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Form factors in $\bar{B}^0 \to \pi^+\pi^0\ell\bar{\nu}_\ell$ from QCD light-cone sum rules

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Abstract

The form factors of the semileptonic $B \to \pi\pi\ell\bar{\nu}$ decay are calculated from QCD light-cone sum rules with the distribution amplitudes of dipion states. This method is valid in the kinematical region, where the hadronic dipion state has a small invariant mass and simultaneously a large recoil. The derivation of the sum rules is complicated by the presence of an additional variable related to the angle between the two pions. In particular, we realize that not all invariant amplitudes in the underlying correlation function can be used, some of them generating kinematical singularities in the dispersion relation. The two sum rules that are free from these ambiguities are obtained in the leading twist-2 approximation, predicting the $\bar{B}^0 \to \pi^+\pi^0$ form factors $F_\perp$ and $F_\parallel$ of the vector and axial $b \to u$ current, respectively. We calculate these form factors at the momentum transfers $0 < q^2 \lesssim 12$ GeV^2 and at the dipion mass close to the threshold $4m_\pi^2$. The sum rule results indicate that the contributions of the higher partial waves to the form factors are suppressed with respect to the lowest P-wave contribution and that the latter is not completely saturated by the ρ-meson term.

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1. Introduction

The current tendency in the studies of the flavour-changing decays of heavy hadrons is to enlarge the set of exclusive processes used for the determination of the fundamental CKM parameters. Probing different exclusive $b \to u$ processes may, in particular, help in the $|V_{ub}|$ determination. The interval of this CKM parameter obtained from the measurements of the $B \to \pi \ell \bar{\nu}_\ell$ decay, combined with the $B \to \pi$ form factors from lattice QCD or from the QCD light-cone sum rules (LCSR), deviates from the results obtained in the inclusive $B \to X_u \ell \bar{\nu}_\ell$ decay studies (see, e.g., the review [1] and references therein).

Alternative exclusive $b \to u$ processes are being actively investigated, among them the $B \to \pi\pi \ell \bar{\nu}_\ell$ decay, where the ρ-meson contribution is prominent. The semileptonic B-decay mode with the two-pion (*dipion*) final state is not only important for the $|V_{ub}|$ determination, but also has a rich set of observables (see e.g., Ref. [2]) which can be used for nontrivial tests of Standard Model. The $B \to \pi\pi \ell \bar{\nu}_\ell$ decay has already been measured, but mainly its resonant, $B \to \rho \ell \bar{\nu}_\ell$ part (see e.g., the BaBar [3] and Belle [4] Collaborations data). Significantly more detailed data on the $B \to \pi\pi \ell \bar{\nu}_\ell$ observables are expected from the Belle-2 experiment in future.

The dynamics of the $B \to \pi\pi \ell \bar{\nu}_\ell$ decay is governed by general $B \to 2\pi$ form factors, hence the calculation of these form factors is becoming the next big task for the practitioners of QCD-based methods. As discussed in Ref. [2] in detail, various non-lattice methods, from heavy-meson chiral perturbation theory to the soft-collinear effective theory are applicable, depending on the region of the Dalitz plot formed by the invariant masses of the lepton pair and dipion.

In this paper, we use the method of LCSRs [5] to calculate the $B \to 2\pi$ form factors relevant for the $\bar{B}^0 \to \pi^+\pi^0\ell^-\bar{\nu}_\ell$ decay. We shall confine ourselves with the charged dipion (isovector) final state, and postpone the case of the neutral (isoscalar) state with related scalar resonances for the future work. The approach we use is applicable in the region of small and intermediate lepton-pair masses, restricting simultaneously the dipion invariant mass by the $\lesssim 1$ GeV region, so that a large hadronic recoil takes place with two energetic and almost collinear pions in the B-meson rest frame.

The technique we use has many similarities with the LCSRs obtained for $B \to \pi$ form factors, but employs a different and more complicated nonperturbative input: the light-cone distribution amplitudes (DAs) of the dipion state. These universal objects have been introduced in Refs. [6,7] to encode the hadronization of the quark-pair in the $\gamma\gamma^* \to 2\pi$ process at large momentum transfer. The properties of dipion DAs were worked out in details in Refs. [8,9]. In a different context, two-meson wave functions in hard exclusive processes were discussed earlier in Ref. [10].

In this paper we aim at the following goals. First, we demonstrate how the method works, deriving the LCSRs for the two of the $B \to \pi\pi$ form factors in the leading twist-2 approximation. The sum rules predict these form factors at large recoil and small mass of the dipion state. Second, based on this calculation, we investigate the role of higher partial waves in the $B \to \pi\pi$ form factors and assess the impact of the contributions beyond the ρ-meson in the lowest P-wave. In what follows, the derivation of LCSRs for $B \to \pi\pi$ form factors is presented in Sect. 2. In Sect. 3 we compare our predictions with the $B \to \rho$ form factors. In Sect. 4 using the available information on the chiral-odd dipion DA, we calculate the form factors numerically. Our conclusions are presented in Sect. 5. The Appendices contain some details (A) on the decay kinematics and (B) on the dipion DAs.

2. Light-cone sum rules with dipion distribution amplitudes

The LCSR derivation starts from defining an appropriate correlation function. We consider the T-product of the $b \to u$ weak current $j_\mu^{V-A}(x) = \bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)$ with the B-meson interpolating current $j_5^{(B)}(0) = im_b\bar{b}(0)\gamma_5 d(0)$. Since we are interested in the final state with two pions, this T-product is then sandwiched between the vacuum and the on-shell dipion state:

$$\Pi_\mu(q,k_1,k_2) = i\int d^4x e^{iqx}\langle \pi^+(k_1)\pi^0(k_2)|T\{j_\mu^{V-A}(x),\, j_5(0)\}|0\rangle\,. \tag{1}$$

The above correlation function has a more complicated kinematics than in the case of the one-pion final state and depends on three independent 4-momenta q, k_1, k_2. We denote by $k = k_1 + k_2$ the total dipion four-momentum and by $p = q + k$ the external four-momentum of the B-meson interpolating current. At fixed $k_{1,2}^2 = m_\pi^2$ these momenta form four independent invariant variables, as such we choose $p^2 = (q+k)^2$, q^2, k^2 and $q\cdot\bar{k}$, where $\bar{k} = k_1 - k_2$. Further details on the kinematics are given in the Appendix A.

The correlation function (1) is decomposed in four independent Lorentz-vectors [1]:

$$\Pi_\mu(q,k_1,k_2) = i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\,\Pi^{(V)} + q_\mu\Pi^{(A,q)} + k_\mu\Pi^{(A,k)} + \bar{k}_\mu\Pi^{(A,\bar{k})}\,, \tag{2}$$

where the first term (the rest) corresponds to the contribution of the vector (axial) part of the $b \to u$ weak current and the invariant amplitudes $\Pi^{(V),(A,q),\dots}$ depend on the four invariant variables: $p^2, q^2, k^2, q\cdot\bar{k}$.

