Analysis of Lightning-Induced Voltages in Overhead Transmission Lines

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Abstract

In this work we study the coupling of lightning induced voltages in Overhead Transmission Lines. The study is led directly in the time domain with hold in account the effect of a finite conductivity of the soil. After discretization by the method so-called FDTD (Finite Difference Time Domain) of the lines equations excited by a lightning wave and the application in every node of the network in current and voltage, we deduct a equations system, of which the resolution permits us to deduct the induced electric quantities in every node of the network. In order to confirm our theoretical work, we present a set of applications that allows validating this study.

1. Introduction

The induced voltages generated by lightning in the transmission of electrical energy are nowadays one of the main causes of poor quality of energy supplied to consume and electromagnetic compatibility. Due to growing a better quality of electricity demand, protection against disturbances caused by lightning has become of paramount importance. Therefore, the evaluation of induced voltages has become essential for the effective protection of electrical and electronic systems. In this work we interested on the calculation of currents and voltages induced by lightning channel on an overhead line, which is usually responsible for the transmission of the disturbance by conduction. In this several works are devoted to this subject [1, 2], [3,4], antenna theory is the most rigorous [5] but implementation remains inadequate and heavy work for long wire frames. Drawing on the work of C.R Paul [6]. We model the coupling lightning-line by the method of transmission line. For the electromagnetic excitation, which is the second of the equations of the lines, we use the formalism of the dipoles for the calculation of the electromagnetic field radiated by the lightning channel.

2. Modelling return stroke channel

To calculate the lightning electromagnetic field, a straight vertical channel over a perfectly conducting ground is assumed. Figure 1 shows the geometry for the calculation nearby overhead line. The electric and magnetic fields generated by the lightning return stroke can be obtained if the spatial temporal distribution of the lightning return stroke current \( i(z_0, t) \) and its velocity along the channel are known. There are several return stroke models that have been...
proposed by earlier researchers that specify these parameters [7]. In the present study the modified transmission line (MTL) model has been adopted. According to this model, the lightning current is allowed to decrease with height while propagating upward along the channel and is described as follows:

\[ i(z', t) = i(0, t - \frac{z'}{v}) \exp \left( -\frac{z'}{\xi} \right) u \left( t - \frac{z'}{v} \right) \]  

(1)

Where \( v \) is the velocity of the return stroke and \( \xi \) is the decay constant that accounts for the effect of the vertical distribution of charge stored in the corona sheath of the leader and subsequent discharge during the return stroke phase.

\[ v \]

\[ \xi \]

Fig. 1. Geometry of the line.

magnetic field is required when using the Cooray–Rubinstein formula. For a current element of length \( dz' \) at a height \( z' \) and carrying a current \( i(z', t) \), the vertical electric field \( dE_z(r, z, t) \), the horizontal electric field \( dE_r(r, z, t) \) and the magnetic field \( dH_z(r, z, t) \) at a point \( P(r, \phi, z) \) over a perfectly conducting ground in cylindrical co-ordinates in the time domain are given by the following expressions:

\[
dE_z(r, z, t) = \frac{1}{4\pi \epsilon_0 R^2} \left[ \frac{2(\epsilon - \zeta)}{\epsilon} \left( \frac{\partial}{\partial t} \right) \frac{\epsilon}{c^2} \left( \frac{\partial}{\partial t} \right) \left( \epsilon R \right) \right] dE_r(r, z, t)
\]

(2)

\[
dE_r(r, z, t) = \frac{1}{4\pi R^2} \left[ \frac{2(\epsilon - \zeta)}{\epsilon} \left( \frac{\partial}{\partial t} \right) \frac{\epsilon}{c^2} \left( \frac{\partial}{\partial t} \right) \left( \epsilon R \right) \right] dE_z(r, z, t)
\]

