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Direct Adaptive Control for Infinite-Dimensional Symmetric Hyperbolic Systems

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Abstract

Given a linear continuous-time infinite-dimensional plant on a Hilbert space and disturbances of known and unknown waveform, we show that there exists a stabilizing direct model reference adaptive control law with certain disturbance rejection and robustness properties. The closed loop system is shown to be exponentially convergent to a neighborhood with radius proportional to bounds on the size of the disturbance. The plant is described by a closed densely defined linear operator that generates a continuous semigroup of bounded operators on the Hilbert space of states.
Symmetric Hyperbolic Systems of partial differential equations describe many physical phenomena such as wave behavior, electromagnetic fields, and quantum fields. To illustrate the utility of the adaptive control law, we apply the results to control of symmetric hyperbolic systems with coercive boundary conditions.

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1. Introduction

Many control systems are inherently infinite dimensional when they are described by partial differential equations. Currently there is renewed interest in the control of these kinds of systems especially in flexible aerospace structures and the quantum control field[1-2]. It is especially of interest to control these systems adaptively via finite-dimensional controllers. In our work[3-6], we have accomplished direct model reference adaptive control and disturbance rejection with very low order adaptive gain laws for MIMO finite dimensional systems. When systems are subjected to an unknown internal delay, these systems are also infinite dimensional in nature. The adaptive control theory can be modified to handle this situation[7]. However, this approach does not handle the situation when partial differential equations describe the open loop system.

This paper considers the effect of infinite dimensionality on the adaptive control approach previously published[4-6]. We will show that the adaptively controlled system is globally stable, but the adaptive error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design. We will prove a robustness result for the adaptive control which extends the published results[4].

Our focus will be on applying our results to Symmetric Hyperbolic Systems of partial differential equations. Such systems, originated by K.O. Friedrichs and P. D. Lax, describe many physical phenomena such as wave behavior, electromagnetic fields, and the theory of relativistic quantum fields[15-18]. To illustrate the utility of the adaptive control law, we apply the results to control of symmetric hyperbolic systems with coercive boundary conditions.

2. Robustness of the error system

We begin by considering the definition of Strict Dissipativity for infinite-dimensional systems and the general form of the "adaptive error system" to later prove stability. The main theorem of this section will be utilized in the following section to assess stability of the adaptive controller with disturbance rejection for linear diffusion systems. Noting that there can be some ambiguity in the literature with the definition of strictly dissipative systems, we modify the suggestion of Wen[8] for finite dimensional systems and expand it to include infinite dimensional systems.

Definition ***1***: *The triple (A_c, B, C) is said to be* ***Strictly Dissipative*** *if* A_c *is a densely defined ,closed operator on* $D(A_c) \subseteq X$ *a complex Hilbert space with inner product* (x,y) *and corresponding norm* $\|x\| \equiv \sqrt{(x,x)}$ *and generates a* C_0 *semigroup of bounded operators* $U(t)$*, and* (B,C) *are bounded finite rank input/output operators with rank M where* $B: R^m \to X$ and $C: X \to R^m$ *. In addition there exist symmetric positive bounded operatorz P,Q* on X *such that* $p_{\min}\|x\|^2 \le (Px,x) \le p_{\max}\|x\|^2$*, i.e. P is bounded and coercive, and* $\alpha\|x\|^2 \le (Qx,x)$ *where* $\alpha > 0$ *and* $e \in D(A_c)$ *with*

$$\begin{cases} \mathrm{Re}(PA_c e, e) \equiv \frac{1}{2}[(PA_c e, e) + (e, PA_c e)] \le -\alpha \|e\|^2 \\ PB = C^* \end{cases} \tag{1}$$

We say that (A,B,C) is *Almost Strictly Dissipative (ASD)* when there exists a $G_* \in \Re^{mxm}$ such that (A_c,B,C) is strictly dissipative with $A_c \equiv A + BG_*C$. Note that if $P = I$ in (1), by the Lumer-Phillips Theorem[11], we would have $\|U_c(t)\| \le e^{-\sigma t};\quad t \ge 0$. The following theorem shows that convergence to a neighborhood with radius determined by the supremum norm of ν is possible for a specific type of adaptive error system. In the following, we denote $\|M\|_2 \equiv \sqrt{\mathrm{tr}(M\gamma^{-1}M^T)}$ as the trace norm of a matrix *M* where $\gamma > 0$.