To guarantee the validity of the operator–product expansion (OPE) for the correlation function (1) near the light-cone ($x^2 \sim 0$), we consider the region $p^2 \ll m_b^2$ and $q^2 \ll m_b^2$, so that the b-quark mass provides the large scale. In this respect, the conditions for the light-cone dominance are practically the same as in the case of the vacuum-to-pion correlation functions used to obtain the LCSRs for $B \to \pi$ form factors (for a detailed derivation of the latter sum rules see, e.g., Ref. [11]). An additional constraint concerns the invariant mass of dipion which is also kept small, $k^2 \lesssim 1\text{GeV}^2 \ll m_b^2$. In this region the two-pion system with isospin one is dominated by the $\rho(770)$ resonance, accompanied by a nonresonant background. In this paper, we only consider the charged dipion state, so that only odd angular momenta contribute in the isospin symmetry limit. This limitation simplifies our analysis, whereas the case of neutral dipion state where also the scalar/isoscalar f^0 resonances contribute, will be considered elsewhere.

Turning to the calculation of the correlation function (1), in the leading-order (LO) approximation ($\alpha_s = 0$), after inserting the free b-quark propagator, we obtain:

$$\Pi_\mu(q,k_1,k_2) = i\int d^4x\int\frac{d^4f}{(2\pi)^4}e^{i(q-f)x}\frac{m_b}{m_b^2 - f^2}$$
$$\times\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu(1-\gamma_5)(\not{f} + m_b)\gamma_5 d(0)|0\rangle. \tag{3}$$

This expression consists of the hard-scattering amplitude – the virtual b-quark propagator – convoluted with the vacuum $\to$ dipion matrix elements of bilocal quark–antiquark operators. These matrix elements absorb long-distance dynamics and are expressed via universal dipion DAs, defined following Ref. [8]. The LO diagram of OPE for the correlation function (1) is shown in Fig. 1.

[1] Here we use the convention $\epsilon^{0123} = -1$.

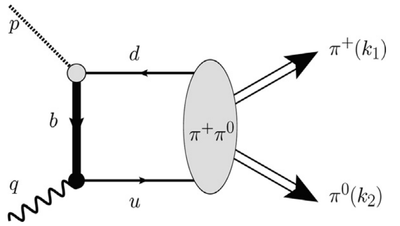

Fig. 1. Diagram representing the correlation function (1) in leading order; the wavy (dotted) line represents the weak (B-meson interpolating) quark current.

In this paper we will confine ourselves to the leading, twist-2 approximation for the nonlocal hadronic matrix elements. We use the following definitions of the twist-2 DAs [8]:

$$\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int\limits_0^1 du e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2)\,, \tag{4}$$

$$\begin{aligned}&\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle\\ &\quad = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int\limits_0^1 du e^{iu(k\cdot x)}\Phi_\perp^{I=1}(u,\zeta,k^2)\,,\end{aligned} \tag{5}$$

where Eq. (4) and Eq. (5) represent, respectively, the chiral-even and chiral-odd terms in the light-cone expansion and $[x,0]$ is the gauge factor. The DAs depend on the dipion mass squared k^2, on the fraction u of the two-pion longitudinal momentum carried by the u-quark (so that $1-u\equiv\bar{u}$ is carried by the $\bar{d}$ quark) and on the parameter ζ related to $q\cdot\bar{k}$ (see Appendix A). The normalization conditions are [8]:

$$\int\limits_0^1 du\, \Phi_\parallel^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{em}(k^2)\,, \qquad \int\limits_0^1 du\, \Phi_\perp^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{t}(k^2)\,, \tag{6}$$

where $F_\pi^{em}(k^2)$ is the standard electromagnetic form factor of the pion in the timelike region (so that $F_\pi^{em}(0)=1$) and $F_\pi^t(k^2)$ is the "tensor" form factor of the pion normalized to the dimensionful parameter introduced in Ref. [8]:

$$F_\pi^t(0) = 1/f_{2\pi}^\perp\,. \tag{7}$$

The definition (4) coincides with the one introduced in Ref. [8], whereas the DA defined in Eq. (5) differs by the above factor. We also use the isospin conventions as defined in Ref. [9] to relate the dipions with definite isospin projections of pions to the $\langle\pi^+\pi^0|$ state. Hereafter we omit the isospin index at DAs, since in this paper we only consider the $I=1$ dipion state. Note that k^2 has to be sufficiently small to avoid large generic $O(k^2x^2)$ terms in the light-cone expansion.

In addition to the matrix elements (4) and (5), one recovers in Eq. (3) also the ones with the Dirac matrices $1,\gamma_\mu\gamma_5$; they correspond to the higher twists and are neglected here, whereas the nonlocal matrix element with γ_5 vanishes due to P-parity conservation. Sorting out the Dirac structures in Eq. (3) and applying the definitions of DAs we obtain, at twist-2 accuracy:

$$\Pi_\mu(q,k_1,k_2) = i\sqrt{2}m_b \int_0^1 \frac{du}{(q+uk)^2 - m_b^2}\Big\{\Big[(q\cdot\bar{k})k_\mu - \Big((q\cdot k)+uk^2\Big)\bar{k}_\mu + i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\Big]\frac{\Phi_\perp(u,\zeta,k^2)}{2\zeta-1} - m_b k_\mu \Phi_\parallel(u,\zeta,k^2)\Big\}. \tag{8}$$

From the above expression one reads off the invariant amplitudes $\Pi^{(r)}$ defined in Eq. (2) with $(r) = (V), (A,q), (A,k), (A,\bar{k})$, and represents them with a generic expression:

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp}\int_0^1 du \frac{f_i^{(r)}(p^2,q^2,k^2,\zeta)\Phi_i(u,\zeta,k^2)}{(q+uk)^2 - m_b^2}, \tag{9}$$

where the coefficient function convoluted with the dipion DA's consists of the b-quark propagator multiplied by a certain kinematical factor $f_i^{(r)}$. Transforming the integration variable u to

$$s(u) = \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{u}, \tag{10}$$

we bring the integral in Eq. (9) to a dispersion form in the variable p^2:

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp} f_i^{(r)}(p^2,q^2,k^2,\xi)\int_{m_b^2}^{\infty}\frac{ds}{s-p^2}\left(\frac{du}{ds}\right)\Phi_i(u(s),\zeta,k^2). \tag{11}$$

The coefficient functions in the above, after transforming the variable: $f_i^{(r)}(p^2,q^2,k^2,\xi) = f_i^{(r)}(p^2 - s + s, q^2, k^2, \xi)$, can be expanded in the powers of $(p^2 - s)$, which will vanish after the Borel transformation of Eq. (11) in p^2 used below. Hence, we can simply replace $p^2 \to s$ in Eq. (11) and put the functions $f_i^{(r)}(s,q^2,k^2,\xi)$ under the integral, as a part of the spectral density. However, due to a more complicated kinematics of the correlation function, this replacement is not legitimate in one particular invariant amplitude multiplying k_μ. In this case the function $f_i^{(A,k)}(p^2,q^2,k^2,\xi)$ contains the factor $q\cdot\bar{k} = 1/2(2\xi-1)\lambda^{1/2}(p^2,q^2,k^2)$ (see Appendix A for details). This factor, after analytical continuation in p^2, generates a cut at the real axis, more specifically at $(\sqrt{q^2}-\sqrt{k^2})^2 < p^2 < (\sqrt{q^2}+\sqrt{k^2})^2$, which does not correspond to any physical intermediate state and represents a typical kinematic singularity. Moreover, after Borel transformation, the contribution of this cut to the dispersion integral is enhanced with respect to the b-quark spectral density. Hence, within the framework of the standard sum rule procedure, we are only in a position to derive the LCSRs for the invariant amplitudes $\Pi^{(V)}$ and $\Pi^{(A,\bar{k})}$.

