Linear Parameter-Varying Modeling for Gain-Scheduling Robust Control Synthesis of Flexible Joint Industrial Robot

Bin Niu*, Hui Zhang

ABB Robotics R&D Center, 201319, Shanghai, China

Abstract

Industrial robot features a multivariable, nonlinear and coupled dynamics property, which is always the inevitable topic for control engineer to face during the design of motion controller to achieve desired performance. In the past half century, motion control of industrial robot has gone through a sustainable development from no model involved to model based, from rigid body model to flexible joint model, and even more delicate models of higher order with flexibility in non-drive-train components are also introduced lately. Involvement of gradually refined dynamic model into control design, as well as manipulator design, has become a dominating approach for robot manufacturer to pursue competitive performance and keep technological leadership. Focusing on dynamic modeling, this paper introduces the formulation and approximation of a novel linear parameter-varying (LPV) model for the three main axes of typical six degrees-of-freedom (DOF) elbow type robot, which converts the strong nonlinear system into a quasi linear one globally dependent on certain scheduling parameters. This modeling method, as the prerequisite step for gain-scheduling robust control synthesis, paves the way for further step towards the implementation of LPV gain-scheduling modeling and control techniques for full robot in the next step.

1. Introduction

Modeling techniques of industrial robots are constantly developed and corresponding dynamic models are increasingly refined to be closer to the actual cases. As a consequence, on one hand, controller designed with advanced model can exert the maximum performance out of the robot with prior...
knowledge of robot’s various limits and vibration modes. But on the other hand, a less conservative controller will become more sensitive and susceptible to potential deviations between ideal and realistic cases, such as individual physical differences between robots, extra flexibilities serially existing in foundation or tool, definition error of payload or armload and disturbances from high process force. All of these non-ideal factors may lead to unexpected degradation of performance and thus bring forward a higher requirement of in-depth understanding and proper operation on engineering personnel on site. But even if models are becoming more and more accurate, it can never fully represent the overall properties of real system with existence of various inevitable inherent nonlinearities, such as hysteresis effect [1] and Stribeck effect [2][3] in gearboxes, which are hard to be modeled and incorporated into control design.

From the perspective of industrial practices, only one or several prototypes are usually identified and verified against ideal nominal model at the initial development stage of a new robot and then a common set of modeling parameters will be deployed for each robot individuals to be produced. As a consequence, robot individuals at following serial production could be probably subject to potential property deviation imported from different batches of raw materials in spite of strict execution of incoming quality check. So this will cause accurate prior knowledge of system not applicable to specific unit and the inconsistency between them will lead to incapability of desired performance. Even if time-consuming parametric identification process is especially conducted for specific robot one by one in production, it is still not an advisable solution since robot, as a nonlinear time-variant system, will also behave differently with operation hours going on and electromechanical components worn out at customer site. So utilization of high order complex models will be a double-edged sword. On one hand, intensive knowledge of system will undoubtedly help to dig out the potential of a robot, but on the other hand, accurate model, if blind to possible parametric perturbation in practice, will not express the actual system properly and even undermine its normal operation oppositely. So how much is the suitable level to which we should refine the dynamic description of system and how robust the system behaves to tolerate the model uncertainty will be a trade-off issue.

During the modeling of industrial robot, it is impossible to take all of the above uncertainties into consideration. Due to the missing information by unmodeled dynamics, robot is actually blind to the existing nonlinear factors and nonideal external conditions, which will degrade its expected stability and performance designed under ideal circumstances. And even if all of necessary dynamics are covered by advanced modeling methods, parametric uncertainty, including deviation or drifting of modeled parameters, can always be unavoidable. Such situation restricts further development of the classical inverse dynamics controllers, which are highly model based and therefore accurate modeling is a prerequisite.

