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A Counter Example to a Conjecture of D. J. Rose on Minimum Triangulation

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D. J. Rose concluded his discussion on the elimination process in graphs [1] with a conjecture saying that there exists a minimum triangulation $\hat{T}$ of a graph $G = (X, E)$ such that S_0 is a separator clique in $\hat{G} = (X, E \cup \hat{T})$, where S_0 is a separator of G with minimum deficiency. In Fig. 1 we show a graph G, such that $S_0 = \{3, 4, 5, 6, 7, 8\}$ is the only separator of G with deficiency less than or equal to 2. Let T be a minimum triangulation of G using S_0 as a separator clique. Then $|T| = 8$. However, elimination of the vertices of G in the order 6, 1, 7, 2, 9, 3, 4, 5, 8, 10, adds only seven edges. All these facts were confirmed by checking all possibilities by means of a computer program. Thus T is not a minimum triangulation, contradicting the conjecture.

In this case a minimum triangulation can be achieved by choosing as a separator clique, a separator with minimum vertices. But this approach does not guarantee a minimum triangulation either. For example, the graph drawn in Fig. 2 has only two separators $S_1 = \{1, 2, 3, 4\}$, $S_2 = \{5, 6, 7, 8, 9\}$. Triangulation using S_1 as a separator clique adds six edges, while triangulation using S_2 adds only four.

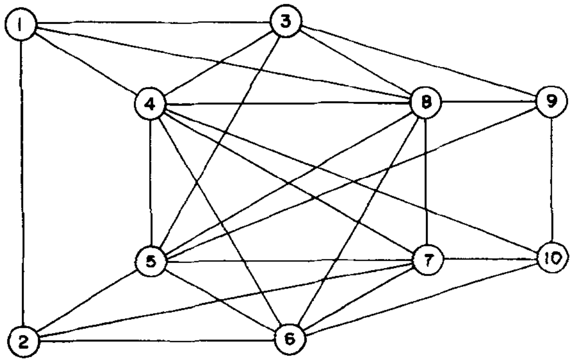

FIGURE 1

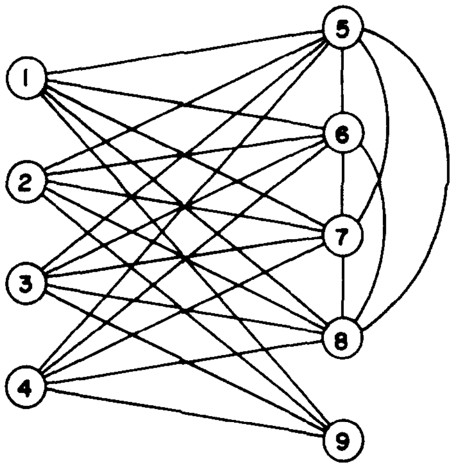

FIGURE 2

As a result of these examples, I do not believe that there exists a method of achieving minimum triangulation through some local criterion for choosing a separator to become a separator clique.

REFERENCE

1. D. J. ROSE, Triangulated graphs and the elimination process, *J. Math. Anal. Appl.* 32 (1970), 597–609.