The derivation of these LCSRs continues along the same lines as in the well-known case of $B\to\pi$ form factor (see e.g., Ref. [11]). Applying the quark–hadron duality approximation, one introduces the effective threshold s_0^B in the B-meson channel, so that the part of the integral in Eq. (11) from s_0^B to ∞ is approximated by its duality-counterpart in the hadronic dispersion relation and subtracted. After that, the Borel transformation with respect to the variable $p^2 \to M^2$ is applied. The result in generic form is:

$$\Pi^{(r)}(M^2,s_0^B,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp}\int_{m_b^2}^{s_0^B} ds\, e^{-s/M^2} f_i^{(r)}(s,q^2,k^2,\xi)\frac{du}{ds}\Phi_i(u(s),\zeta,k^2), \tag{12}$$

At this stage it is convenient to return to the original integration variable u, using the inverse transformation of Eq. (10):

$$u(s) = \frac{k^2 + q^2 - s + \sqrt{4k^2(m_b^2 - q^2) + (s - k^2 - q^2)^2}}{2k^2} \,. \tag{13}$$

The following expressions for the Borel-transformed and subtracted invariant amplitudes are obtained:

$$\Pi^{(V)}(M^2, s_0^B, q^2, k^2, \zeta) = -\frac{2\sqrt{2} i m_b}{2\zeta - 1} \int\limits_{u_0}^{1} \frac{du}{u} e^{-\frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}} \Phi_\perp(u, \zeta, k^2) \,, \tag{14}$$

$$\begin{aligned}&\Pi^{(A,\bar{k})}(M^2, s_0^B, q^2, k^2, \zeta)\\ &= \frac{\sqrt{2} i m_b}{2(2\zeta - 1)} \int\limits_{u_0}^{1} \frac{du}{u^2} e^{-\frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}} \Big(m_b^2 - q^2 + k^2 u^2\Big) \Phi_\perp(u, \zeta, k^2) \,,\end{aligned} \tag{15}$$

where $u_0 = u(s_0)$. In addition, the condition:

$$\Pi^{(A,q)}(M^2, s_0^B, q^2, k^2, \zeta) = 0 \,, \tag{16}$$

is valid at the twist-2 order.

To proceed, we use the hadronic dispersion relation for the correlation function in the variable p^2 where we only retain the ground B-meson state contribution:

$$\Pi_\mu(q, k_1, k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + \dots \,, \tag{17}$$

with the decay constant of B-meson defined via $\langle \bar{B}^0(p)|\bar{b}\, i m_b \gamma_5 d|0\rangle = f_B m_B^2$. In Eq. (17) the ellipses denote the contributions of radially excited and continuum states with B-meson quantum numbers, approximated employing the quark–hadron duality approximation.

We then decompose the $B \to \pi\pi$ transition matrix element in the form factors we are interested in. We use the definition similar to the one in Ref. [2][2]:

$$\begin{aligned} i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= -F_\perp \frac{4}{\sqrt{k^2\lambda_B}} i\epsilon^{\mu\alpha\beta\gamma} q_\alpha k_{1\beta} k_{2\gamma} \\ &\quad + F_t \frac{q^\mu}{\sqrt{q^2}} + F_0 \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k^\mu - \frac{k\cdot q}{q^2} q^\mu\right) \\ + F_\parallel \frac{1}{\sqrt{k^2}} &\left(\bar{k}^\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B} k^\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B} q^\mu\right), \end{aligned} \tag{18}$$

where

$$\lambda_B \equiv \lambda(m_B^2, q^2, k^2), \quad q\cdot k = \frac{1}{2}(m_B^2 - q^2 - k^2), \quad q\cdot\bar{k} = \frac{1}{2}(2\zeta - 1)\lambda_B \,. \tag{19}$$

[2] Our definition of the form factors differs from the one in Ref. [2] only by some phase factors, caused by a difference in the conventions for the ϵ-tensor and for the phase of the dipion state.

The $B \to 2\pi$ form factors $F_\perp$ and $F_{t,0,\parallel}$ depend on the variables q^2, k^2 and $q \cdot \bar{k}$, and parametrize the transition matrix element of the vector and axial weak $b \to u$ currents, respectively. Hereafter, we replace in the form factors the variable $q \cdot \bar{k}$ by ζ, using the relation (19). The form factors defined in Eq. (18) can be expanded in partial waves:

$$F_{0,t}(q^2, k^2, \zeta) = \sum_{\ell=0}^{\infty} \sqrt{2\ell+1} F_{0,t}^{(\ell)}(q^2, k^2) P_\ell^{(0)}(\cos\theta_\pi), \tag{20}$$

$$F_{\perp,\parallel}(q^2, k^2, \zeta) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1} F_{\perp,\parallel}^{(\ell)}(q^2, k^2) \frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi}, \tag{21}$$

where $P_l^{(m)}$ are the (associated) Legendre polynomials, and θ_π is the angle between the pions in their c.m. frame, related to the parameter ζ via:

$$(2\zeta - 1) = \beta_\pi \cos\theta_\pi, \quad \beta_\pi \equiv \sqrt{1 - 4m_\pi^2/k^2}. \tag{22}$$

Substituting the decomposition (18) in Eq. (17), we match the hadronic dispersion relation to the OPE result for the correlation function Π_μ. For each invariant amplitude in the decomposition (2) a separate equation is obtained relating it to one of the form factors or to their linear combination. For the OPE result after the subtraction of higher than B-meson states and Borel transformation, we can directly use the expressions given in Eqs. (14) and (15). For the vector-current form factor we obtain the following LCSR in the adopted LO and twist-2 approximation:

$$\frac{F_\perp(q^2, k^2, \zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = -\frac{m_b}{\sqrt{2} f_B m_B^2 (2\zeta - 1)} \int_{u_0}^{1} \frac{du}{u} \Phi_\perp(u, \zeta, k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}}. \tag{23}$$

Furthermore, equating the coefficients at $\bar{k}_\mu$ in the OPE and hadronic representations of the correlation function, yields the LCSR for the one of the axial-current form factors:

$$\frac{F_\parallel(q^2, k^2, \zeta)}{\sqrt{k^2}} = -\frac{m_b}{\sqrt{2} f_B m_B^2 (2\zeta - 1)} \times \int_{u_0}^{1} \frac{du}{u^2} \left(m_b^2 - q^2 + k^2 u^2\right) \Phi_\perp(u, \zeta, k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}}. \tag{24}$$

Finally, since the invariant amplitude multiplying q_μ vanishes, an additional relation between the axial-current form factors emerges:

$$F_t(q^2, k^2, \zeta) = \frac{1}{\sqrt{\lambda_B}} \Big[(m_B^2 - q^2 - k^2) F_0(q^2, k^2, \zeta) - 2\sqrt{k^2}\sqrt{q^2}(2\zeta - 1) F_\parallel(q^2, k^2, \zeta)\Big]. \tag{25}$$

Note that the remaining invariant amplitude multiplying k^μ contains irreducible kinematical singularities mentioned above, hence, the additional sum rule which could yield the form factor F_0 cannot be derived with the same method. Hence, in the following we confine ourselves by analyzing in detail the LCSRs for the form factors $F_\perp$ and $F_\parallel$. Interestingly, both sum rules depend on the single, chiral-odd dipion DA defined in Eq. (5).

