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Procedia Engineering 15 (2011) 1947 – 1951

Procedia Engineering

www.elsevier.com/locate/procedia

Advanced in Control Engineeringand Information Science

Research on Rough Set Model Based on Golden Ratio

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Abstract

How to make decision with pre-defined preference-ordered criteria also depends on the environment of the problem. Dominance rough set model is suitable for preference analysis and probabilistic rough set introduces probabilistic approaches to rough sets. In this paper, new dominance rough set rough set models are given by taking golden ratio into account. Also, we present steps to make decision using new dominance rough set models.

Selection and/or peer-review under responsibility of [CEIS 2011]

Keywords: rough set, dominance rough set, golden ratio, decision making;

1. Introduction

Decision-making is choosing a strategy among many different projects in order to achieve some purposes. Many decision-making problems are characterized by the ranking of objects according to a set of criteria with pre-defined preference-ordered decision classes, such as credit approval[1], stock risk estimation[2], mobile phone alternatives estimation[3], etc.. Models and algorithms were proposed for extracting and aggregating preference relations based on distinct criteria. The underlying objectives are to understand the decision process, to build decision models and to learn decisions rules from data. According to various decision criteria, the decision-making problem is formulated as three different models: high risk decision, usual risk decision and low risk decision. Meanwhile, with different attitude of decision makers for different types of decision models, decision criteria are formulated as five different

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doi:10.1016/j.proeng.2011.08.363

models: optimistic-decision criterion, pessimistic-decision criterion, evenness-decision criterion, minimum-risk-decision criterion, compromise-decision criterion.

Rough set as a mathematical methodology for data analysis were introduced by Pawlak[4,5] with the concept of equivalence classes as its basic principle. Rough set theory adopts the concept of equivalence classes to partition the training instances according to some criteria. Two kinds of partitions are formed in the mining process: lower approximations and upper approximations, from which certain and possible rules are easily derived. It operates only on the data and does not require any added information; it is completely data-driven. The theory of rough sets has been successfully applied to diverse areas, such as pattern recognition, artificial intelligent, machine learning, knowledge acquisition, economy forecast, data mining and so on[6,7]. Pawlak rough set model is constructed based on equivalence relations. These relations are viewed by many to be one of the main limitations when employing the model to complex decision tasks. In multiple criteria decision-making problems, there are preference structures between conditions and decisions. Greco et al.[8] introduced a dominance rough set model that is suitable for preference analysis. They examined the decision-making problem with multiple attributes and multiple criteria, where dominance relations were extracted from multiple criteria and similarity relations were constructed from numerical attributes and equivalence relations were constructed from nominal features. An extensive review of multi-criteria decision analysis based on dominance rough sets is given in [9]. Dominance rough sets have also been applied to ordinal attribute reduction and multi-criteria classification.

In real life, how to make decision with pre-defined preference-ordered criteria also depends on the environment of the problem. This paper is organized as follows. In Section 2, we recall Pawlak rough set , dominance rough set and golden ratio. In section 3, new dominance rough set rough set models are given by taking golden ratio into account. Also, we present steps to make decision using new dominance rough set rough set models. Finally, a brief summary of this paper and some advice about further development of rough set are advanced.

2. Preliminaries

2.1. Pawlak rough set and Dominance rough set

Rough set theory provides a powerful tool for data analysis and knowledge discovery from imprecise, uncertain and incomplete information. In Pawlak rough set, a binary relation is an equivalence relation iff it is reflexive, symmetric and transitive. For every equivalence relation there is a partition and vice versa. Let U be the (finite or infinite) universe of discourse. Let R be a given equivalence relation on U. The family of all equivalence classes is a set which is called the quotient set and denoted by U/R. We use $[x]_R$ to denote an equivalence class in R containing an element $x \in U$.

Definition 1(Rough set) Let U be a non-empty set and R an equivalence relation on U. The pair (U,R) is called an approximation space. $\forall X \subseteq U$, we define operations $\underline{R}$ and $\overline{R}$ as:

$$\underline{R}X = \{x \mid [x]_R \subseteq X\}, \overline{R}X = \{x \mid [x]_R \bigcap X \neq \varnothing\} \quad (1)$$

If $\underline{R}X = \overline{R}X$, then X is a crisp set on U about R. If $\underline{R}X \neq \overline{R}X$, then X is a rough set on U about R. $\underline{R}X$ is called the lower approximation of X and $\overline{R}X$ is the upper approximation of X.

Dominance-based rough set is an extension of rough set theory for multi-criteria decision analysis, introduced by Greco, e.t.al. The main change comparing to the classical rough sets is the substitution of the indiscernibility relation by a dominance relation. The definitions about dominance relation are presented as follows.

Definition 2(dominance relation) Let U be the universe of discourse and $x, y \in U$. Let $\succeq_q$ be a dominance relation to U with reference to criterion $q \in Q$, such that $x \succeq_q y$ means that " x is at least as good as y with respect to criterion q ".

Definition 3 Let U be the universe of discourse and $x \in U$. If $x \succeq_q y$ for all $q \in P$,then $xD_P y$. Given $P \subseteq C$ and $x \in U$, let

$$D_P^+(x) = \{y \,|\, y \in U, yD_P x\},\ D_P^-(x) = \{y \,|\, y \in U, xD_P y\} \quad (3)$$

represent the P-dominating set and the P-dominated set with respect to x, respectively.

