ll 1/m$) one has to analyse $\gamma(F,\xi)$. Considering it as a function of F it is easy to see that $\gamma(F,\xi)$ is negative for all values of F if $\xi > 3/16$, positive for all values of F if $\xi \leqslant 1/6$ and its sign depends on F otherwise: $\gamma(F,\xi) > 0$ if $F \in (F_p, 1 - F_p)$ and $\gamma(F,\xi) < 0$ if $F \in (0, F_p) \cup (1 - F_p, 1)$, where

$$F_p = \frac{1 - \sqrt{3(3 - 16\xi)}}{2}. \tag{4.2}$$

Using above, one can conclude that there are three different types of gravitational potential behavior:

Type a: $\xi \in (1/4, \infty)$, $\quad F \in (0,1)$.

In this case, $\gamma(F,\xi) < 0$ at all F. So at small distances $\phi_1(\rho)$ behaves as $-1/\rho^2$ and asymptotically approaches to zero from below at large ρ. As it cannot intersect zero more than once, it cannot intersect it at all. Analogous we conclude that it has not extremes. So, it is monotonic and attractive function at all distances.

Type b: $\begin{cases} \xi \in (3/16, 1/4), & F \in (0,1), \\ \xi \in (1/6, 3/16], & F \in \bigl(0, F_p\bigr) \cup (1 - F_p, 1). \end{cases}$

In this range of parameters, $\gamma(F,\xi) < 0$ and gravitational potential is attractive at small distances, but since $\xi < 1/4$ it is repulsive at large ρ. So, with increasing ρ, $\phi_1(\rho)$ increases as $-1/\rho^2$ at $\rho \ll 1/m$, then at some ρ intersects zero, reaches its maximum value (at $\rho \sim 1/m$ according to numerical computation) and decreases to zero from above.

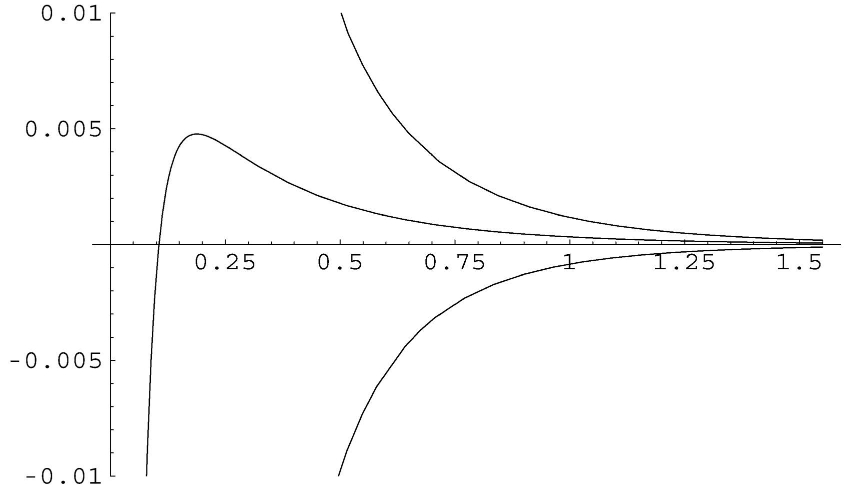

Fig. 1. Gravitational potential in F-independent regions for $F = 1/2$ and $\xi = 0.26$ (type a), 0.19 (type b), 0.14 (type c) correspondingly from down to up.

Type c: $\begin{cases} \xi \in (-\infty, 1/6], & F \in (0,1), \\ \xi \in (1/6, 3/16), & F \in (F_p, 1 - F_p). \end{cases}$

In this case, $\gamma(F, \xi) > 0$ and this means that at small distances $\phi_1(\rho)$ behaves as $1/\rho^2$. Since it is the case of $\xi < 1/4$, then $\phi_1(\rho)$ asymptotically approaches to zero from above at large distances. Analogously to the previous case we conclude that gravitational potential in this range of parameters is repulsive, droningly decreasing function.

Our argumentation fails if $\xi = 1/4$ or $F = F_p$, $F = 1 - F_p$, because in this case asymptotical expressions (3.8), (3.9) vanishes and we need the next terms of expansion. Frequently considered in the literature cases $\xi = 0$ (so-called minimal coupling) and $\xi = 1/6$ (conformal coupling) belong to type c that corresponds to repulsion at all distances.

