Rates and asymmetries in $B \to K \pi$ decays

Michael Gronau $^1$, Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, IL 60637, USA

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Abstract

We discuss a potential discrepancy in an approximate relation among $B \to K \pi$ rates which, with increased statistical significance, would imply new physics in $\Delta I = 1$ transitions. An approximate relation between CP-violating rate differences in $B^0/\overline{B}^0 \to K^{\pm}\pi^\mp$ and $B^\pm \to K^{\pm}\pi^0$ is used to combine these rate differences to reduce upper limits on the two CP asymmetries. These rates and asymmetries are used to update bounds on the CKM phase $\gamma$.

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In Ref. [1] we proposed separate relations among decay rates and among direct CP asymmetries in $B \to K \pi$ decays, following from a model-independent hierarchy among various contributions to decay amplitudes. At that time three of these decays, $B^0 \to K^+\pi^-$, $B^+ \to K^+\pi^0$ and $B^+ \to K^0\pi^+$, had been observed, while a fourth, $B^0 \to K^0\pi^0$, still remained to be seen. The question of direct CP asymmetries in these decays remained very much an open one. We noted the conditions under which one expected the following sum rule to hold [1,2]:

$$2\Gamma(B^+ \to K^+\pi^0) + 2\Gamma(B^0 \to K^0\pi^0) 
\approx \Gamma(B^+ \to K^+\pi^+) + \Gamma(B^0 \to K^+\pi^-),$$

(1)

and derived an approximate relation between the rate differences in the decays involving $K^+$:

$$\Delta(K^+\pi^-) 
\equiv \Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-) 
\simeq 2\Delta(K^+\pi^0)$$

$$\equiv 2[\Gamma(B^- \to K^-\pi^0) - \Gamma(B^+ \to K^+\pi^0)]$$

(2)

which would allow one to combine such rate differences to improve the statistical accuracy of either one. In the present note we update these analyses, as well as one [3] in which decay rates and asymmetries are combined in order to obtain limits on phases of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. (Preliminary accounts of some of this last work have appeared in Ref. [4].)

The decay rates and CP asymmetries which we use are summarized in Table 1. We use averages of CLEO [5], BaBar [6], and Belle [7] measurements compiled in Ref. [8], and new BaBar results [9] on $B^+ \to K^0\pi^+$. To relate branching ratios to decay rates we have used $\tau^+ = (1.656 \pm 0.014)$ ps and $\tau^0 = (1.539 \pm 0.014)$ ps [10] for the respective $B^+$ and $B^0$. 

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$^1$ Permanent address: Physics Department, Technion, Israel Institute of Technology, 32000 Haifa, Israel.

E-mail address: rosner@bquark.uchicago.edu (J.L. Rosner).
Table 1
CP-averaged branching ratios, CP-averaged decay rates, and CP rate asymmetries for $B \to K\pi$ decays. Branching ratios and CP asymmetries are based on averages in Ref. [8] except for $B^+ \to K^0\pi^+$, where we have used new BaBar results [9] in our averages.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Branching ratio ($10^{-6}$)</th>
<th>Partial width ($10^{-9}$ eV)</th>
<th>$A_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to K^+\pi^-$</td>
<td>18.16 ± 0.79</td>
<td>7.77 ± 0.35</td>
<td>−0.088 ± 0.040</td>
</tr>
<tr>
<td>$B^0 \to K^0\pi^0$</td>
<td>11.21 ± 1.36</td>
<td>4.79 ± 0.58</td>
<td></td>
</tr>
<tr>
<td>$B^+ \to K^0\pi^+$</td>
<td>20.62 ± 1.35</td>
<td>8.19 ± 0.54</td>
<td>0.003 ± 0.059</td>
</tr>
<tr>
<td>$B^+ \to K^+\pi^0$</td>
<td>12.82 ± 1.07</td>
<td>5.10 ± 0.43</td>
<td>0.035 ± 0.071</td>
</tr>
</tbody>
</table>

The terms $|t/p|$ and $|c/p|$ are second order in the small ratios $|t/p|$ and $|c/p|$. It holds separately for the decays shown and their CP-conjugates. In Ref. [8] the last two terms are found to contribute at most 4% of the dominant $2|p|^2$ terms.