(3)

\[
dH_z(r, z, t) = \frac{1}{4\pi R^2} \left[ \frac{2(\epsilon - \zeta)}{\epsilon} \left( \frac{\partial}{\partial t} \right) \frac{\epsilon}{c^2} \left( \frac{\partial}{\partial t} \right) \left( \epsilon R \right) \right] dH_r(r, z, t)
\]

(4)

where \( \epsilon_0 \) is the permittivity of free space and \( c \) is the velocity of light. \( R = \sqrt{r^2 + (z-z')^2} \) is the distance from the current element to the observation point. The above equations use the MTL lightning return stroke model discussed in the previous Section. The total vertical and horizontal electric field and magnetic field are obtained by integrating along the lightning return stroke channel and its image. The horizontal component of the electric field with the finite conductivity of the ground is calculated using the Cooray–Rubinstein formula [8]. The horizontal electric field \( E_{\phi}(z = h, r) \) as per the above formula is given by

\[
E_{\phi}(z = h, r) = E_z(z = h, r) - H_y(z = 0, r) \frac{\sqrt{\mu_0}}{\sqrt{\epsilon_0 + \sigma_0 / j\omega}}
\]

(5)

where \( E_z(z = h, r) \) is the Fourier-transform of the horizontal electric field at height \( h \), \( H_y(z = 0, r) \) is the Fourier transform of the azimuthal component of the magnetic field at ground level, \( \mu_0 \) is the permeability of air, and \( \epsilon \) and \( \sigma \) are the permittivity and conductivity of the ground, respectively. Both \( E_z(z = h, r) \) and \( H_y(z = 0, r) \) are calculated assuming a perfect conducting ground using (2) and (4).
3. Modelling of coupling of electromagnetic field to overhead line

In this work, the coupling model as proposed by Taylor et al. has been used. The presence of finite ground conductivity is included in the coupling equation by two terms, namely (i) series per-unit length impedance, which accounts for earth return, and (ii) transverse ground admittance. Both these terms are frequency dependent. The transverse ground admittance is generally neglected for overhead power distribution lines. The frequency-dependent series ground impedance is represented in the time-domain coupling equation by a convolution integral [9]. Hence the coupling equations, including the ground impedance in the time domain, get modified as follows:

\[
\frac{d[v_i(x,t)]}{dx} + \left[ L_{ij} \right] \frac{\partial}{\partial t} [v_i(x,t)] + \int_0^h [\sigma_{\text{so}}(x,\tau)] \frac{\partial}{\partial \tau} [i_j(x,\tau)] d\tau = - \frac{\partial}{\partial t} \left[ \int_0^h B_i^x(x,z,t) dz \right]
\]

(6)

\[
\frac{d[i_j(x,t)]}{dx} + \left[ C_{ij} \right] \frac{\partial}{\partial t} [v_i(x,t)] + \left[ G_{ij} \right] [v_j(x,t)] = - \frac{\partial}{\partial t} \left[ C \int_0^h E_z^x(x,z,t) dz \right]
\]

(7)

where \( \int_0^h B_i^x(x,z,t) dz \) and \( \int_0^h E_z^x(x,z,t) dz \) is the vector of the incident magnetic field and electric field along the conductor at conductor height \( h \); the sub-index \( i \) denotes the particular wire of the multiconductor line; \( [L] \), \( [G] \) and \( [C] \) are the inductance, conductance and capacitance matrices per unit length of the line, respectively; \( [i] \) is the line current vector; \( [v] \) is the line voltage vector; \( [^G] \) is the transient ground resistance matrix and is equal to the inverse Fourier-transform of \( \frac{\sigma_{\text{so}}}{\sigma_{\text{so}}} \), i.e. \(\left[ \sigma_{\text{so}} \right] = F^{-1} \left[ \frac{\sigma_{\text{so}}}{\sigma_{\text{so}}} \right] \) and \( [Z_{\text{so}}] \) is the ground impedance matrix. The internal impedance of the line is neglected.