We plot $\phi_1(\rho)$ (see Figs. 1, 2) to see general features of the gravitational potential behavior patently. Here variable $m\rho$ is along x-axis and dimensionless gravitational potential $\phi_1(\rho)/m^2$ is along y-axis. In the regions where type of the gravitational potential behavior does not depend on the magnetic flux F, the ξ-dependence of the gravitational potential is presented in Fig. 1. In the region where type of gravitational potential behavior is sensitive to the magnetic flux ($\xi \in (1/6, 3/16)$), we illustrate the gravitational potential as F function in Fig. 2.

The maximum amplitude of the local gravitational potential is at half-integer value of flux ($F = 1/2$). In the F-sensitive alternating-sign part of the region ($1/6 < \xi < 3/16$, $F \in (0, F_p) \cup (1 - F_p, 1)$), effect is increasing under going from zero flux to the border points $F = F_p$.

Condition (2.17) of validity of our linear approximation is violated near the string. Using asymptotical expression (3.8) for the gravitational potential at small distances and recovering the dimension, one can rewrite the condition of validity in the form

$$\frac{l_p^2}{\rho^2} \ll 1, \tag{4.3}$$

where $l_p \sim 10^{-33}$ cm is Planck length. This is what we expected since, at Planck scales, the semiclassical approach (2.1) is violated, and we cannot consider the gravitational field as a background of quantum processes.

Finishing this section it will be useful to note that since gravitational force (3.4) is a derivative of (3.5), the gravitational force behavior is similar to the behavior of gravitational potential.

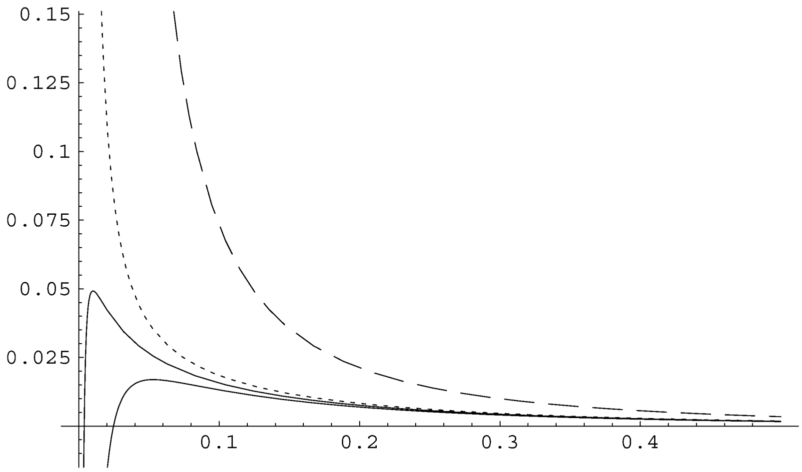

Fig. 2. Gravitational potential at the F-sensitive region ($\xi \in (\frac{1}{6}, \frac{3}{16})$) for the cases of $\xi = 0.18$, $F_p = 0.2$. Solid lines correspond F from the region $F \in (0, F_p) \cup (1 - F_p, 1)$: $F = 0.19$, 0.1995 accordingly from down to up. For the region $F \in (F_p, 1 - F_p)$ dotted line corresponds to $F = 0.21$, dashed line to $F = 1/2$.

5. Deficit angle

The middle distance behavior of the deficit angle (3.6) is not so clear as for the gravitational potential (3.5). So one can differentiate the deficit angle behavior at three types depending on the asymptotical behavior. Using (3.10), (3.11) we obtain:

Type 1: $\xi \in (1/4, \infty)$, $\quad F \in (0, 1)$.

Deficit angle is positive at small and large distances. At small distances it behaves as $1/\rho^2$ and exponentially decreases at $\rho \gg 1/m$.

Type 2: $\begin{cases} \xi \in [1/6, 1/4), & F \in (0, 1), \\ \xi \in (1/8, 1/6), & F \in (F_d, 1 - F_d). \end{cases}$

At distances $\rho \ll 1/m$ the deficit angle is positive and behaves as $1/\rho^2$, but at large distances it change its sign (at least once) and approaches to zero from below.

Type 3: $\begin{cases} \xi \in (-\infty, 1/8), & F \in (0, 1), \\ \xi \in [1/8, 1/6), & F \in (0, F_d) \cup (1 - F_d, 1). \end{cases}$

Deficit angle is negative at small and large distances.