Following Ref. [8], we represent this DA in a form of the double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^\perp}\sum_{n=0,2,..}^{\infty}\sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\perp(k^2)C_n^{3/2}(2u-1)\beta_\pi P_\ell^{(0)}\Big(\frac{2\zeta-1}{\beta_\pi}\Big), \tag{26}$$

with multiplicatively renormalizable coefficients $B_{n\ell}^\perp(k^2)$ (see Appendix B for more details). Note that the index n (l) goes over even (odd) numbers and the normalization conditions (6), (7) yield for the lowest coefficient $B_{01}^\perp(0)=1$. The coefficients with $n \geq 2$ play the same role as the Gegenbauer moments of the twist-2 pion DA. The values of $B_{(n\geq 2)\ell}^\perp(k^2)$ at a low scale determine the nonasymptotic part of the DA, logarithmically decreasing at large scales. Importantly, if one adopts a certain approximation for the nonasymptotic part of DA, that is, truncates the expansion (26) at a given n_{max}, the values of ℓ are restricted to $n_{max}+1$. The coefficients $B_{nl}^\perp(k^2)$ are complex functions of the dipion invariant mass, with the imaginary part at $k^2 > 4m_\pi^2$, due to the unitarity relation. Note that the function $B_{01}^\perp(k^2)$ is reduced to the timelike "tensor" form factor of the pion, which cannot be simply extracted from experiment.

Furthermore, we substitute the partial wave expansion (21) in l.h.s. and the double expansion (26) in r.h.s. of the LCSRs (23) and (24), replacing in the r.h.s. the argument of the Legendre polynomial by $\cos\theta_\pi$, according to Eq. (22). Multiplying both parts of the resulting relation by $\sin\theta_\pi P_{\ell'}^{(1)}(\cos\theta_\pi)$ and integrating over $\cos\theta_\pi$ we use the orthogonality relation:

$$\int_{-1}^{+1} dz P_\ell^{(1)}(z)P_{\ell'}^{(1)}(z) = \frac{2(\ell+1)!}{(2\ell+1)(\ell-1)!}\delta_{\ell\ell'} \tag{27}$$

and obtain the sum rules for the ℓ-th partial wave contribution to the $B\to 2\pi$ form factors ($\ell = 1,3,\dots$):

$$F_\perp^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2)J_n^\perp(q^2,k^2,M^2,s_0^B), \tag{28}$$

$$F_\parallel^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{m_b^3}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,4,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2)J_n^\parallel(q^2,k^2,M^2,s_0^B), \tag{29}$$

where the short-hand notation is introduced for the angular integral:

$$I_{\ell\ell'} \equiv -\frac{\sqrt{2\ell+1}(\ell-1)!}{2(\ell+1)!}\int_{-1}^{+1}\frac{dz}{z}\sqrt{1-z^2}P_\ell^{(1)}(z)P_{\ell'}^{(0)}(z), \tag{30}$$

so that, e.g., $I_{1,1} = 1/\sqrt{3}$, $I_{1,3} = -1/\sqrt{3}$, $I_{1,5} = 4/(5\sqrt{3})$, and the integrals over the quark-momentum fraction are defined as

$$J_n^\perp(q^2,k^2,M^2,s_0^B) = 6\int_{u_0}^{1} du(1-u)C_n^{3/2}(2u-1)e^{-\frac{m_b^2-q^2\bar{u}+k^2u\bar{u}}{uM^2}}, \tag{31}$$

and

$$J_n^{\parallel}(q^2,k^2,M^2,s_0^B) = 6\int\limits_{u_0}^{1}\frac{du}{u}(1-u)C_n^{3/2}(2u-1)\Big(1-\frac{q^2-k^2u^2}{m_b^2}\Big)e^{-\frac{m_b^2-q^2\bar{u}+k^2u\bar{u}}{uM^2}}\,. \tag{32}$$

Note that $I_{ll'}=0$ at $\ell>\ell'$, hence, in the limit of the asymptotic DA, that is, when all coefficients $B_{n\ell}$, except B_{01}, vanish, only the $\ell=1$ (partial P-wave) term remains in the form factors. Altogether, the LCSRs (28) and (29) allow us to assess the relative importance of the higher partial waves with $\ell=3,5,..$ in the $B\to\pi\pi$ form factors. One simply has to calculate the ratio:

$$R^{(\ell)}_{\perp,\parallel}(q^2,k^2)=\frac{F^{(\ell)}_{\perp,\parallel}(q^2,k^2)}{F^{(1)}_{\perp,\parallel}(q^2,k^2)}\,. \tag{33}$$

3. How much ρ the $B\to 2\pi$ form factors contain?

Having at our disposal the LCSR calculation of the $\bar{B}^0\to\pi^+\pi^0$ form factors, we now address another important question: the dominance of the ρ-meson contribution to these form factors. This knowledge is indispensable for an accurate interpretation of the $B\to\pi\pi\ell\nu_\ell$ measurements. With more data on this decay available in future, the angular analysis can in principle isolate the final-state dipion in the P-wave from other partial waves. It is then important to clarify if the events in the interval of dipion invariant mass around the ρ-meson mass, at $\sqrt{k^2}\sim m_\rho\pm\Gamma_\rho^{tot}/2$, originate predominantly from the $B\to\rho$ transition, or there is a noticeable interference with excited ρ resonances and/or 2π (P-wave) continuum background. Strictly speaking, the answer to this question relies on a (model-dependent) parametrization of the ρ resonance and nonresonant background. An approach to the $B\to\pi\pi$ form factors at low dipion masses that is independent of the resonance model and employs the hadronic dispersion relation in the variable k^2 was suggested in Ref. [12] where the $\pi\pi$ rescattering effects, as well as the effect of the ρ meson, were taken into account employing the Omnés representation and the data on the pion scattering phases.

Within the LCSR framework, a similar approach would correspond to using a hadronic dispersion relation for the coefficients $B^{\perp}_{nl}(k^2)$ treated as analytical functions of k^2. An attempt in this direction was already made in Ref. [8] where these coefficients at low mass ($k^2>\;4m_\pi^2$) were calculated in the instanton model of QCD vacuum and the Omnés representation including the ρ-resonance effect was used to extrapolate them towards $k^2\sim 1$ GeV^2. We postpone a more detailed study along these lines to a future work.