Using the experimental values for CP-averaged partial widths in Table 1, the sum rule reads

\[ (19.8 ± 1.4) \times 10^{-9} \text{ eV} = (16.0 ± 0.6) \times 10^{-9} \text{ eV}, \]

The left-hand side differs from the right-hand side by $3.8 ± 1.6 \times 10^{-9}$ eV, or $(24 ± 10\%)$ of the better-known right-hand side. This discrepancy is too large to be accounted for by the neglected standard-model terms. If it is not caused by new physics effects, the most likely source is a systematic underestimate of the efficiency for $\pi^0$ detection in each experiment.

An enhancement of $B \to K\pi$ modes involving a neutral pion would be interpreted as a new physics amplitude in $\Delta I = 1$ transitions. Written in terms of isospin amplitudes, the sum rule (1) reads [1]

\[ |B_{1/2}|^2 + |A_{1/2} - 2A_{3/2}|^2 \approx |B_{1/2}|^2 + |A_{1/2} + A_{3/2}|^2, \]

where $A$ and $B$ are $\Delta I = 1$ and $\Delta I = 0$ amplitudes and subscripts denote the isospin of $K\pi$. They are related to $p$, $c$, and $t$ by [1]

\[ B_{1/2} = p + \frac{t}{2}, \quad A_{1/2} = \frac{2c - t}{6}, \quad A_{3/2} = \frac{c + t}{3}. \]

The sum rule holds when

\[ 3|A_{3/2}|^2 - 2 \text{Re}(A_{1/2}A_{3/2}^*) \ll |B_{1/2}|^2. \]
beyond a few percent (which could be associated with isospin violations stemming from $m_u \neq m_d$) would imply $\Delta I = 1$ contributions from physics beyond the Standard Model. Models involving such amplitudes and several other manifestations in $B \to K\pi$ decays were studied in [12].

The validity or violation of the sum rule affects interpretations of various $B \to K\pi$ rate ratios [3,13, 15–17]. Three rate ratios provide useful information and use the binomial expansion for $R_n$ stands for a CP-averaged partial width.

where we have used the averages of Table 1, and $\overline{R}$ stands for a CP-averaged partial width.

To first order in terms of order $|t/p|$ and $|c/p|$ (where $c$ is dominated by $P_{EW}$), the ratios (14) and (15) should be equal. In fact, at this order the equality of the two ratios of rates holds separately for $B^+$ and $B^0$ and for $B^-$ and $\overline{B}^0$. To see this, we write

$$2\overline{\Gamma}(B^+ \to K^+\pi^-)/\overline{\Gamma}(B^+ \to K^0\pi^+) = \left|{p + c + t\over p}\right|^2,$$

$$2\overline{\Gamma}(B^0 \to K^+\pi^-)/\overline{\Gamma}(B^0 \to K^0\pi^+) = \left|{p + t\over p - c}\right|^2,$$

and use the binomial expansion for $(p - c)^{-1} = p^{-1}(1 - {c\over p})^{-1}$ in the second relation. Alternatively, we can show that $R_c = R_n$ to this order by writing

$$2\overline{\Gamma}(B^+ \to K^+\pi^-) = \overline{\Gamma}(B^+ \to K^0\pi^+)(1 + \epsilon_{+0}).$$

and

$$2\overline{\Gamma}(B^0 \to K^0\pi^+) = \overline{\Gamma}(B^+ \to K^0\pi^+)(1 + \epsilon_{+0}).$$

The sum rule (1) implies $\epsilon_{+0} + \epsilon_{00} = \epsilon_{+-}$, or to first order in small quantities $\epsilon$,

$$R_c = (1 + \epsilon_{+0}) = {1 + \epsilon_{+-} \over 1 + \epsilon_{00}} = R_n.$$

The fact that $R_c$ and $R_n$ differ so much [18], being nearly $2\sigma$ above and below 1, respectively, is directly related to the large violation of the sum rule (1). Thus, until the source of the sum rule violation is clarified, one should view results based on either ratio with some caution. We shall show below that one may cancel out effects of imperfectly determined $\pi^0$ detection efficiency by considering the quantity $(R_c R_n)^{1/2}$.