Let $Cl = \{Cl_t \,|\, t \in T\}$, $T = 1, 2, \cdots, n$ be a set of classes of U such that each $x \in U$ belongs to one and only one class $Cl_t \in Cl$. We assume that all r, s, such that $r > s$, and each element of Cl_r is preferred to each element Cl_s. In other words, if $\succeq$ is a comprehensive outranking relation on U, then it is supposed that

$$[x \in Cl_r, y \in Cl_s, r > s] \Rightarrow x \succ y; \quad (2)$$

Where $x \succ y$ means $x \succeq y$ and not $y \succeq x$.

We can define unions of classes relative to a particular dominated or dominating class; these unions of classes are called upward and downward unions of classes and are defined, respectively, as

$$Cl_t^{\succeq} = \bigcup_{s \geq t} Cl_s, Cl_t^{\preceq} = \bigcup_{s \leq t} Cl_s. \quad (3)$$

Definition 4(dominance rough set) Let U be the universe of discourse and P be a set of criteria. The P-lower and P-upper approximations of $Cl_t^{\succeq}$, $t \in \{1, 2, \cdots, n\}$, with respect to $P \subseteq C$ (denotation $\underline{P}(Cl_t^{\succeq})$ and $\overline{P}(Cl_t^{\succeq})$, respectively), are defined as

$$\underline{P}(Cl_t^{\succeq}) = \{x \,|\, x \in U, D_P^+(x) \subseteq Cl_t^{\succeq}\},$$

$$\overline{P}(Cl_t^{\succeq}) = \bigcup_{x \in Cl_t^{\succeq}} D_P^+(x) = \{x \,|\, x \in U, D_P^-(x) \cap Cl_t^{\succeq} \neq \varnothing\}. \quad (4)$$

2.2. Golden Ratio

The golden ratio, usually denoted by the Greek letter Phi (φ), is not only the fascinating history but also remarkable physical phenomena observed by critical minds of scientists, artists, architects, engineers, naturalists and spiritualists[17,18]. It was introduced to solve a geometrical problem called the problem of division of a line segment in extreme and mean ratio. It is an irrational number, the value of which is given by the proportion AC:AB = AB:BC, where A and C are the endpoints of a line segment, and B is the point on the line segment between A and C such that AC:AB=AB:BC. The Golden Ratio is $\varphi = (1+\sqrt{5})/2 \approx 1.6180339887$. Alternatively, the ratio of the minor sub segment and the major sub segment equals 0.618. The discovery of the number φ can be attributed to Pythagoras in the 5th century B.C. And Plato predicated it should be the best beautiful thing if it accord to golden ratio in the universe.

Golden ratio is widely used in many fields, such as in industry, architecture and upholsters, agriculture and daily life. In agriculture, the famous mathematician Hua Luogeng spread "optimization method"[19] which is named "0.168 method". This method can obtain the excellence blue print if triers choose the first experiment area in the place where it just in the 0.168 of the whole area. And this "0.168 method" bring much value in economy. In stock market, Eliot Wave Theory which is adopted broadly by investors was presented rooting in the golden ratio.

Usually, 70:30 which is close to 5:8 also denotes golden ratio. For researching simply, 2/3 denotes golden ratio sometimes and this is the origin of 2/3, 1/3 in the paper.

3. New dominance rough set model based on golden ratio

Since rough set theory was proposed, many researches were done about its theory and applications. An along with the actual need, extended models of rough set were established.

Definition 5 Let U be the universe of discourse. $P \subseteq C$ and P is a set of criteria. For each subset $Cl_t^{\geq}$ of U, we associate two subsets, lower and upper approximations:

$$(1)\ \begin{cases} \underline{P_1}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq \frac{2}{3}\} \\ \overline{P_1}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq 0\} \end{cases},$$

$$(2)\ \begin{cases} \underline{P_2}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| = 1\} \\ \overline{P_2}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq 0\} \end{cases},$$

$$(3)\ \begin{cases} \underline{P_3}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq \frac{2}{3}\} \\ \overline{P_3}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq \frac{1}{3}\} \end{cases},$$

$$(4)\ \begin{cases} \underline{P_4}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| = 1\} \\ \overline{P_4}(Cl_t^{\geq}) = \{x \,\middle|\, x \in U, \left| D_P^+(x) \cap Cl_t^{\geq} \right| \Big/ \left| D_P^+(x) \right| \geq \frac{1}{3}\} \end{cases}.$$

Steps to make decision using new dominance rough set rough set models are presented as follows.

Step 1: Discretion of continuous attributes. In literatures, research communities have given countless efforts to discrete continuous attributes[10,11].

Step 2: Sorting. Sort every criteria based on object from decision making.

Step 3: Attribute Reduction. The attribute reduction algorithm is key technologies of rough set.

Step 4: Decision Rules acquisition. Rule acquisition for decision-making also is a research focus of concern.

4. Conclusions

This paper introduces new models to make decision considering five decision criterions and multiple criteria. New dominance rough sets models are based on golden section have parameters as 1/3 and 2/3. Meanwhile, the starting point of the probability idea is introduced to construct rough set model based on golden section. Decision models are expressed by means of rough set and golden section. These models deals with partial classification by different environmental conditions and can be used in many fields.

Although the proposed method works well in dealing with problems in real life, it is just a beginning. There is still much work to be done in this field. The attribute reduction should be researched further.

Acknowledgements

This work is supported by Doctorate Foundation of Donghua University(NO.BC200923) and Doctorate Foundation of Shanghai(No. 11692191400)

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