Here we used notation

$$F_d = \frac{1 - \sqrt{3(8\xi - 1)}}{2}. \tag{5.1}$$

We plot $\triangle\varphi(\rho)$ (see Figs. 3, 4) to see the general features of the deficit angle behavior evidently. Here variable $m\rho$ is along x-axis and the dimensionless deficit angle $\triangle\varphi(\rho)/m^2$ is along y-axis. In the regions where type of the deficit angle behavior does not depend on the magnetic flux F, the ξ-dependence of the gravitational potential is presented in Fig. 3. In the region where type of the deficit angle behavior is sensitive to the magnetic flux ($\xi \in (1/8, 1/6)$), we illustrate the deficit angle as F function in Fig. 4.

The maximum amplitude of local deficit angle is at the half-integer value of the flux ($F = 1/2$) except the F-sensitive part of region ($1/8 < \xi < 1/6$) where if $F \in (F_d, 1 - F_d)$ the effect is minimal at $F = 1/2$ and take

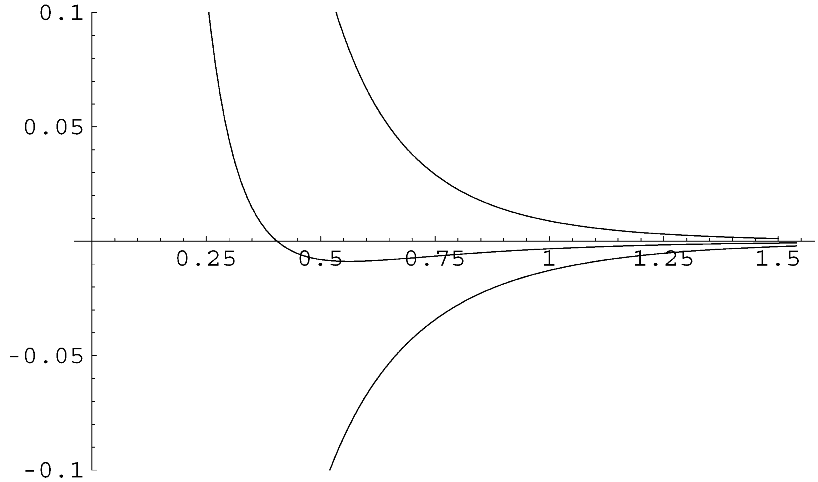

Fig. 3. Deficit angle in F-independent regions for $F = 1/2$ and $\xi = 0.26$ (type 1), 0.17 (type 2), 0.1 (type 3) correspondingly from up to down.

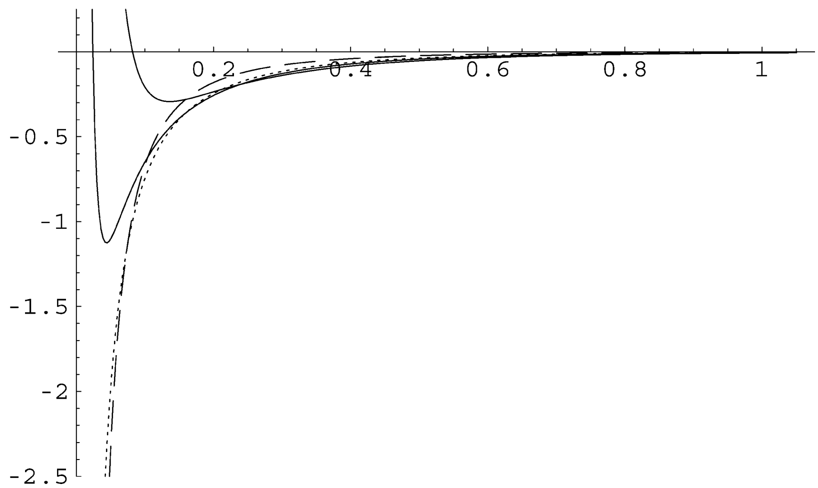

Fig. 4. Deficit angle at the F-sensitive region ($\xi \in (\frac{1}{8}, \frac{1}{6})$) for the cases of $\xi = 0.14$, $F_d = 0.2$. Solid lines correspond F from the region $F \in (F_c^d, 1 - F_d)$: $F = 0.5$, 0.25 correspondingly from up to down. For the region $F \in (0, F_d) \cup (1 - F_d, 1)$ dotted line corresponds to $F = 0.19$, dashed line to $F = 0.1$.

its maximum peak at the border points $F = F_d$; for the case $F \in (0, F_d) \cup (1 - F_d, 1)$ the lines corresponding different values of F goes to negative infinity near the string and intersects under moving away from one.

