Non-uniform slot suction/injection into mixed convective MHD flow over a vertical wedge with chemical reaction

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Abstract

The effect of non-uniform slot suction/injection into steady mixed convection MHD boundary layer flow over a vertical wedge with chemical reaction has been investigated. The governing boundary layer equations are transformed into non-dimensional form by suitable coordinate transformation. Non-similar solutions are obtained numerically using an implicit finite difference scheme in combination with the quasi-linearization technique. Numerical results are presented graphically and in tabular form to display the effects of suction/injection, Prandtl number, Schmidt number, buoyancy and chemical reaction parameters. Comparison with previously published works are performed and are found be in excellent agreement.

Keywords: mixed convection; non-uniform slot suction; MHD; chemical reaction

1. Introduction

The study of boundary layer flow over a vertical wedge has received considerable attention of researchers due to its frequent occurrence in many branches of science and technology. In many investigations, notable contribution on convection flows over a wedge was made by Watanabe [1], who studied the thermal boundary layer over a wedge with uniform suction or injection in forced flow. Later, Watanabe et al. [2] have investigated the theoretical analysis on mixed convection flow over a wedge with uniform suction or injection. A uniform suction or injection effect on wedge flow was studied by many researchers, for example, Yih [3] and Ishak et al. [4]. Kumari et al. [5] have investigated the mixed convection flow over a vertical wedge embedded in a highly porous medium. Later, unsteady mixed convection flow over a vertical wedge was studied by Singh et al. [6]. Recently, the uniform suction or injection effect on steady and unsteady mixed convection flow over a vertical cone was studied by Ravindran et al. [7] and Patil and Pop [8]. All the above investigations dealt with uniform surface mass transfer (suction or injection).

In many cases, mass transfer from a wall slot into the boundary layer is of interest for various eventual applications. If we choose uniform mass transfer, the discontinuities arise at the leading and trailing edges of the slot and those can be avoided by choosing a non-uniform slot mass transfer. Recently, different studies [9–15] were reported

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non-uniform slot suction/injection into steady boundary layer flow over different geometries such as sphere, rotating sphere, cylinder, yawed cylinder, slender cylinder and cone. As a step towards the eventual development in the study of mass transfer into the boundary layer flows, it is interesting as well as useful to investigate the effect of non-uniform slot suction/injection into steady boundary layer flow over a vertical wedge. The problem becomes more interesting in the hydromagnetic case as the fluid is electrically conducting and the flow field is influenced appreciably by the presence of an applied magnetic field. There has been a renewed interest in studying magnetohydrodynamic flow heat and mass transfer aspects in the presence of chemical reaction [16]. Therefore, the aim of this work is to study the effect of non-uniform slot suction or injection into steady mixed convection boundary layer flow of an electrically conducting fluid over a vertical wedge in the presence of chemical reaction, authors are motivated for present study which may have useful applications such as chemical factories, crude oil extractions, ground water pollution, thermal insulation, solid matrix heat exchangers and the storage of nuclear wastes etc.

2. Mathematical Formulation

Consider a steady mixed convection laminar boundary layer flow of an electrically conducting fluid over a vertical wedge with half angle \( \frac{\pi}{2} \). It is assumed that the flow moves parallel to the axis of the wedge in the upward direction with free stream velocity \( u_e \) and the gravitational acceleration \( g \) acts downward parallel to the axis of the wedge. The wall temperature \( T_w \) and the free stream temperature \( T_\infty \) are taken as constants, where \( T_w > T_\infty \) corresponds to a heated wedge (assisting flow), \( T_w < T_\infty \) corresponds to a cooled wedge (opposing flow). Also, the wall of the wedge maintained at constant concentration \( C_w \) with \( C_w > C_\infty \), where \( C_\infty \) is the free stream concentration. The uniform transverse magnetic field is applied normal to the wedge surface. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. All the fluid properties are assumed to be constant, except for the density variation with temperature and concentration which is considered only in the body force term. The governing boundary layer equations under the Boussinesq approximation have the form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \left[ g \beta_r (T - T_\infty) + g \beta_c (C - C_\infty) \right] \cos \left( \frac{\pi y}{2} \right) - \frac{\sigma B^2}{\rho} (u - u_e), \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu \partial^2 T}{\partial y^2}, \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{v \partial^2 C}{S_c \partial y^2} - k_c (C - C_\infty). \tag{4}
\]

The boundary conditions are

\[
\begin{align*}
    u &= 0, & v &= v_w, & T &= T_w, & C &= C_w & \text{at} & y = 0, \\
    u &= u_e, & T &= T_\infty, & C &= C_\infty & \text{as} & y \to \infty. \tag{5}
\end{align*}
\]