Theorem 1: Consider the coupled system of differential equations

$$\begin{cases} \dfrac{\partial e}{\partial t} = A_c e + B\underbrace{\left(G(t) - G^*\right)}_{\Delta G} z + \nu; \quad e_y = Ce \\ \dot{G}(t) = -e_y z^{\mathrm{T}} \gamma - aG(t) \end{cases} \tag{2}$$

where $e, \nu \in D(A_C), z \in R^m$ *and* $\begin{bmatrix} e & G \end{bmatrix}^{\mathrm{T}} \in \bar{X} \equiv XxR^{mxm}$ *is a Hilbert space with inner product* $\left(\begin{bmatrix} e_1 \\ G_1 \end{bmatrix}, \begin{bmatrix} e_2 \\ G_2 \end{bmatrix}\right) \equiv (e_1, e_2) + \mathrm{tr}\left(G_1 \gamma^{-1} G_2\right)$, *norm* $\left\| \begin{bmatrix} e \\ G \end{bmatrix} \right\| \equiv \left(\|e\|^2 + \mathrm{tr}(G\gamma^{-1}G)\right)^{\frac{1}{2}}$ *and where* $G(t)$ *is the mxm adaptive gain matrix and* γ *is any positive definite constant matrix, each of appropriate dimension.*

Assume the following:

i.) (A, B, C) *is ASD with* $A_c \equiv A + BG_* C$

ii.) there exists $M_G > 0$ *such that* $\sqrt{\mathrm{tr}(G^* G^{*T})} \le M_G$

iii.) there exists $M_\upsilon > 0$ *such that* $\sup_{t \ge 0} \|\nu(t)\| \le M_\nu < \infty$

iv.) there exists $\alpha > 0$ *such that* $a \le \dfrac{\alpha}{p_{\max}}$, *where* $p_{\max}$ *is defined in Definition 1*

v.) the positive definite matrix γ *satisfies* $\mathrm{tr}(\gamma^{-1}) \le \left(\dfrac{M_\nu}{aM_G}\right)^2$,

then the gain matrix, G(t), is bounded, and the state, e(t) exponentially with rate e^{-at} approaches the ball of radius

$$R_* \equiv \frac{\left(1 + \sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_\nu$$

Proof of Theorem 1: This has been proven by the authors[21].

3. Robust adaptive regulation with disturbance rejection

In order to accomplish some degree of disturbance rejection in a MRAC system, we make use of a definition[7]:

Definition 2: *A disturbance vector* $u_D \in R^q$ *is said to be* ***persistent*** *if it satisfies disturbance generator equations:*

$$\begin{cases} u_D(t) = \theta z_D(t) \\ \dot{z}_D(t) = F z_D(t) \end{cases} \quad \text{or} \quad \begin{cases} u_D(t) = \theta z_D(t) \\ z_D(t) = L\phi_D(t) \end{cases} \tag{3}$$

where F is a marginally stable matrix and $\phi_D(t)$ is a vector of known functions forming a basis for all the possible disturbances. This is known as "disturbances with known waveforms but unknown amplitudes".

Consider the Linear Infinite Dimensional Plant with Persistent Disturbances given by:

$$\frac{\partial x}{\partial t}(t) = Ax(t) + Bu(t) + \Gamma\, u_D(t) \tag{4a}$$

$$Bu \equiv \sum_{i=1}^{m} b_i u_i \tag{5b}$$

$$y(t) = Cx(t), y_i \equiv (c_i, x(t)), i = 1...m \tag{5c}$$

where $x(0) \equiv x_0 \in D(A)$, $x \in D(A)$ is the plant state, $b_i \in D(A)$ are actuator influence functions, $c_i \in D(A)$ are sensor influence functions, $u, y \in \Re^m$ are the control input and plant output m-vectors respectively, u_D is a disturbance with known basis functions ϕ_D . *We assume the columns of* Γ *are linear combinations of the columns of B (denoted Span(*Γ*)* $\subseteq$ *Span(B)).* The above system must have *output regulation to a neighborhood:*

$$y \xrightarrow[t \to \infty]{} N(0, R) \tag{6}$$

Since the plant is subjected to unknown bounded signals, we cannot expect better regulation than (6). The adaptive controller will have the form:

$$\begin{cases} u = G_e y + G_D \phi_D \\ \dot{G}_e = -yy^T \gamma_e - aG_e \\ \dot{G}_D = -y\phi_D^T \gamma_D - aG_D \end{cases} \tag{7}$$