The boundary conditions for the voltage vector

\[
\begin{align*}
\left[ v_i \right](0,t) &= -[Z_A][i_i(0,t)] \\
\left[ v_i \right](L,t) &= [Z_B][i_i(L,t)]
\end{align*}
\]

(8)

(9)

where \( [Z_A] \) and \( [Z_B] \) are the terminating impedance matrices. A single-wire overhead line equivalent circuit using this model is shown in Fig. 2.

![Fig. 2. Equivalent circuit of single-wire overhead line excited by lightning return-stroke field (conductance neglected).](image)

4. Time-domain approximation for transient ground resistance

For the case of multiconductor lines, the ground impedance is a full matrix with diagonal (self-impedance) and off diagonal terms. Accurate approximations for the ground impedance corresponding to single-wire lines have been presented by Sunde and Vance. The expression for the mutual ground impedance between the conductors in low-frequency approximation i.e. assuming \( \sigma_g \gg \omega \varepsilon_x \), is given by [10]:
where $r_i$ is the distance between the two conductors in the horizontal plane. The general expression for the ground impedance matrix does not have an analytical inverse Fourier transform. However, a low-frequency approximation of the inverse Fourier transform of the ground self impedance is given by [10]:

$$
\tilde{\zeta}_i(t) = \frac{\mu_0}{4\pi r_{si}} \left[ \exp\left(\frac{r_{si}}{t}\right) - 1 - \sum_{n=1}^{\infty} \frac{a_n}{n} \left( \frac{r_{si}}{t} \right)^{2n+1} \right]
$$

where $r_{si} = h_i^2 \mu_0 \sigma_g$ and $a_n = \frac{2^n}{1.3... (2n+1)}$

The off-diagonal terms of the transient ground resistance matrix are given by

$$
\tilde{\zeta}_{ij}(t) = \frac{\mu_0}{\pi \xi (t)} \left[ \frac{1}{2\xi h} \cos\left(\frac{\pi h}{2\xi}\right) + \frac{j\xi h}{2\xi} \cos\left(\frac{\pi h}{2\xi}\right) \right] - \frac{1}{2\xi h} \sum_{n=1}^{\infty} a_n \left( \frac{r_{ij}}{t} \right)^{2n+1} \frac{\cos\left(\frac{2n-1}{2}\theta_i\right)}{4}
$$

with

$$
\left( \frac{h_i + h_j}{2} + \frac{j v_i}{2} \right) = T_i e^{i\phi_i}
$$

The elements of the transient ground resistance matrix $[\tilde{\zeta}_{ij}(t)]$ present singularity as $t \to 0$. This singularity is due to the low-frequency approximation used in deriving equation (11) and equation (12) and it is not present otherwise [11]. Indeed, considering the diagonal terms of the general expression equation (10) it can be shown [12] that

$$
\lim_{t \to 0} \tilde{\zeta}_{ii} = \frac{1}{2\pi h} \sqrt{\frac{\mu_0}{\xi_i E_{rs}}}
$$

and applying the initial value theorem, we get

$$
\tilde{\zeta}_{ii}(t = 0) = \lim_{t \to 0} j\omega \tilde{\zeta}_{ii} = \frac{1}{2\pi h} \sqrt{\frac{\mu_0}{\xi_i E_{rs}}}
$$

In a similar way, considering the mutual terms of equation (10), it can be easily demonstrated that

$$
\lim_{t \to 0} \tilde{\zeta}_{ij} = \frac{1}{2\pi h} \sqrt{\frac{\mu_0}{\xi_i E_{rs}}}
$$

in which

$$
h = \frac{h_i + h_j + r_{ij}^2}{(h_i + h_j)}
$$

and therefore

$$
\tilde{\zeta}_{ij}(t = 0) = \lim_{t \to 0} j\omega \tilde{\zeta}_{ij} = \frac{1}{2\pi h} \sqrt{\frac{\mu_0}{\xi E_{rs}}}
$$
Equation (14) and equation (16) show that the ground transient resistance tends to an asymptotic value when $t \to 0$. It is interesting to note that this asymptotic value is expressed in terms of the line height and the ground relative permittivity and is independent of the ground.