We now turn to the relation (2). At a leading order in $|T/P|$, $|P_{EW}/P|$ and $|C/T|$, the two rate differences are equal [1], since they involve a common interference term of $p$ and $t$ (namely $P$ and $T$). The rate difference $\Delta(K^+\pi^0)$ contains also a higher order interference of $p$ and $c$ (namely, $P$ and $C$) dominating $\Delta(K^0\pi^0)$, and an even higher order interference of $P_{EW}$ and $C$. Using the partial widths in Table 1, we find

$$\Delta(K^+\pi^-) = (-0.67 \pm 0.31) \times 10^{-9} \text{ eV},$$

$$2\Delta(K^0\pi^0) = (0.36 \pm 0.71) \times 10^{-9} \text{ eV}.$$  (19)

These two partial rate differences are consistent with each other and with zero. If constrained to be equal and averaged, they give

$$\Delta(K^+\pi^-) = 2\Delta(K^0\pi^0) = (-0.52 \pm 0.29) \times 10^{-9} \text{ eV}.$$  (20)

The $K^+\pi^-$ asymmetry clearly carries more weight. The implied CP asymmetries are then

$$A_{CP}(B^0 \to K^+\pi^-) = -0.066 \pm 0.037,$$

$$A_{CP}(B^+ \to K^+\pi^-) = -0.051 \pm 0.028.$$  (21)

These can be used, if desired, in updated analyses along the lines of Ref. [3], to interpret experimental ranges of $R$ and $R_c$ in terms of limits on the weak CKM phase $\gamma$. Instead, we shall use the observed CP asymmetries separately in each channel, since, as we shall show, neither analysis is very sensitive to small changes in the CP asymmetries as long as these are already small.

The decay $B^+ \to K^0\pi^+$ is a pure penguin ($p$) process, while the amplitude for $B^0 \to K^+\pi^-$ is proportional to $p + t$, where $t$ is a tree amplitude. The ratio $t/p$ has magnitude $r$, weak phase $\gamma \pm \pi$ (depending on convention), and strong phase $\delta$. The ratio $R$ of these two rates (averaged over a process and its CP conjugate) is

$$R = 1 - 2r \cos \gamma \cos \delta + r^2.$$  (22)
The CP asymmetry in $B^0 \to K^+\pi^-$ is $A_{\text{CP}}(B^0 \to K^+\pi^-) = -2r (\sin \gamma \sin \delta)/R$. One may eliminate $\delta$ between this equation and Eq. (22) and plot $R$ as a function of $\gamma$ for the allowed range of $A_{\text{CP}}(B^0 \to K^+\pi^-)$. The value of $r$, based on present branching ratios and arguments given in Refs. [3,13], is $r = 0.17 \pm 0.04$. The average in Table 1 implies $|A_{\text{CP}}(B^0 \to K^+\pi^-)| \leq 0.13$ at the $1\sigma$ level. Curves for $A_{\text{CP}}(B^0 \to K^+\pi^-) = 0$ and $|A_{\text{CP}}(B^0 \to K^+\pi^-)| = 0.13$ are shown in Fig. 1. The lower limit $r = 0.13$ is used to generate these curves since the limit on $\gamma$ will be the most conservative.

At the $1\sigma$ level, using the constraints that $R$ must lie between 0.873 and 1.022 and $|A_{\text{CP}}(B^0 \to K^+\pi^-)|$ must lie between zero and 0.13, one finds $\gamma \gtrsim 50^\circ$. (We consider only those values of $\gamma$ allowed at 95% confidence level by fits to other observables [14], $38^\circ \leq \gamma \leq 80^\circ$. Thus although values of $\gamma \lesssim 31^\circ$ are allowed in Fig. 1, we do not consider them further. We adopt a similar restriction for other bounds to be presented below.) No bound can be obtained at the 95% confidence level, however. If one were to use the improved bound on $A_{\text{CP}}(B^0 \to K^+\pi^-)$ implied by Eq. (2), a slight improvement on the $1\sigma$ lower bound on $\gamma$ would result. Reduction of errors on $R$ and improvement of the estimate of $r$ would have a much greater impact.