Here we address a different aspect that has an immediate importance for the LCSR approach: are the $B\to\pi\pi$ form factors predicted from LCSRs at low dipion masses $k^2\sim 4m_\pi^2$ conform and/or consistent with the $B\to\rho$ form factors calculated from the LCSRs with the ρ-meson DAs defined in the zero-width approximation. To this end, we employ the hadronic dispersion relation for the P-wave ($l=1$) part of the $B\to\pi\pi$ form factors in k^2 and retain only the intermediate ρ-resonance contribution. A more detailed derivation of these relations can be found in Ref. [2]. For the two form factors considered above we obtain:

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}}=\frac{g_{\rho\pi\pi}}{m_\rho^2-k^2-im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B+m_\rho}+\dots \tag{34}$$

and

$$\frac{\sqrt{3}F_{\parallel}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}(m_B+m_\rho)A_1^{B\to\rho}(q^2) + \dots \tag{35}$$

where the ellipses denote the contributions of excited states such as $\rho(1450)$ as well as the possible subtraction terms. Note that here we prefer to use dispersion relations for the complete invariant amplitudes multiplying the four-momenta in the Lorentz-decomposition (18) of the $B\to\pi\pi$ matrix element,[3] treating these amplitudes as analytical functions of k^2 and avoiding unnecessary kinematic singularities. To make the ρ-resonance description complete, in Eqs. (34), (35) an energy dependent total width is added, defined as:

$$\Gamma_\rho(k^2) = \frac{m_\rho^2}{k^2}\left(\frac{k^2-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2}\theta(k^2-4m_\pi^2)\Gamma_\rho^{tot}, \tag{36}$$

(see e.g., the discussion in Ref. [14]), however it does not play a role at $k^2 \sim 4m_\pi^2$. The residues of the ρ-pole in the dispersion relations (34) and (35) contain the $\rho\to 2\pi$ strong coupling defined as $\langle \pi^+(k_1)\pi^0(k_2)|\rho^+(k)\rangle = -g_{\rho\pi\pi}\epsilon^{(\rho)}\cdot(k_1-k_2)$ ($\epsilon^{(\rho)}$ is the polarization vector of ρ meson) and the $B\to\rho$ form factors $V^{B\to\rho}(q^2)$ and $A_1^{B\to\rho}(q^2)$. For the latter we use the standard definition:

$$\begin{aligned}\langle \rho^+(k)|\bar u\gamma_\mu(1-\gamma_5)b|\bar B^0(p)\rangle &= \epsilon_{\mu\alpha\beta\gamma}\epsilon_\alpha^{*(\rho)}p^\beta k^\gamma\frac{2V^{B\to\rho}(q^2)}{m_B+m_\rho}\\ &\quad - i\epsilon_\mu^{*(\rho)}(m_B+m_\rho)A_1^{B\to\rho}(q^2)+\dots\end{aligned} \tag{37}$$

where ellipses denote the remaining form factors related to the axial current. The above decomposition is the same as e.g., in Ref. [15]. There one can also find a detailed derivation of LCSRs for these form factors in terms of the ρ-meson DAs in the same, leading twist-2 approximation:

$$V^{B\to\rho}(q^2) = \frac{(m_B+m_\rho)m_b}{2m_B^2 f_B}f_\rho^\perp e^{\frac{m_B^2}{M^2}}\int_{u_0}^1 \frac{du}{u}\,\phi_\perp^{(\rho)}(u)\,e^{-\frac{m_b^2-q^2\bar u+m_\rho^2 u\bar u}{uM^2}}, \tag{38}$$

$$A_1^{B\to\rho}(q^2) = \frac{m_b^3}{2(m_B+m_\rho)m_B^2 f_B}f_\rho^\perp e^{\frac{m_B^2}{M^2}}\int_{u_0}^1 \frac{du}{u^2}\,\phi_\perp^{(\rho)}(u)\left(1-\frac{q^2-m_\rho^2u^2}{m_b^2}\right)e^{-\frac{m_b^2-q^2\bar u+m_\rho^2 u\bar u}{uM^2}}. \tag{39}$$

Note that both sum rules are also determined by the chiral-odd DA defined via vacuum $\to\rho$ hadronic matrix element:

$$\langle \rho^+(k)|\bar u(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = -if_\rho^\perp\left(\epsilon_\mu^{*(\rho)}k_\nu - k_\mu\epsilon_\nu^{*(\rho)}\right)\int_0^1 du e^{iuk\cdot x}\phi_\perp^{(\rho)}(u), \tag{40}$$

and having the Gegenbauer polynomial expansion:

$$\phi_\perp^{(\rho)}(u) = 6u(1-u)\left(1+\sum_{n=2,4,\dots} a_n^{(\rho)\perp}C_n^{3/2}(2u-1)\right), \tag{41}$$

[3] Our choice is similar to the standard form-factor decomposition for K_{e4} decay (see e.g., Ref. [13]).

where the coefficients $a_n^{(\rho)\perp}$ have the same scale-dependence as the ones in the Gegenbauer expansion of the dipion chiral-odd DA.

The simplest and rather straightforward way to assess the dominance of the ρ-meson contribution to r.h.s. of the dispersion relations (34) and (35) is to compare numerically both parts of these relations at $k^2 \sim 4m_\pi^2$ where we can evaluate the l.h.s. knowing the coefficients $B_{nl}^{\perp}(k^2)$ at low dipion masses. A noticeable difference between both sides of these relations will clearly indicate the importance of the heavier than ρ states and/or continuum nonresonant background. The known higher-twist contributions and gluon radiative corrections to the sum rules (38) and (39) (see e.g., Ref. [16]), can be added in future if also the corresponding contributions in the LCSRs for $B \to 2\pi$ form factors are worked out.

4. Numerical analysis

To specify the numerical input for the LCSRs (28) and (29), first of all we have to adopt a quantitative ansatz for the dipion DAs. This task is more complicated than for the single-pion or ρ-meson DAs, because the coefficients $B_{nl}^{\perp}(k^2)$ are now complex functions of dipion invariant mass. More is known on the functions $B_{nl}^{\parallel}(k^2)$, for which the lowest ("asymptotic") one is directly related to the well measured pion form factor in the timelike region: $B_{01}^{\parallel}(k^2) = F_\pi^{em}(k^2)$. In addition, some relations between $B_{nl}^{\parallel}(k^2)$ and the Gegenbauer moments of the single-pion DAs are available [8] via soft-pion limit at $k^2 \to 0$. The only available information on the coefficients $B_{nl}^{\perp}(k^2)$ are the estimates at low k^2 based on the instanton model of QCD vacuum [8,17], up to $n = 4$. We list them in the Appendix B. For the ρ-meson DA we use the same ansatz as the one used in Ref. [15]: $a_2^{\perp} = 0.2 \pm 0.1$, $a_{n>2} = 0$ and $f_\rho^{\perp} = 160 \pm 10$ MeV.

The rest of the input parameters entering LCSRs concerns: (a) the short-distance part of the correlation function, (b) the B-meson decay constant and (c) the quark–hadron duality approximation for the B-meson channel. In the following we comment on these points:

(a) Although here the correlation function is known only at LO, and the choice of the renormalization scale cannot be optimized without gluon radiative corrections, in anticipation of the future NLO improvement, we adopt the same default scale $\mu = 3$ GeV for all scale-dependent parameters including the ones in DAs, following the analyses of LCSRs for the $B \to \pi$ form factor in Refs. [11,18]. We also use the b-quark mass in $\overline{MS}$ scheme $\bar{m}_b(\bar{m}_b) = 4.18 \pm 0.03$ GeV [19] and adopt the central value $m_b = \bar{m}_b(3\ \text{GeV}) = 4.47$ GeV, neglecting a small uncertainty.