Using Theorem 1, we have the following corollary about the corresponding direct adaptive control strategy:

Corollary 1: *Assume the following:*

i.) *There exists a gain,* G_e^* *such that the triple* $(A_C \equiv A + BG_e^* C, B, C)$ *is SD, i.e.* (A, B, C) is ASD,

ii.) A *is a densely defined ,closed operator on* $D(A) \subseteq X$ *and generates a* C_0 *semigroup of bounded operators* $U(t)$ *,*

iii.) *Span*$(\Gamma) \subseteq$ *Span*(B)

Then the output $y(t)$ *exponentially approaches a neighborhood with radius proportional to the magnitude of the disturbance,* υ *, for sufficiently small* α *and* γ_i *. Furthermore, each adaptive gain matrix is bounded.*

Proof*:* Proof is omitted due to space limitations.

Corollary 1 provides a control law that is robust with respect to persistent disturbances and unknown bounded disturbances, and, exponentially with rate e^{-at} , produces: $\overline{\lim}_{t\to\infty} \|y(t)\| \le \frac{1+\sqrt{p_{max}}}{\alpha\sqrt{p_{min}}} \|B\| M_v$

4. Symmetric hyperbolic systems

The above robust adaptive controller is illustrated on an *m* input, *m* output Symmetric Hyperbolic Problem*:*

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + B(u + u_D) + v \\ y = Cx \equiv \left[(c_1, x) \quad (c_2, x) \quad (c_3, x) \quad \cdots \quad (c_m, x) \right]^{\mathrm{T}} \end{cases} \tag{8}$$

with inner product $(v,w) \equiv \int_{\Omega} (v^T w) dz$ and Ω is a bounded open set with smooth boundary, and where $B \equiv \begin{bmatrix} b_1 & b_2 & b_3 & \cdots & b_m \end{bmatrix} : \Re^m \to X$ linear; $b_i \in D(A)$, $x(0) \equiv x_0 \in D(A) \subseteq X \equiv L_N^2(\Omega)$, and $C : X \to \Re^m$ linear; $c_i \in D(A)$. For this application we will *assume the disturbances are step functions*. Note that the disturbance functions can be any basis function as long as φ_D is bounded, in particular sinusoidal disturbances are often applicable. So we have $\varphi_D \equiv 1$ and $\begin{cases} u_D = (1) z_D \\ \dot{z}_D = (0) z_D \end{cases}$ which implies $F = 0$ and $\theta_D = 1$.

Let the adaptive control law be $u = G_e y + G_D$ with $\begin{cases} \dot{G}_e = -y y^T \gamma_e - \alpha G_e \\ \dot{G}_D = -y \gamma_D - \alpha G_D \end{cases}$. Define the closed linear operator A with domain $D(A)$ dense in the Hilbert space $X \equiv L^2(\Omega)$ with inner product $(v_1, v_2) \equiv \int_{\Omega} (v_1^T v_2) dz$ as:

$Ax \equiv \sum_{i=1}^{N} A_i \frac{\partial x}{\partial z_i} + A_0 x$ where A_i are *NxN* symmetric constant matrices, A_0 is a real *NxN* constant matrix, and x is an *Nx1* column vector of functions.Thus (8) is a symmetric Hyperbolic System of first order partial differential equations with $A(\xi) \equiv \sum_{i=1}^{N} \xi_i A_i$ which is an NxN symmetric matrix[15]. The Boundary Conditions which define the operator domain $D(A)$ will be *coercive,* i.e. $h^T n = 0$ where $h(x) \equiv 0.5 \begin{bmatrix} x^T A_1 x & x^T A_2 x & x^T A_3 x & \ldots & x^T A_N x \end{bmatrix}$ and $n(z)$ is the outward normal vector on boundary $\partial\Omega$ of the domain $\Omega \subseteq \Re^N$. Now use $u = G_e y + G_D \phi_D = G_e^* y + \underbrace{G_D^* \phi_D}_{u_D} + \underbrace{\Delta G_D \eta}_{w}$ where $\eta \equiv \begin{bmatrix} y & \phi_D \end{bmatrix}^{\mathrm{T}}$ which implies $x_t = \underbrace{[Ax + B G_e^* C x]}_{A_c x} + Bw + v$ which implies $A_c = A + B G_e^* C$. Since the boundary conditions are coercive, we use the Divergence Theorem to obtain