2. LPV modeling

H-infinity control, as one of the typical methodologies in the framework of robust control conception, attracts our attention for its feasibility study of application in industrial robot motion control, not only because it favours multivariable model-based control design but also due to its inherent consideration of system robustness to unmodeled dynamics and parametric uncertainty. In recent years, as the extension of linear optimal H-infinity control methods and Linear Matrix Inequalities (LMI), LPV gain-scheduling techniques have evolved into a promising and efficient tool for solutions to modern nonlinear control applications. Industrial robot motion control cannot be more suitable for this method since, besides the unknown dynamics uncertainty discussed above, industrial robot also owns another well known dynamics problem, that is, rapidly varying dynamics resulting from changing configurations during quick movement in entire work envelope will bring forward a tricky issue for the controller regarding properly handling of such a parameter-varying system in an effective way. For example, as shown in Fig.1, with
robot configurations varying from (a) to (b) then to (c), angles of axis 2 and 3, $q_{a2},q_{a3}$ are gradually changing and accordingly the inertias felt by axis 1 and 2 are becoming higher and higher. As studied by [4], one constant-gain H-infinity controller synthesized with $q_{a2}$ and $q_{a3}$ taken as parametric deviation is not sufficient to control the varying system globally. During such movement, controller has to own an adaptive ability to apply a scheduled control gain and meanwhile attenuate the varying-frequency resonant modes in real time. To fulfill these requirements, LPV modeling and corresponding gain-scheduling controller synthesis turn out to be the appropriate methods.

In the remainder of this paper, LPV modeling based on the flexible joint models will be addressed and corresponding gain-scheduling H-infinity controller synthesis will be particularly discussed in another paper.

2.1. LPV model formulation

LPV formulation can be derived for many nonlinear systems, where the scheduling parameters can include system inputs, outputs, states and external signals. Usually there are two limitations on the application of LPV gain-scheduling methods to practical control problems. One is overbounding in the convex scheduling parameter set, which may lead to conservative controller design. And the other is that the number of LMIs to be solved for standard H-infinity controller synthesis increases exponentially with the number of scheduling parameters. [5]

Consider an LPV model in the state space form

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t)$$
$$y(t) = C(\theta(t))x(t) + D(\theta(t))u(t)$$
(1)

where the mappings $A(.)$, $B(.)$, $C(.)$ and $D(.)$ are continuous functions of time-variant scheduling parameter vector $\theta(t) \in \mathbb{R}^l$. This model can also be represented by a linear input-output map

$$P(\theta) = \begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix}$$
(2)

The parameter vector $\theta(t)$ depends on measurable signals $\rho(t) \in \mathbb{R}^l$ referred to as scheduling signals,

$$\theta(t) = f(\rho(t)),$$
(3)

where $f: \mathbb{R}^l \rightarrow \mathbb{R}^l$ is a continuous mapping.
The LPV system is called parameter-affine, if the state space model depends affinely on the parameters.

\[ P(\theta) = \sum_{i=0}^{l} \theta_i P_i = P + \theta_1 P_1 + \ldots + \theta_l P_l \]  

(4)

Since \( \theta \) can be expressed as a convex combination of \( L \) vertices \( \theta_{ii} \),

\[ \theta \in \text{Co}\{\theta_{i1}, \theta_{i2}, \ldots, \theta_{il}\} \]

(5)

where \( L = 2^l \) is the number of vertices, if (4) holds, the system can be represented by a linear combination of linear time-invariant (LTI) models at the vertices, which is called a polytopic LPV system.

\[ P(\theta) = \sum_{i=1}^{l} \alpha_i P(\theta_{ii}) \in \text{Co}\{P(\theta_{i1}), P(\theta_{i2}), \ldots, P(\theta_{il})\} \]

(6)

where \( \sum_{i=1}^{l} \alpha_i = 1 \), and \( \alpha_i \geq 0 \) are the convex coordinates.