The comparison of rates for $B^+ \to K^+\pi^0$ and $B^+ \to K^0\pi^+$ gives similar information on $\gamma$. The amplitude for $B^+ \to K^+\pi^0$ is proportional to $p + t + c$, where $c$ contains a color-suppressed amplitude. Originally it was suggested that this amplitude be compared with $p$ from $B^+ \to K^0\pi^+$ and $t + c$ taken from $B^+ \to \pi^+\pi^0$ using flavor SU(3) [19] using a triangle construction to determine $\gamma$. However, electroweak penguin (EWP) amplitudes contribute significantly in the $t + c$ term [20]. It was noted subsequently [15] that since the combination $t + c$ corresponds to isospin $I(K\pi) = 3/2$ for the final state [see Eq. (11)], the strong-interaction phase of its EWP contribution is the same as that of the rest of the $t + c$ amplitude and the ratio of the two contributions is given in terms of known Wilson coefficients and CKM factors. This permits a calculation of the EWP correction.

New data on branching ratios and CP asymmetries permit an update of previous analyses [3,15]. The expressions for the rate ratio and CP asymmetry are

$$R_c = 1 - 2r_c \cos \delta_c (\cos \gamma - \delta_{\text{EW}}) + r_c^2 (1 - 2\delta_{\text{EW}} \cos \gamma + \delta_{\text{EW}}^2),$$

$$A_{\text{CP}}(K^+\pi^0) = -2r_c \sin \delta_c \sin \gamma/R_c,$$

where $r_c \equiv |(T + C)/P| = 0.20 \pm 0.02$, and $\delta_c$ is a strong phase, eliminated by combining (23) and (24). One must also use an estimate [15] of the electroweak penguin parameter $\delta_{\text{EW}} = 0.65 \pm 0.15$. One obtains the most conservative (i.e., weakest) bound on $\gamma$ for the maximum values of $r_c$ and $\delta_{\text{EW}}$ [3]. The resulting plot is shown in Fig. 2. One obtains a bound at the $1\sigma$ level very similar to that in the previous case: $\gamma \gtrsim 52^\circ$. The bound is set by the curve for zero CP asymmetry, as emphasized in Ref. [15]. Consequently, the improved estimate of $A_{\text{CP}}(B^+ \to K^+\pi^0)$ has no impact on this bound.

If the deviations from unity of $R_c$ and $R_n$ are a consequence of an underestimate of the efficiency for $\pi^0$ detection, one may compensate for this effect by considering their geometric mean:
probably amounts to corrections of a few percent in $R$ second order terms, as well as of rescattering effects, provide an equally valid limit on $R_c$. Since we have argued that to first order in small quantities $R_c$ and $R_n$ should be equal, this ratio should also be given in this approximation by Eq. (14), and should provide an equally valid limit on $\gamma$. The neglect of second order terms, as well as of rescattering effects, probably amounts to corrections of a few percent in $R$, $R_c$, and $R_n$, and hence of a few degrees in $\gamma$.

Since $(R_c R_n)^{1/2}$ is so close to unity, it turns out that the most conservative bound occurs for the smallest values of $r_c$ and $\delta_{\Pi W}$, respectively 0.18 and 0.50, and for $|A_{\text{CP}}(B^+ \rightarrow K^\pi^0)|$ at its upper limit of 0.11. The resulting plot is shown in Fig. 3. One obtains an upper limit in this case: $\gamma \lesssim 80^\circ$ at the 1σ level. The 1σ limits $50^\circ \lesssim \gamma \lesssim 80^\circ$, obtained from $R$ and $(R_c R_n)^{1/2}$ and from the CP asymmetries in $B^0 \rightarrow K^+ \pi^-$ and $B^+ \rightarrow K^+ \pi^0$ are to be compared with those from a global fit to CKM parameters [14]: $44^\circ \leq \gamma \leq 72^\circ$ at 68% c.l. or $38^\circ \leq \gamma \leq 80^\circ$ at 95% c.l.