Condition of validity of our result for the deficit angle coincides with (4.3) and, hence, do not lead to the new restrictions.

6. Concluding remarks

In the semiclassical approach, we computed the gravitational effect caused by the vacuum polarization of the massive charged scalar field in the background of a singular massless carrying magnetic flux cosmic string. Corrections (3.5), (3.6) to the metric components (3.14) depend periodically on the cosmic string flux (Φ) (i.e., depends on only its fractional value F), has a symmetry $F \leftrightarrow 1 - F$ and vanish at its integer value ($\Phi = n$).

It turned out, that behavior of the gravitational potential can be divided at 3 types depending on F and coupling constant ξ. This three types are: attractive behavior over all distances, repulsive behavior[3] over all distances and alternating-sign behavior (gravitational potential is negative near the string but change its sign and becomes positive under moving away from it). The areas of F and ξ parameters for different types are pointed out in Section 4. Gravitational potential was found to be repulsive in the commonly considered cases of $\xi = 0$ and $\xi = 1/6$.

To see general features of gravitational potential behavior patently we plot $\phi_1(\rho)$ (see Figs. 1, 2). Here variable $m\rho$ is along x-axis and dimensionless gravitational potential $\phi_1(\rho)/m^2$ is along y-axis. In the regions where type of the gravitational potential behavior does not depend on the magnetic flux F, the ξ-dependence of the gravitational potential is presented in Fig. 1. In the region where type of the gravitational potential behavior is sensitive to the magnetic flux ($\xi \in (1/6, 3/16]$), we illustrate the gravitational potential as F function in Fig. 2.

The behavior of the deficit angle can be also divided in 3 types. The areas of F and ξ parameters for different types are pointed out in Section 5. To see general features of the deficit angle behavior patently we plot $\triangle\varphi(\rho)$ (see Figs. 3, 4). Here variable $m\rho$ is again along x-axis and dimensionless deficit angle $\triangle\varphi(\rho)/m^2$ is along y-axis. In the regions where type of the deficit angle behavior does not depend on the magnetic flux F, the ξ-dependence of the deficit angle is presented in Fig. 3. In the region where type of the deficit angle behavior is sensitive to magnetic flux ($\xi \in [1/8, 1/6)$), we plot the deficit angle as F function in Fig. 4.

It is interesting to note that regions of the different types of the gravitational potential and deficit angle behavior strictly speaking are different. From the classical point of view we could expect that for the attractive-type potentials deficit angle is positive and for repulsive-type potentials one is negative (positive deficit angle lead to the bending of light as like it attracts to the string and vice versa). This is a fact for the regions $\xi > 1/4$ (attractive-type potential and positive deficit angle) and $\xi < 1/8$ (repulsive-type potential and negative deficit angle). But at the region $\xi \in (1/8, 1/4)$ our classical reasons fail. For example, if $\xi = 1/6$ (conformal coupling), the gravitational potential of a string is repulsive at all distances, while the deficit angle is alternating-sign.

Near a string the potential and deficit angle behaves as like the scalar field is massless and we recover the result of [22] (for the zero linear mass density of string). But in comparing with [22][4] (see asymptotic expressions (3.8), (3.10)), the massiveness of field gives an essentially new type of behavior that allow gravitational potential and deficit angle change its sign under moving away from the string. One another difference is that the massless scalar field produces a long-range power decreasing potential (3.8) and deficit angle (3.10), while in our case it are short-range exponentially decreasing functions (3.9) and (3.11).

Condition of validity of semiclassical approximation (4.3) is violated near the string at Planck length. It should be noted that we considered simplified analytically solved case of massless ($\mu = 0$) string with zero radius. For realistic string which radius is of the order of the Compton wavelength of the Higgs bosons involved in the phase transitions, the condition (4.3) is satisfied everywhere outside the string.

As was pointed in [22], the contribution to the gravitational effect coming from the Aharonov–Bohm interaction dominates over one coming from the non-zero linear mass density μ of the string. So we can expect that our results will be not changed significantly with taking in to account μ.