To derive the LPV model for the three main axes of 6-DOF flexible joint robot shown in Fig.1, we should firstly obtain the state space expression of the dynamic model. Considering a robot at invariant configuration, the linear model for control design is simplified from its complete model [6] by ignoring nonlinear components. So an ideal model of multiple joints is derived as below.

\[ u = M\ddot{q} + D\dot{q} + Kq \]

(7)

The state space formulation of the system will be

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

(8)

where,

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} O \\ M^{-1} \end{bmatrix}, \quad C = \begin{bmatrix} I & O \end{bmatrix}, \quad D = O,
\]

\[ x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad q = [q_{a1}, q_{a2}, q_{a3}, q_{m1}, q_{m2}, q_{m3}]^T. \]

Inputs are three applied motor torques and outputs are three motor positions.

Such a model can be naturally extended to LPV model only if the original invariant matrices \( A(.), B(.), C(.) \) and \( D(.) \) become parameter dependent based on varying configurations, i.e., varying combinations of scheduling signals

\[ \rho = [q_{a2}, q_{a3}]^T \]

(9)

In such case, the expression of LPV system model (2) can be derived dependent on \( \rho \). The complicated symbolic expressions of all the matrix elements will not be detailed here due to length limitation. But we can easily find out only the six elements \( [A_{a1}, A_{a2}, A_{a3}, A_{a4}, A_{a5}] \) are independent variables and the others are either induced variables or constant values. Since no obvious affine relationship can be found out between the six elements and \( \rho \), as shown in Fig.2, we can take all the six elements as scheduling parameters for the moment.

\[ \theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T = [A_{a1}, A_{a2}, A_{a3}, A_{a4}, A_{a5}]^T \]

(10)
Here an LPV model in polytopic formulation is constructed such that the state matrices are dependent of the scheduling parameters in an affine manner. The parameter set contains L=64 vertices and 129 LMIs have to be simultaneously solved for a polytopic LPV controller synthesis. With such a large number of scheduling parameters, this model will make the LPV controller synthesis conservative and computationally costly. Therefore, parameter set mapping has to be applied to convert the system expression with a smaller number of scheduling parameters without sacrificing too much modeling accuracy.

2.2. LPV model approximation

Parameter set mapping based on principal component analysis (PCA) can approximate and simplify the LPV model with less overbounding by using a tighter parameter set. Meanwhile, correlation between scheduling parameters can be detected and insignificant directions in the parameter space can be ignored without losing much information about the control plant.

Principal component analysis is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or
equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it should be orthogonal to (i.e., uncorrelated with) the preceding components. The operation of PCA can be thought of as revealing the internal structure of the data in a way which best explains the variance in the data. If a multivariate dataset is visualized as a set of coordinates in a high-dimensional data space, PCA can supply the user with a lower-dimensional picture, a "shadow" of this object when viewed from its most informative viewpoint. This is done by using only the first few principal components so that the dimensionality of the transformed data is reduced. [7]

The objective of parameter set mapping is to find out a new mapping \( \Phi: \mathbb{R}^n \rightarrow \mathbb{R}^m \) such that \( m \leq l \), and yields a model
\[
\dot{x}(t) = A(\phi(t))x(t) + B(\phi(t))u(t) ,
\]
\[
y(t) = C(\phi(t))x(t) + D(\phi(t))u(t)
\]
which provides a satisfactory approximation of (1).

Firstly, a typical trajectory of the scheduling signals will be generated so that all expected operating regions of the plant are covered. Here we will examine the typical path moving from configuration (a) to configuration (c) with maximum inertia variations for both the first and second joints. The whole path is interpolated with \( N=100 \) data points by time interval \( T \).