We comment further on what would be required to enhance $\tilde{\Gamma}(B^+ \rightarrow K^+ \pi^0)$ and $\tilde{\Gamma}(B^0 \rightarrow K^0 \pi^0)$ by $\mathcal{O}(25\%)$, leading to the observed deviations of $R_c$ and $R_n$ from unity. It is sufficient to take a suitable linear combination of the $\Delta I = 1$ amplitudes $A_{1/2}$ and $A_{3/2}$ such that only neutral-pion emission, and not charged-pion emission, is affected. This corresponds to an amplitude transforming as $c$ in Eqs. (4) and (5). The new amplitude cannot enhance both decay rates by interfering with the dominant $\Delta I = 0$ amplitude $B_{1/2}$ (which is the only one receiving a contribution from the dominant penguin term $p$), since the interference terms are of opposite sign. One can see this from the relative $p$ and $c$ contributions in Eqs. (4). The new amplitude has to be nearly half in magnitude and $90^\circ$ out of phase with respect to $B_{1/2}$ so that its absolute square would give the needed enhancement for both $B \rightarrow K \pi^0$ decays.
We imagine two types of operators. The first, transforming as an electroweak penguin $P_{EW}$ contributing to the $c$ amplitude in Eq. (5), could contribute to $b \to \bar{d}\pi^0$. One would expect the corresponding $b \to d\pi^0$ amplitude to be suppressed by a factor of $|V_{cd}/V_{us}| \simeq 0.23$ and hence to have little effect in $\Delta S = 0$ $B$ decays. On the other hand, if there were a term transforming as the $C$ part of $c$ in Eq. (5), for example due to a serious mis-estimate of a rescattering contribution to the color-suppressed amplitude, one would expect the $\Delta S = 0$ process to be enhanced by a factor of $|V_{cd}/V_{us}|$ with respect to the $|\Delta S| = 1$ contribution. Such an enhanced color-suppressed amplitude would certainly have been noticed in $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$, and can be ruled out.

In conclusion, the $B \to K\pi$ decay rates are approximately in the ratios of $2:1:2:1$ expected for the $K^+\pi^-, K^0\pi^0, K^0\pi^+, K^+\pi^0$ modes if the penguin amplitude ($p$) is dominant. However, the deviations from these rates that one would expect due to interference with the smaller tree ($t$) and electroweak penguin ($P_{EW}$) amplitudes do not follow the expected pattern. Rather, there appears to be a slight enhancement of both modes involving a $\pi^0$ with respect to the penguin-dominance expectation. As a result, the sum rule (1) is poorly satisfied. This suggests that the use of such ratios as $R_c$ and $R_n$ to constrain CKM phases will be viewed with some caution, if the problem lies with estimates of $\pi^0$ detection efficiency. In such a case the ratio $R$ may be more reliable. We find that by combining it with the CP asymmetry in $B^{0} \to K^+\pi^-$ one can place a $1\sigma$ lower bound $\gamma \gtrsim 50^\circ$. The corresponding $1\sigma$ bound obtained by considering $R_n$ is $\gamma \gtrsim 52^\circ$. An upper $1\sigma$ bound $\gamma \lesssim 80^\circ$ is obtained from the geometric mean $(R_c R_n)^{1/2}$, in which neutral-pion detection efficiencies cancel one another.

We have shown that the relation (2) between CP-violating rate differences is satisfied, and that a modest improvement on errors in CP asymmetries $A_{CP}$ may be obtained by assuming it to hold. However, somewhat surprisingly, further progress in the study of $B \to K\pi$ decays may depend more on the resolution of the puzzle surrounding the sum rule (1) than on more precise determinations of CP asymmetries.

If the discrepancy in the sum rule (1) persists at a higher level of statistical significance, one would be forced to consider its origin in physics beyond the Standard Model. The most likely interpretation of an enhancement of $B^+ \to K^+\pi^0$ and $B^0 \to K^0\pi^0$ would be that it originates in a new effective Hamiltonian transforming as $\Delta I = 1$.

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References

[18] This observation was also made recently by T. Yoshikawa, hep-ph/0306147.