(b) The two-point QCD sum rule for f_B at LO is used, which is consistent with our approximation for the LCSRs, schematically:

$$f_B^2 = [f_B^2]_{2ptSR}(m_b, \langle \bar{q}q \rangle, \dots, \mu, \bar{M}^2, \bar{s}_0^B)\,, \tag{42}$$

where the ellipses indicate the vacuum condensate densities of higher dimensions. The expression for this sum rule is well known, hence, for brevity we do not repeat it here; the values of vacuum condensate densities and other parameters are taken the same as in the recent analysis [20] (see Table I there). In particular, we use: for the quark condensate density $\langle \bar{q}q \rangle(2\ \text{GeV}) = (-277\ \text{MeV})^3$, for the optimal Borel parameter $\overline{M}^2 = 5.5\ \text{GeV}^2$ and for the effective threshold $\bar{s}_0^B = 34.0\ \text{GeV}^2$, chosen to reproduce the mass of B-meson from the sum rule.

(c) We anticipate that the typical Borel parameter values for a low dipion mass are in the same ballpark as for the LCSRs for the $B \to \pi$ or $B \to \rho$ form factors. For definiteness we take the interval $M^2 = 16.0 \pm 4.0\ \text{GeV}^2$ and the corresponding threshold values $s_0^B = 37.5 \pm 2.5\ \text{GeV}^2$

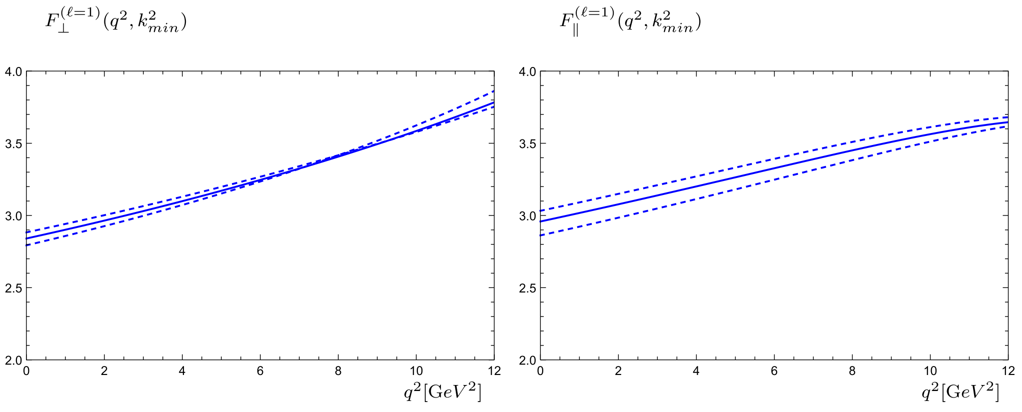

Fig. 2. P-wave contributions to $B \to \pi^+\pi^0$ form factors, $F_\perp^{(\ell=1)}(q^2, k^2_{min})$ (left panel) and $F_\parallel^{(\ell=1)}(q^2, k^2_{min})$ (right panel), calculated from LCSRs at central values of the input. Dashed lines indicate the uncertainty due to the variation of the Borel parameter.

from the analysis in Refs. [11,18]. We expect also that LCSRs with dipion DAs are valid in the same region as the conventional LCSRs with DAs of single hadron, that is at $0 \le q^2 \lesssim 12\ \mathrm{GeV}^2$.

Note that the above input will only serve for numerical illustration and we postpone the overall analysis of uncertainties, having in mind the lack of precision in the new sum rules. Only the Borel-mass dependence will be shown for an assessment of the typical sum rule uncertainties. On the other hand, in all ratios of LCSRs used below, the parametrical uncertainties are expected to be smaller than in the individual sum rules, due to mutual correlations.

Inserting the adopted input in the LCSRs (28) and (29), we calculate first the numerical results for the P-wave contribution $F_\perp^{(\ell=1)}(q^2, k^2_{min})$ and $F_\parallel^{(\ell)}(q^2, k^2_{min})$ at $k^2_{min} = 4m_\pi^2$ and at $q^2 = 0 - 12.0\ \mathrm{GeV}^2$. They are shown in Fig. 2. In Fig. 3 the ratios (33) of F-wave ($l = 3$) and P wave form factors are displayed as a function of q^2. We realize that in the adopted approximation the LCSRs predicts a very small contribution of the higher partial waves in both form factors. The missing higher-twist effects[4] and NLO corrections as well a more elaborated ansatz for the Gegenbauer coefficients $B^\perp_{n\ell}$ can change this ratio, but probably not its order of magnitude.

Finally, in Fig. 4 we plot the ratios obtained dividing the ρ-meson contributions on r.h.s. of Eqs. (34) and (35), by the LCSR results for l.h.s of these relations. As we see, there is up to 20–30% “deficit” which has to be covered by other than ρ contributions to the dispersion relations for the $B \to \pi\pi$ form factors. A more detailed identification of these contributions demands a dispersion relation analysis of DAs in the LCSRs as already mentioned above.

5. Conclusion

In this paper we presented the first systematic derivation of LCSRs for the form factors of $B \to \pi\pi$ semileptonic transitions in terms of dipion light-cone DAs. We considered the case

[4] In fact, one has to mention that the twist 3,4 effects in $B \to \rho$ form factors are rather small, at the level of a few percent as, for example, found in Ref. [21] (see discussion and Fig. 5 there in which the contributions of various twists to the LCSR for $A_1^{B\to\rho}$ form factor are plotted). The situation there is markedly different from the LCSRs for $B \to \pi$ form factors where the twist-3 part is strongly enhanced by the normalization parameter $m_\pi^2/(m_u + m_d)$.

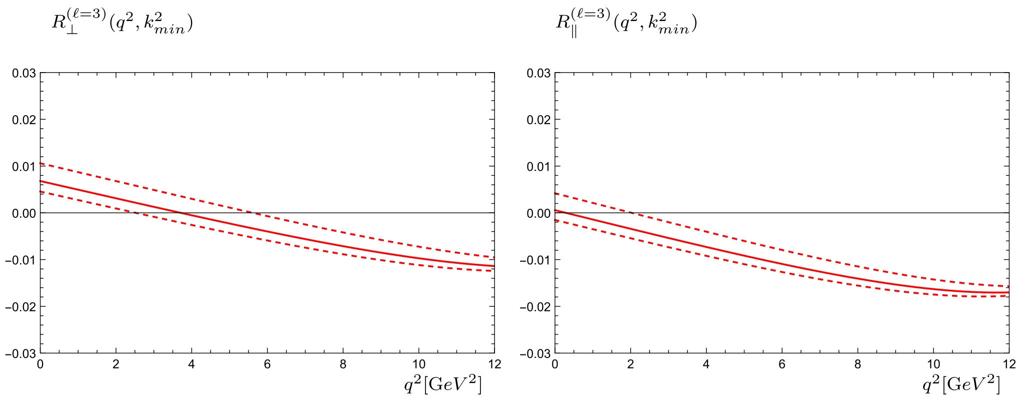

Fig. 3. The ratio of F-wave and P-wave contributions to $B \to \pi^+\pi^0$ form factors $F_\perp$ (left panel) and $F_\parallel$ (right panel) calculated from LCSRs at central values of the input. Dashed lines indicate the uncertainty due to the variation of the Borel parameter.