The coupling equations are solved using the finite-difference time domain (FDTD) method. The conductors are subdivided into successive and equally spaced voltage and current nodes. Two successive nodes of the same type are separated by an interval $\Delta x$. The two end points of the conductors are defined as voltage nodes.

The recurrence Equations for Current and Voltage are:

\[
\begin{aligned}
\left[ v_{k+1}^{n+1} \right] &= \left( \frac{[C] - [G]}{2} \right) \left( \frac{[C] - [G]}{2} \right) \left[ v_k^n \right] - \\
&\quad - \left[ G \left[ h \right] \left[ E_{G, k}^{n+1} \right] - \left[ E_{G, k}^{n} \right] \right] \\
&\quad - \left[ C \left[ h \right] \left[ E_{G, k}^{n+1} \right] - \left[ E_{G, k}^{n} \right] \right] \\
&\quad - \left[ \frac{L}{\Delta t} \right] \left[ \frac{i_{k+1}^{n+1} \Delta t}{2} \right] - \left[ \frac{L}{\Delta t} \right] \left[ \frac{i_{k}^{n+1} \Delta t}{2} \right] - \\
&\quad - \frac{1}{2} \sum_{j=0}^{n+1} \left[ \left[ \xi_g (n-j) \Delta t \right] + \left[ \xi_g (n+1-j) \Delta t \right] \right] \\
&\quad - \left[ \frac{\xi_g (n-j) \Delta t}{2} \right] - \left[ \frac{\xi_g (n+1-j) \Delta t}{2} \right] \\
&\quad - \left[ \frac{E_{G, k+1}^{n+1} \Delta t}{2} \right] - \left[ \frac{E_{G, k}^{n+1} \Delta t}{2} \right] \\
&\quad - \left[ \frac{E_{G, k+1}^{n+1} \Delta t}{2} \right] - \left[ \frac{E_{G, k}^{n+1} \Delta t}{2} \right].
\end{aligned}
\]
\[
\begin{align*}
[x^{n+1}] &= \left[ \begin{array}{cc}
[\Delta] & [G] \\
2 & 2
\end{array} \right]^{-1} \left[ \begin{array}{c}
[\Delta] \\
2
\end{array} \right] [x^n] \\
&= \left[ \begin{array}{c}
\frac{n+1}{2} \\
\frac{n-1}{2}
\end{array} \right] \Delta t - \frac{[G][h]}{\Delta t} \left[ \frac{E^{n+1}_{x,k \text{ max}}}{2} + \frac{E^n_{x,k \text{ max}}}{2} \right]
\end{align*}
\]

(19)

\[
\begin{align*}
[U^n_{k \text{ max}}] &= \left[ \begin{array}{cc}
[\Delta] & [G] \\
2 & 2
\end{array} \right]^{-1} \left[ \begin{array}{c}
[\Delta] \\
2
\end{array} \right] [U^n_{k \text{ max}}] \\
&= \left[ \begin{array}{c}
\frac{n+1}{2} \\
\frac{n-1}{2}
\end{array} \right] \Delta t - \frac{[G][h]}{\Delta t} \left[ \frac{E^{n+1}_{x,k \text{ max}}}{2} + \frac{E^n_{x,k \text{ max}}}{2} \right]
\end{align*}
\]

(20)

The determination of the disturbances induced in Overhead transmission lines illuminated by a lightning wave, is to solve the system of equations of the type

\[
[A][X] = [B]
\]

(21)

The first step is the definition of the matrix \([A]\) consists of two sub-matrices.

\[
[A] = \left[ \begin{array}{c}
A_1 \\
A_2
\end{array} \right] : \text{matrix topology representation of the line;}
\]

The sub-matrix \([A_1]\) is obtained after the writing equations (19) an (20) for each line length \(\ell\). The sub-matrix \([A_2]\) is derived from the classical laws of Kirchhoff. For each network \(m\).