Applying the following transformations:

\[
\eta = y \left( \frac{m + 1}{2} \frac{u_e}{x} \right)^{1/2}, \quad \psi(x, y) = \left( \frac{2}{m + 1} \frac{x u_e}{y} \right)^{1/2} f(\tilde{x}, \eta), \quad u_e = u_\infty(\tilde{x}^m), \quad \tilde{x} = \frac{x}{L},
\]

\[
m = \tilde{x} \frac{d u_e}{u_e} d \tilde{x} = \frac{y}{2 - \gamma}, \quad G(\tilde{x}, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad H(\tilde{x}, \eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad f_\psi(\tilde{x}, \eta) = F(\tilde{x}, \eta),
\]

\[
u = -\frac{\partial \psi}{\partial \eta}, \quad u = -\frac{\partial \psi}{\partial \tilde{x}}, \quad v = u_e f_\psi(\bar{x}, \eta) = u_e F,
\]

\[
v = -\left( \frac{2}{m + 1} \frac{u_e}{\tilde{x}} \right)^{1/2} \times \left[ (m + 1)f + 2\tilde{f} \tilde{x} + (m - 1)\eta F \right]. \tag{6}
\]
Eq. (1) is identically satisfied, and Eqs. (2) - (4) reduce to non-dimensional form, as follows

\[ F_{\eta\eta} + f F_\eta + \frac{2m}{m+1} (1-F^2) + \frac{2}{m+1} \lambda N_1 [G + S H] \cos \left( \frac{\pi \xi}{2} \right) + \frac{2}{m+1} M N_2 (1-F) = \left( \frac{1-m}{1+m} \right) \xi \left( F F_\xi - F \eta f_\xi \right), \]  

(7)

\[ Pr^{-1} G_{\eta\eta} + f G_\eta = \left( \frac{1-m}{1+m} \right) \xi \left( F G_\xi - G_\eta f_\xi \right), \]  

(8)

\[ Sc^{-1} H_{\eta\eta} + f H_\eta - \frac{2}{m+1} \Delta N_2 H = \left( \frac{1-m}{1+m} \right) \xi \left( F H_\xi - H_\eta f_\xi \right), \]  

(9)

where

\[ \xi = (\tilde{x})^{(1-m)}, \quad N_1 = \xi^{\frac{1+m}{1-m}}, \quad N_2 = \xi^2, \quad S = \frac{\lambda^*}{\lambda}, \quad \lambda = \frac{Gr_L}{Re_L^2}, \quad \lambda^* = \frac{Gr_L^*}{Re_L^2}, \quad Re_L = \frac{u_\infty L}{\nu}, \]  

\[ Gr_L = \frac{g\beta T L (T - T_\infty)}{\nu^2}, \quad Gr_L^* = \frac{g\beta C L^3 (C_w - C_\infty)}{\nu^2}, \quad M = \frac{Ha^2}{Re_L}, \quad Ha^2 = \frac{\sigma B^2 L^2}{\mu}, \quad \Delta = \frac{k_c}{\nu}. \]  

The boundary conditions become

\[ f = f_w, \quad F = 0, \quad G = 1, \quad H = 1 \quad \text{at} \quad \eta = 0 \]  

\[ F \to 1, \quad G \to 0, \quad H \to 0 \quad \text{as} \quad \eta \to \infty \]  

(10)

here

\[ f = \int_0^\eta F d\eta + f_w, \quad f_w \text{ is given by} \]

\[ \left( \frac{m+1}{2} \right) f_w + \left( \frac{1-m}{2} \right) \xi (f_\xi)_w = -\frac{v_w}{u_\infty} \left( \frac{m+1}{2} \right) Re_L \xi, \]  

the boundary condition \( v_w \) is taken as

\[ v_w = \begin{cases} -u_\infty \left( \frac{1-m}{2} \right) \left( \frac{2}{m+1} \right)^{1/2} (Re_L)^{-1/2} A \xi^{-\left( \frac{1+m}{1-m} \right)} \omega^* \sin \left( \omega^* (\xi - \xi_0) \right), & \xi_0 \leq \xi \leq \xi_0^* \\ 0, & \text{otherwise} \end{cases} \]  

\[ f_w = \begin{cases} 0, & \xi \leq \xi_0^* \\ A \xi^{-\left( \frac{1+m}{1-m} \right)} \left[ 1 - \cos \left( \omega^* (\xi - \xi_0) \right) \right], & \xi_0 \leq \xi \leq \xi_0^* \\ A \xi^{-\left( \frac{1+m}{1-m} \right)} \left[ 1 - \cos \left( \omega^* (\xi_0 - \xi_0) \right) \right], & \xi \geq \xi_0^* \end{cases} \]  

(11)

Here, \( \omega^* \) and \( \xi_0 \) are the two free parameters which determine the slot length and slot location, respectively. The surface mass transfer parameter \( A > 0 \) or \( A < 0 \) indicates the suction or injection, respectively.