Then, corresponding scheduling parameters are calculated to generate the data matrix
\[
\Theta = [\theta(0) \ \theta(T) \ \ldots \ \theta((N-1)T)] \in \mathbb{R}^{d \times N}
\]
To put the same weight on each \( \theta_i \), all rows of the data matrix are normalized with zero mean and unity standard deviation.
\[
\tilde{\Theta}^i = N_{ \Theta_i } , \Theta_i = N_{ \Theta_i }^{-1}(\Theta_i)
\]
PCA is applied to the normalized data by conducting the singular value decomposition (SVD) of \( \Theta_i \)
\[
\Theta_i = [U_i \ \Sigma_i \ \Omega_i] = [U_i \ \Sigma_i \ \Omega_i] \begin{bmatrix} \Sigma_i \sigma_i \vdots \sigma_i \end{bmatrix}
\]
where \( \Sigma_i = \text{diag}(\sigma_i \ldots \sigma_i) \), \( \Sigma_i = \text{diag}(\sigma_i \ldots \sigma_i) \) and assume that \( U_i , \Sigma_i , \Omega_i \) correspond to the m significant singular values, such that
\[
\tilde{\Theta}^i = U_i \Sigma_i V_i^T \approx \Theta^i
\]
is a reasonable approximation of the given data. Accuracy of the approximated model is evaluated by the fraction of total variation
\[
\tilde{V}_m = \frac{\sum_{i=1}^{m} \sigma_i^2}{\sum_{i=1}^{n} \sigma_i^2}
\]
where \( \sigma_i \) denotes the singular values in (15). By choosing the number \( m \) of scheduling parameters, the accuracy of the model can be traded against complexity.

The matrix \( U_i \) represents a basis of the significant column space of the data matrix \( \Theta_i \), and can be used to obtain a reduced mapping by computing
\[
\phi(t) = g(\rho(t)) = U_i^T N(\theta(t)) = U_i^T N(f(\rho(t)))
\]
The approximated model in (12) is related to (1) by
where \( N^{-1} \) denotes the row-wise rescaling. The approximated LPV model can be produced at any time by (20).

The above steps will be applied to the LPV model derived in chapter 2.1. As the consequence, \( \bar{\phi} = [\bar{\phi}_1, \bar{\phi}_2]^T \) is calculated as the new scheduling parameter vector, which travels along the new trajectory shown in Fig.3 (a) during the examined movement, such that the elements of system matrices can be expressed affinely by the new parameters. The mapping relationship is shown in (21).

\[
\bar{\theta}(t) = N^{-1}(U, \phi(t)) = N^{-1}(U, U^T N(\theta(t)))
\]

Fig.3 (b) illustrates the plot of \( v_m \) versus \( m \), where \( m=2 \) is selected since 99.5% of the information of original system is remaining. In such case, the substitutes of original scheduling parameters will travel along the trajectories shown in Fig.4, where good matching is achieved by compare against the original ones. And what’s more, the LPV model has only four vertices and nine LMIs should be solved for LPV controller synthesis. The mapped parameter space with two dimensions has much less overbounding than the original one, leading to a less conservative controller.

3. Conclusion

Modern industrial robot is a parameter-varying nonlinear system with presence of dynamics uncertainty and flexibility eigenfrequency within bandwidth. Description and handling of such features are major tasks of modeling and control. LPV gain-scheduling technique, together with robust control concept, can help to reach the goal. Especially, industrial robot with nonlinearity in terms of configuration...
variation can be modeled in the LPV formulation. In this paper, the LPV model for three main axes of 6-DOF robot is derived and ends up with six initial scheduling parameters. Then parameter set mapping based on PCA method is used to find out two new scheduling parameters so that computation cost and conservatism of the controller are much reduced. With such an LPV model, corresponding linear controllers at the vertices can be synthesized in the standard robust control design framework and LPV gain-scheduling controller can be obtained in the polytopic form dependent on the same scheduling parameter vector as the model accordingly.

![Graph showing trajectories comparison](image)

Fig.4. Compare of trajectories between original scheduling parameters with the substitutes expressed by new scheduling parameters after parameter set mapping

References

[1] Reducer for Precision Control: General Catalog of HarmonicDrive, No.0603-0R-HD, March 2006