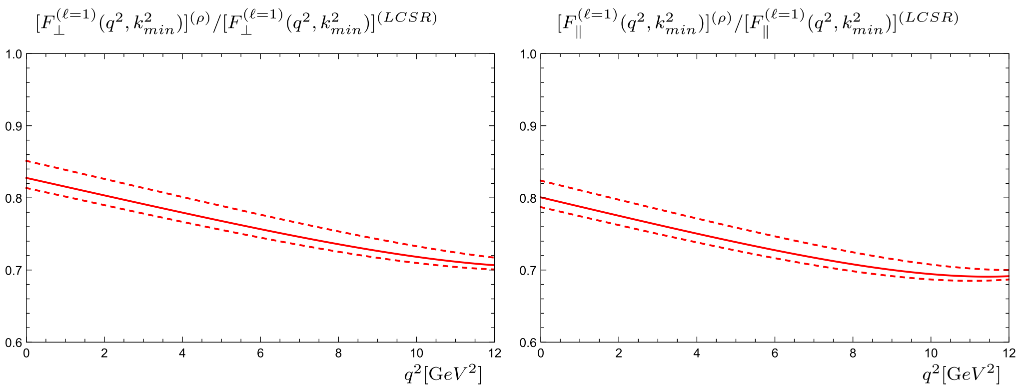

Fig. 4. The relative contribution of ρ-meson to the P-wave $B \to \pi^+\pi^0$ form factors $F_\perp^{(\ell=1)}(q^2,k^2_{min})$ (left panel) and $F_\parallel^{(\ell=1)}(q^2,k^2_{min})$ (right panel) calculated from LCSRs at central values of the input. Dashed lines indicate the uncertainty due to the variation of the Borel parameter.

with an odd angular-momentum (isospin one) dipion state, so that the dependence on the angle θ_π (or equivalently on the invariant variable $q \cdot \bar{k}$) becomes essential. As we have shown, the presence of this variable complicates the derivation of sum rules, producing in separate cases kinematical singularities in the underlying correlation function. We concentrated on two particular form factors for which the sum rules are free from ambiguities. In the twist-2 approximation, the resulting LCSRs are determined by a single, chiral-odd dipion DA. We obtained numerical predictions at small k^2 employing the available nonperturbative estimate of the coefficients in the expansion for this DA.

Apart from the two sum rules for the $F_\perp$ and $F_\parallel$ form factors, we also found a relation between two remaining $\bar{B}^0 \to \pi^+\pi^0$ form factors F_0 and F_t in twist-2 approximation. The remaining question is: how to circumvent the problem of kinematical singularities and derive an additional LCSR for one of the latter form factors, in order to be able to predict their full set. One possibility, a subject of a future investigation, is to modify the correlation function, e.g., by employing a different interpolating heavy–light current for B meson, so that the form factor we need is contained in a kinematical structure free from singularities.

After partial wave expansion, the new sum rules quantify the contributions of higher partial waves to the $B \to \pi\pi$ form factors. These contributions turn out to be very small with respect to the lowest P-wave form factors. Furthermore, in the latter, according to LCSRs, the dominance of the ρ-meson terms parametrized using the LCSRs for $B \to \rho$ form factors is violated at the level of 20–30%.

The question of ρ-meson dominance in the $B \to \pi\pi$ form factors was recently discussed in Ref. [22] where the LCSRs for $B \to \rho, K^*$ form factors were updated. There it was argued that the ρ-state effectively includes the nonresonant background in the P-wave dipion state in the experimental as well as the LCSR prediction for $B \to \rho$. Concerning experimental determination of the ρ-meson decay constant, this statement does not reflect, e.g., the most up-to-date experimental analyses of $e^+e^- \to 2\pi$ and $\tau \to \pi\pi\nu_\tau$ done by CMD-2 [23] and Belle [24] Collaborations, respectively. In both cases the experimentalists use a model of the timelike pion form factor, explicitly taking into account the excited states, e.g., adding a separate $\rho(1450)$-resonance contribution to the ρ-meson contribution and then fitting the resonance parameters. In the similar way, one can assess the ρ-meson dominance in $B \to \pi\pi$ form factors at a quantitative level, including the $B \to \rho(1450)$ transition in the dispersion relations (34) and (35), so that in the $k^2 \lesssim m_\rho^2$ region this contribution represents a nonresonant $B \to \pi\pi$ background interfering with the $B \to \rho$ contribution. We emphasize that the dominance of the ρ-meson and the shape of the nonresonant background are important issues for the $B \to \pi\pi\ell\nu_\ell$ decays. They will be addressed in future using available LCSR results for the $B \to \rho$ form factors and more accurate LCSR analyses of $B \to \pi\pi$ form factors.

In the literature, an earlier attempt to use the dipion DAs in the LCSRs for $B \to \pi\pi$ form factors can be found in Ref. [25]. However, in that analysis an expansion of the correlation function, including the factor $\lambda^{1/2}(p^2, q^2, k^2)$, in powers of the dipion mass k^2 was used. We doubt that in the presence of kinematical singularities, discussed above, such an expansion is legitimate, also in the resulting form factors presented in Ref. [25], the most important contribution of the chiral-odd DA was neglected.

Recently, the LCSRs for $B \to K\pi$ form factors were obtained in Ref. [26] employing the DAs of the $K\pi$ system in the S-wave state, in this case the generalized DAs have the same form as the DAs for a light scalar meson, with no dependence on the variable ζ. In Ref. [26], the twist-2 and twist-3 contributions are taken into account and their common normalization is related to the main input, the scalar $K\pi$ form factor calculated within the chiral perturbation theory framework in [27]. This result provides an estimate for the S-wave contribution to the form factors of the FCNC $B \to K\pi\ell^+\ell^-$ decays.

The calculation presented in our paper can also be extended to the dimeson states with strangeness. If one removes the S-wave constraint on the $K\pi$ state chosen in [26], it is possible to access the $B \to K\pi$ transition form factors with a kaon-pion state in the P-wave and higher partial waves, quantifying the contribution of K^*-resonance in the $B \to K\pi\ell^+\ell^-$ decays. All axial-vector and tensor $B \to K\pi$ form factors can in principle be calculated, choosing an appropriate $b \to s$ transition current in the vacuum $\to K\pi$ correlation function similar to Eq. (1). Here however one needs additional studies of kaon-pion DAs, taking into account the $SU(3)_{flavour}$ violating asymmetry in the Gegenbauer expansion, and establishing the accurate inputs for the coefficients which will involve various timelike $K\pi$ form factors.

Further improvements of LCSRs obtained in this paper are possible in several directions: (1) working out and taking into account the higher-twist components for the vacuum $\to$ dipion bilocal matrix elements, most importantly the twist-3 DAs; (2) calculating the gluon radiative corrections to the hard-scattering amplitude and (3) performing a dispersion relation analysis for the coefficients of DAs considered as analytic functions of the dipion mass.