Fig. 3. Illustration of nodes interconnections junctions".
The vector $[\chi]$ contains the unknown currents and voltages at both ends of each line. This vector $[\beta]$ is composed of two sub-vectors $[\beta_1]$ and $[\beta_2]$. In the vector $[\beta_1]$ will contain the second member of the equations (19) and (20). The second vector in $[\beta_2]$ contains nonlinear relationships currents and voltages and / or generators in each node $m$.

5. Results and discussions

To validate our theoretical developments for coupling wave lightning-line, we treat an application whose results are published in [12]. We consider a three-phase line vertical of 1 km in length at a height of 10m above the finished ground conductivity $\sigma = 0.001 \text{s/m}$, illuminated by an electromagnetic wave radiated by a lightning channel of 7.5km, the point of impact is considered symmetrical at both ends and at a distance of 50m from the latter, the electromagnetic field emitted by the lightning wave is calculated using the return-stroke to the MTL model with a typical value of $v=1.3 \times 10^8 \text{m/s}$, a decay rate of the current intensity and $\lambda = 2000 \text{km F. Heidler}$ expression for the current at the base of the channel with the data in Table 1.

Table 1. Lightning parameters.

<table>
<thead>
<tr>
<th>$I_{01}$ (kA)</th>
<th>$\tau_{11}$ ($\mu$s)</th>
<th>$\tau_{21}$ ($\mu$s)</th>
<th>$n_1$</th>
<th>$I_{02}$ (kA)</th>
<th>$\tau_{12}$ ($\mu$s)</th>
<th>$\tau_{22}$ ($\mu$s)</th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.7</td>
<td>0.25</td>
<td>2.5</td>
<td>2</td>
<td>6.5</td>
<td>2.1</td>
<td>230</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 4. Geometry considered.

![Fig. 4](image)

Fig. 5. (a) voltages induced at the extremity of phase 1 of the line, (b) voltages induced at the extremity of phase 1 of the line [12].

Our calculation results correspond to those published by [12].

Calculation results that we present in this section are those obtained for different configurations of the line (figure 6).
Fig. 6. Different configurations of the line.

Fig. 7. (a) voltages induced at the extremity of conductor 1 « triangle line », (b) voltages induced at the extremity of conductor 2 « triangle line ».

Fig. 8. (a) voltages induced at the extremity of conductor 3 « triangle line », (b) voltages induced at the extremity of conductor 1 « vertical line ».

Fig. 9. (a) voltages induced at the extremity of conductor 2 « vertical line », (b) voltages induced at the extremity of conductor 3 « vertical line ».
We also note that the effect of the finite conductivity of the ground is manifested by the appearance of a negative peak and a reduction of the induced voltage due to losses incurred in the line by the finite conductivity of the ground.

6. Conclusion

This set of applications, shows that it is possible to quantify by calculating the voltages induced resulting from the electromagnetic effect of lightning on an overhead distribution network. This is an advantage for the insulation coordination and the right choice in voltage (reference voltage) and location of surges arresters. By the classic method poses a problem of digital heaviness linked to the spatial discretization and taking into account the electrical conditions at the ends of nodes and connections. The formalism that we propose overcomes the spatial discretization taking into account the electromagnetic assault along each conductor and the electrical conditions at the ends. The particular interest of this formalism is that it allows us to deduce the currents and voltages induced in every node of the network directly. This advantage then makes it possible to study the effect of indirect disturbance on the electrical devices located in a meshed network. This formalism has a certain advantage in terms of computation time view.

References


[3] Lightning Interaction with 132 kV Transmission Line Protected by Surge Arresters UPEC 2011 · 46th International Universities' Power Engineering Conference · 5-8th Soest, Germany, September


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