$$
\begin{aligned}
(A_c x, x) &= (Ax, x) + (B G_e^* C x, x) = \int_{\Omega} (x^T \sum_{i=1}^{N} A_i \frac{\partial x}{\partial z_i}) dz + (A_0 x, x) + (B G_e^* C x, x) \\
&= \frac{1}{2} \int_{\Omega} (\underbrace{\nabla \circ h}_{Div(h)}) dz + (A_0 x, x) + (B G_e^* C x, x) = \frac{1}{2} \int_{\Omega} (\underbrace{h^T n}_{=0}) dz + (A_0 x, x) + (B G_e^* C x, x) \\
&= (A_0 x, x) + (B G_e^* C x, x)
\end{aligned}
$$

Assume $b_i = c_i$ or $B^* = C$ and $G_e^* \equiv -g_e^* < 0$. Then we have

$$
(A_c x, x) = (A_0 x, x) + (B G_e^* C x, x) = (A_0 x, x) - g_e^* (Cx, B^* x) = (A_0 x, x) - g_e^* \|Cx\|^2 \le 0
$$

which implies $\mathrm{Re}(A_c x, x) = (A_0 x, x) - g_e^* \|Cx\|^2$ and $B^* = C$ which is **not** quite strictly dissipative. But we have the following result:

Theorem 2: *A_c is a normal operator with compact resolvent; hence it has discrete spectrum, in the sense that it consists only of isolated eigenvalues with finite multiplicity.*

Proof: Proof is omitted due to space limitations.

Consider that $X = E_s \oplus E_u$ where E_s is the stable eigenspace and E_u is the unstable eigenspace with corresponding projections P_s, P_u. Assume that $\dim E_u \equiv N_u$ and $E_u^{\perp} = E$. This implies that P_s, P_u are bounded self adjoint operators. Choose $C \equiv P_u$; this is possible when the unstable subspace is finite-dimensional.

Then we have the following result:

Theorem 3:

$\mathrm{Re}(A_0 x, x) \le -\alpha \left\| P_s x \right\|^2$ for all $x \in D(A)$ implies that (A, B, C) is almost strictly dissipative (ASD).
Proof: Proof is omitted due to space limitations.

Here is a simple first order symmetric hyperbolic system example to illustrate some of the above:

$$\begin{cases} x_t = \underbrace{\begin{bmatrix} \varepsilon & 1 \\ 1 & 0 \end{bmatrix}}_{A_1} x_z + \underbrace{\begin{bmatrix} -\varepsilon & 0 \\ 0 & 0 \end{bmatrix}}_{A_0} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} (u + u_D) \\ y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} x \end{cases}$$

where $\varepsilon > 0$ is small. If we use $G_e^* = -g_* < 0$ where $x \equiv \begin{bmatrix} q_1 & q_2 \end{bmatrix}^{\mathrm{T}}$ this implies that

$$\mathrm{Re}\left(A_c x, x\right) = (A_0 x, x) - g_e^* \left\| Cx \right\|^2 = -\varepsilon \left\| q_1 \right\|^2 - g_* \left\| q_2 \right\|^2 \le -\underbrace{\min(g_*, \varepsilon)}_{\alpha > 0} \left(\left\| q_1 \right\|^2 + \left\| q_2 \right\|^2 \right) \le -\alpha \left\| x \right\|^2$$

Then $(A_c = A + BG_e^* C, B, C)$ is strictly dissipative with $P = I$ and we can apply Theo. 1 and Cor. 1.

5. Conclusions

In Theo. 1 we proved a robustness result for adaptive control under the hypothesis of almost strict dissipativity for infinite dimensional systems. This idea is an extension of the concept of m-accretivity for infinite dimensional systems[9]. In Cor 1, we showed that adaptive regulation to a neighborhood was possible with an adaptive controller modified with a leakage term. This controller could also mitigate persistent disturbances. The results in Theo. 1 can be easily extended to cause model tracking instead of regulation. Also we can relax the requirement that the disturbance enters through the same channels as the control. We applied these results to general symmetric hyperbolic systems using m actuators and m sensors and adaptive output feedback. We showed that under some limitations on operator spectrum that we can accomplish robust adaptive control. This allows the possibility of rather simple direct adaptive control which also mitigates persistent disturbances for a large class of applications in wave behavior, electromagnetic fields, and some quantum fields.

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