The physical quantities of interest are the local skin friction coefficient, the local Nusselt and Sherwood numbers which represent the wall shear stress, heat transfer rate and mass transfer rate, respectively. These are defined as

\[ C_{f_w} (Re_x)^{1/2} = 2 \left( \frac{m+1}{2} \right)^{1/2} (F_\eta)_w, \]  

(12)

\[ Nu_w (Re_x)^{-1/2} = - \left( \frac{m+1}{2} \right)^{1/2} (G_\eta)_w, \]  

(13)

\[ Sh_w (Re_x)^{-1/2} = - \left( \frac{m+1}{2} \right)^{1/2} (H_\eta)_w. \]  

(14)
3. Method of Solution

First, the non-linear partial differential equations (7) - (9) were linearised with the help of quasi-linearization technique [17], and hence the following set of linear partial differential equations was obtained:

\[ F^{i+1}_\eta + X^i_1 F^{i+1}_\eta + X^i_2 F^{i+1}_\xi + X^i_3 G^{i+1}_\eta + X^i_4 G^{i+1}_\xi = X^i_6, \]  

\[ G^{i+1}_\eta + Y^i_1 G^{i+1}_\eta + Y^i_2 G^{i+1}_\xi + Y^i_3 G^{i+1}_\xi = Y^i_5, \]  

\[ H^{i+1}_\eta + Z^i_1 H^{i+1}_\eta + Z^i_2 H^{i+1}_\xi + Z^i_3 H^{i+1}_\xi = Z^i_5. \]  

The coefficient functions with iterative index \((i)\) are known and the functions with iterative index \((i + 1)\) are to be determined.

The corresponding boundary conditions are

\[ F^{i+1} = 0, \quad G^{i+1} = 1, \quad H^{i+1} = 1 \quad \text{at} \quad \eta = 0 \]  

\[ F^{i+1} \to 1, \quad G^{i+1} \to 0, \quad H^{i+1} \to 0 \quad \text{as} \quad \eta \to \infty \]  

The coefficients in Eqs. (15) - (17) are as follows:

\[ X^i_1 = f + \left( \frac{1 - m}{1 + m} \right) \xi f \xi, \]  

\[ X^i_2 = -\left( \frac{1 - m}{1 + m} \right) \xi F \xi - \left( \frac{4m}{m + 1} \right) F - \left( \frac{2}{m + 1} \right) M \xi^2, \]  

\[ X^i_3 = -\left( \frac{1 - m}{1 + m} \right) \xi F, \]  

\[ X^i_4 = \left( \frac{2}{m + 1} \right) \lambda \xi \frac{\Delta - 1}{m - 1}, \]  

\[ X^i_5 = \left( \frac{2}{m + 1} \right) \lambda S \xi \frac{\Delta - 1}{m + 1}, \]  

\[ X^i_6 = -\left( \frac{2m}{m + 1} \right) \left( 1 + F^2 \right) - \left( \frac{1 - m}{1 + m} \right) \xi F \xi \xi - \frac{2}{m + 1} M \xi^2; \]  

\[ Y^i_1 = Pr \left[ f + \left( \frac{1 - m}{1 + m} \right) \xi f \xi \right], \]  

\[ Y^i_2 = -Pr \left( \frac{1 - m}{1 + m} \right) \xi F, \]  

\[ Y^i_3 = 0, \]  

\[ Y^i_4 = -Pr \left( \frac{1 - m}{1 + m} \right) \xi G \xi, \]  

\[ Y^i_5 = -Pr \left( \frac{1 - m}{1 + m} \right) \xi G \xi F; \]  

\[ Z^i_1 = Sc \left[ f + \left( \frac{1 - m}{1 + m} \right) \xi f \xi \right], \]  

\[ Z^i_2 = -Sc \left( \frac{1 - m}{1 + m} \right) \xi F, \]  

\[ Z^i_3 = -Sc \Delta \left( \frac{2m}{m + 1} \right) \xi^2. \]
\[
Z_4^i = -Sc \left( \frac{1 - m}{1 + m} \right) \xi H_\xi, \\
Z_5^i = -Sc \left( \frac{1 - m}{1 + m} \right) \xi H_\xi F
\]