Let us particularly discuss the future perspectives to go beyond the twist-2 approximation in the LCSRs, such as Eqs. (23) and (24). To that end, one has to retain all operator structures in the vacuum $\to$ dipion matrix element (3) and identify their twist-3,4 components. The latter have to be parametrized in terms of new DAs for which a double (conformal and spatial partial-wave) expansion has to be worked out, similar to Eq. (26) used for the twist-2 DAs. For the isospin-one dipion system, a systematic study of higher-twist effects should go along the similar lines as in the analysis of ρ-meson DAs of twist-3,4 (see e.g., [29]), so that the role of the polarization four-vector of the vector meson will be played by the difference of four-momenta $\bar{k}$. The emerging coefficients of twist-3,4 DAs – analogs of the Gegenbauer coefficients $B_{n\ell}^{\perp,\parallel}(k^2)$ – will represent new timelike pion form factors of certain local (twist-3,4) operators. Note that similar to the twist-2 coefficients, these will be complex functions at $k^2 \geq 4m_\pi^2$. Hence, as opposed to the parameters of one-pion DAs, one cannot access the dipion DAs using QCD sum rules with local OPE. The only timelike form factor available from experiment is the pion electromagnetic form factor $F_\pi(k^2)$ determining the coefficient $B_{10}^{\parallel}(k^2)$. To obtain the remaining coefficients $B_{n>1,\ell}^{\parallel}(k^2)$, $B_{n,\ell}^{\perp}(k^2)$ of twist-2 DAs and the new emerging coefficients of the twist-3,4 DAs one has to combine theoretical methods with the data on two-pion scattering in different partial waves. Apart from the low-energy QCD calculations such as the instanton model at low k^2 [17] we used for the DA coefficients here, a promising strategy to access the larger $k^2 \lesssim 1$ GeV^2 region is to apply hadronic dispersion relations for the coefficients of DAs in the variable k^2, as suggested already in [8]. These relations will involve known resonance structure (positions and widths of two-pion resonances) and can make use of pion scattering phases (via Omnes representation, see e.g., [8] and [12]), but need additional input for normalization of the resonance residues and/or subtraction constants. One possibility to fix the normalizations is to employ dedicated LCSRs with one-pion DAs and the pion interpolating current, similar to the LCSRs for the pion electromagnetic form factor [30]. These auxiliary sum rules will allow one to calculate the new form factors related to the coefficients of dipion DAs in the spacelike region of k^2. Afterwards, one fits the parameters in the hadronic dispersion relations matching the latter at $k^2 < 0$ to the LCSR calculation. This kind of matching between LCSR results and dispersion representation works for the pion electromagnetic form factor, as discussed in [14]. We plan a dedicated study along these lines.

With the LO and twist-2 accuracy, the sum rules for $B \to 2\pi$ form factors obtained in this paper, represent the first exploratory step towards further development of the new LCSR method and towards its extensions to the other important hadronic heavy-to-light form factors with two mesons in the final state.

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Appendix A. Details on kinematics

The correlation function (1) can formally be viewed as an amplitude of a $2 \to 2$ process, in which the two initial particles with the squared masses p^2 and q^2 produce a final dipion state, the dipion mass squared $k^2 = (p-q)^2$ plays then the role of the Mandelstam variable s, so that $k^2 \geq 4m_\pi^2$, whereas $q \cdot \overline{k} = p \cdot \overline{k} = (t-u)/2$, and the standard condition for the sum of the three variables reads: $s + t + u = 2m_\pi^2 + q^2 + p^2$. The following kinematical limits for the variable $q \cdot \overline{k}$ are then derived using a general inequality for the Mandelstam variables:

$$-\lambda^{1/2}(p^2, q^2, k^2)\sqrt{1 - \frac{4m_\pi^2}{k^2}} \leq 2(p \cdot \overline{k}) \leq \lambda^{1/2}(p^2, q^2, k^2)\sqrt{1 - \frac{4m_\pi^2}{k^2}}\,, \tag{43}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

It is convenient to decompose the momenta k, $k_{1,2}$ near the light cone:

$$k^\mu = \frac{1}{2}(k^+ n^{+\mu} + k^- n^{-\mu}) + k^{\perp\mu}\,, \tag{44}$$

where $n^{\pm\mu} = (1, 0, 0, \pm 1)$.

The parameter ζ determines the light-cone momentum fractions carried by the two pions in the final state [6,8]:

$$\zeta = k_1^+/k^+, \quad 1-\zeta = k_2^+/k^+\,, \quad \zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}\,. \tag{45}$$

To relate this parameter to the invariant variable $q \cdot \overline{k}$, it is convenient to choose the kinematical configuration where the four-momenta p and q of external currents in the correlation function are aligned with the z-direction, so that $k^{\perp\mu} = 0$. The relation has then a form of quadratic equation with a solution:

$$q \cdot \bar{k} = \frac{1}{2}(2\zeta - 1)\lambda^{1/2}(p^2, q^2, k^2)\,, \tag{46}$$

At $p^2 = m_B^2$ we recover the relation (46) for $B \to \pi\pi\ell\nu_\ell$ decay (see e.g., [2]). The parameter ζ is related via Eq. (22) to the angle between the pions in their c.m. frame. The latter relation substituted in Eq. (46) reproduces the limits (43).

The origin of the imaginary part of the $\lambda^{1/2}$-function in the variable p^2 mentioned in Sect. 2 is evident from the following form:

$$\lambda^{1/2}(p^2, q^2, k^2) = (p^2 - (\sqrt{q^2} - \sqrt{k^2})^2)^{1/2}(p^2 - (\sqrt{q^2} + \sqrt{k^2})^2)^{1/2}. \tag{47}$$

Appendix B. Details on dipion DA's

The coefficient functions of the double polynomial expansion of dipion DAs are multiplicatively renormalized in the one-loop approximation:

$$B_{nl}^{\parallel,\perp}(k^2, \mu) = B_{nl}^{\parallel,\perp}(k^2, \mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_n^{\parallel,\perp} - \gamma_0^{\parallel,\perp})/\beta_0}\,, \tag{48}$$

where $\beta_0 = 11 - 2/3n_f$, and the anomalous dimensions are [28]:

$$\gamma_n^{\parallel} = C_F\Big(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{k=2}^{n+1}\frac{1}{k}\Big), \quad \gamma_n^{\perp} = \frac{8}{3}\Big(1 + 4\sum_{k=2}^{n+1}\frac{1}{k}\Big). \tag{49}$$

For the chiral-odd dipion DA these functions are taken from [8] where they are calculated at small k^2 in the instanton model of QCD vacuum at the scale $\mu \simeq 600$ MeV:

$$\begin{aligned}
&B_{01}^{\perp}(k^2) = 1 + \frac{k^2}{12M_0^2},\\
&B_{21}^{\perp}(k^2) = \frac{7}{36}\left(1 - \frac{k^2}{30M_0^2}\right), \quad B_{23}^{\perp}(k^2) = \frac{7}{36}\left(1 + \frac{k^2}{30M_0^2}\right),\\
&B_{41}^{\perp}(k^2) = \frac{11}{225}\left(1 - \frac{5k^2}{168M_0^2}\right), \; B_{43}^{\perp}(k^2) = \frac{77}{675}\left(1 - \frac{k^2}{630M_0^2}\right),\\
&B_{45}^{\perp}(k^2) = \frac{11}{135}\left(1 + \frac{k^2}{56M_0^2}\right).
\end{aligned} \tag{50}$$

The normalization constant is related to the key mass parameter of the instanton model $M_0 \simeq 350$ MeV via $f_{2\pi}^{\perp} = 4\pi^2 f_\pi^2/3M_0 \simeq 650$ MeV, where $f_\pi = 132$ MeV is the pion decay constant.

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