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[3] There is no wonder in a repulsive behavior of the gravitation potential at some values of parameters because of violation of the strong and week energy conditions for components of the induced energy–momentum tensor of the massive charged scalar field (3.1)–(3.3) (see [20]).

[4] In [22], the gravitational potential and the deficit angle can be only positive or negative over all distances from string.

Appendix A

The deficit angle is given by (2.19). Consider separately two corresponding terms (in our case $\mu = 0$). Using recurrent relation

$$\frac{K_1(z)}{z} = -K_0(z) - \partial_z K_1(z) \tag{A.1}$$

and Eq. (3.5) for $\phi_1(\rho)$ one can easily obtain

$$\frac{1}{\rho}\int_\infty^\rho \phi_1(\rho')\,d\rho' = -\frac{\sin(F\pi)}{2\pi^2}\cdot\frac{m}{\rho}\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]\upsilon^{-4}\bigl(1-4\xi\upsilon^2\bigr)$$
$$\times\bigl[G(2m\rho\upsilon) + K_1(2m\rho\upsilon)\bigr], \tag{A.2}$$

where $G(z)$ is defined in (3.7).

To compute $\psi_1(\rho)$ one need expression under integral operation in (2.13):

$$2\langle T_t^t\rangle - \langle T_\varphi^\varphi\rangle = -\frac{\sin(F\pi)}{4\pi^3}\left(\frac{m}{\rho}\right)^2\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\mathrm{arccosh}\,\upsilon\bigr]$$
$$\times\upsilon^{-3}\bigl\{\bigl[1+4(1-3\xi)\upsilon^2\bigr]K_2(2m\rho\upsilon) + \bigl[1-2(1-2\xi)\upsilon^2\bigr]\cdot 2m\rho\upsilon\cdot K_3(2m\rho\upsilon)\bigr\}. \tag{A.3}$$

Using relation

$$\int\bigl[b_1K_2(z) + b_2zK_3(z)\bigr]dz = \int\bigl[(b_1+2b_2)K_2(z) - b_2z\partial_zK_2(z)\bigr]dz$$
$$= -\bigl\{b_2zK_2(z) + (b_1+3b_2)\bigl[G(z)+2K_1(z)\bigr]\bigr\}, \tag{A.4}$$

and Eq. (A.3) one can get:

$$\int_\infty^{\rho'} d\rho''\cdot\rho''^2\bigl[2\langle T_t^t(\rho'')\rangle - \langle T_\varphi^\varphi(\rho'')\rangle\bigr]$$
$$= \frac{\sin(F\pi)}{8\pi^3}m\int_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]\upsilon^{-4}\bigl\{\bigl[1-2(1-2\xi)\upsilon^2\bigr]\cdot 2m\rho'\upsilon\cdot K_2(2m\rho'\upsilon)$$
$$+ 2\bigl(2-\upsilon^2\bigr)\bigl[G(2m\rho'\upsilon) + 2K_1(2m\rho'\upsilon)\bigr]\bigr\}. \tag{A.5}$$

Using expression

$$\int\frac{dz}{z^2}\bigl[f_1zK_2(z) + f_2\bigl(G(z)+2K_1(z)\bigr)\bigr]$$
$$= \int dz\left[\frac{K_2(z)}{z}(f_1+f_2) + f_2\left(\frac{G(z)}{z^2} - \frac{K_0(z)}{z}\right)\right]$$
$$= \int dz\left[\frac{K_2(z)}{z}(f_1+f_2) - f_2\partial_z\left(\frac{G(z)}{z}\right)\right] = -\frac{1}{z}\bigl[(f_1+f_2)K_1(z) + f_2G(z)\bigr] \tag{A.6}$$

and Eqs. (A.5), (2.13) one can easily obtain:

$$\psi_1(r) = -\frac{\sin(F\pi)}{2\pi^2}\,\frac{m}{\rho}\int\limits_1^\infty \frac{d\upsilon}{\sqrt{\upsilon^2-1}}\cosh\bigl[(2F-1)\,\mathrm{arccosh}\,\upsilon\bigr]$$
$$\times\,\upsilon^{-4}\bigl\{\bigl(5-4(1-\xi)\upsilon^2\bigr)K_1(2m\rho\upsilon)+2\bigl(2-\upsilon^2\bigr)G(2m\rho\upsilon)\bigr\}. \tag{A.7}$$

After substitution (A.2), (A.7) into (2.13) one gets (3.6).

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