The linear partial differential equations (15) - (17) were expressed in implicit finite difference form using central difference scheme in \( \eta \)-direction and backward difference scheme in \( \xi \)-direction. The resulting linear equations together with the boundary conditions (18) were then reduced to a system of linear algebraic equations with a block tri-diagonal matrix, which is solved by using Varga’s algorithm [18]. The step sizes in \( \eta \) and \( \xi \)-directions have been chosen as 0.01 and 0.005, respectively. The convergence criteria based on the relative difference between the current and previous iteration values is employed. The solution is assumed to have converged and the iterative process is terminated when the difference reaches less than \( 10^{-5} \).

4. Results and Discussion

In order to verify the accuracy of our numerical method, we have compared the present results with existing literature. The results are found in excellent agreement and the comparison is shown in Table 1.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( Pr = 0.73 )</th>
<th>( Pr = 7.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>0.0</td>
<td>0.0141</td>
</tr>
<tr>
<td>[2]</td>
<td>0.46960</td>
<td>0.50461</td>
</tr>
<tr>
<td>[3]</td>
<td>0.46960</td>
<td>0.50461</td>
</tr>
<tr>
<td>[4]</td>
<td>0.46960</td>
<td>0.50461</td>
</tr>
<tr>
<td>Present results</td>
<td>0.46969</td>
<td>0.50480</td>
</tr>
</tbody>
</table>

Fig. 1. Effect of \( \lambda \) and \( Pr \) on velocity profile

Fig. 2. Effect of \( \lambda \) and \( Pr \) on temperature profile

The effect of Prandtl number \( (Pr) \) and buoyancy parameter \( (\lambda) \) on velocity and temperature profiles are shown in Figs. 1 and 2. It is seen in Fig. 1 that the velocity overshoot is observed for lower Prandtl number fluid \( (Pr = 0.72, \text{ air}) \) while the overshoot is not observed for higher Prandtl number fluid \( (Pr = 7.0, \text{ water}) \). High Prandtl fluid has low thermal conductivity which leads to thinner thermal boundary layer, as can be seen from Fig. 2. It is observed from Fig. 3 that the velocity and thermal boundary layer thicknesses are reduced by suction while they opposite for injection. Figure 4 shows the fluid flow velocity decreases with increasing magnetic parameter \( M \). Figure 5 depicts the variation in concentration profiles for different values of Schmidt number \( (Sc) \) and chemical reaction parameter
It is evident from figure that the concentration boundary layer thickness decreases with increase of $Sc$ due to the high value of $Sc$ has a low mass diffusivity. Moreover, the concentration boundary layer thickness reduces with species generation ($\Delta > 0$). Figures 6 – 8 show the effect of non-uniform slot suction/injection on skin friction coefficient, the local Nusselt and Sherwood numbers. The skin friction coefficient, the local Nusselt and Sherwood numbers are increased with suction while they decrease with increasing of injection. However, suction and injection profiles are not a mirror reflection of each other. The effect of Prandtl number ($Pr$) and buoyancy parameter ($\lambda$) on skin friction coefficient and the local Nusselt number are shown in Figs. 9 and 10. It is observed that the skin friction coefficient decreases with $Pr$ while the local Nusselt number increases with $Pr$. Both skin friction coefficient and the
local Nusselt number enhance with $\lambda$. Buoyancy force effect is large on skin friction coefficient than the local Nusselt number.

5. Conclusion

Non-uniform slot suction/injection into steady mixed convection MHD boundary layer flow over a vertical wedge has been studied. The main conclusions of this study are as follows:

- The buoyancy assisting force causes velocity overshoot for lower Prandtl number fluid ($Pr = 0.72$, air) but the overshoot is not observed for higher Prandtl number fluid ($Pr = 7.0$, water).
- The fluid flow velocity is decreased by an increasing magnetic parameter.
- The skin friction coefficient is more affected by the buoyancy parameter while the local Nusselt number is more affected by the Prandtl number.
- The velocity, thermal and concentration boundary layer thicknesses are reduced by the increase of suction but it is opposite for injection.
- The species generation and high Schmidt number cause a thinner concentration boundary layer.
- The local skin friction coefficient, the local Nusselt and Sherwood numbers increase with suction while these decrease by an increasing of injection.
- Non-uniform slot injection helps to reduce the skin friction coefficient, the local Nusselt and Sherwood numbers at a particular stream-wise location on the body surface.

References


