Object-oriented approach to design of the complex mechanical system dynamics mathematical models

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Abstract

The article presents the results of an object-oriented approach applied to modeling the dynamics of complex mechanical systems when it is used in the educational process. The results of formalization the mathematical model design in the form of the second kind (conventional) Lagrange equations and the resulting Cauchy problem are considered. The article contains the authors’ presentation of the corresponding information flows diagram. The ways to use of object-oriented approach discussed in this article are proposed to be used in the educational process to establish interdisciplinary connections, as well as to create e-learning resources for students of applied mathematics, information technology and engineering profiles. The examples of the display forms for designed software used in the simulation of a transport vehicle mechanical system are presented.

Keywords: Modeling; computer simulation; object-oriented approach; complex mechanical system; formal method; e-leaning

1. Introduction

Computer simulation of the dynamics of complex mechanical systems is an essential task in many practical applications. A task to teach students of technical and informational training areas is very relevant to create new CAD/CAM systems, including specialized software with support the functions of modeling and simulation to design of mechanical systems. The basis of student training to construct of mathematical models of mechanical systems is the study of the basic principles and methods for solving classical problems of theoretical mechanics [1-4]. It should

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be noted that at present there are a number of software systems designed for computer simulation of the kinematics and dynamics of certain types of mechanical systems such as Russian software systems "Euler" [5], "Universal Mechanism" [6], as well as a specialized module "Simulink" [7] of software system "Matlab", and also software tools [8] for implementing simulation technology using the international standard VHDL-AMS. Mathematical modeling units of these software systems are based on classical methods of theoretical mechanics [1-4], and is used also methods, algorithms, and mathematical models [9-13], which are based on the principles of graph theory, affine transformations, tensor analysis and the existence of electromechanical analogy with the use of elements of the circuitry.

The main shortcoming for application of above-mentioned software of educational process is fact that the demanded mathematical model, which is a basis of the analysis of characteristics of mechanical system, is usually not available to the user to direct adjustment of parameters. Software mathematics is hidden in internal representation of a program complex in most cases.

2. Design of mathematical model

The aim of teaching to the principles of mathematical and computer modeling of mechanical systems is: a student should understand the impact of the type of motion of each system element, its inertial and geometric characteristics, the way of interaction of solids in the system to the contributions of the solids and relationships in the system of differential equations describing the system dynamics.

As mathematical tools to describe the dynamics of the classical problems [1-4] to modeling of mechanical systems it is convenient to use the method of conventional Lagrange equations (Lagrange equations of the second kind), the algorithm of which is described in [1, 3]. The mathematical tools and method for forming the conventional Lagrange equations are tools, original intended to describe dynamics of complex mechanical systems with many degrees of freedom, what is clear considered in article [15].

The relationship of data sources and stages of constructing a mathematical model that allows implement adequately a solution of standard educational tasks within a model of a solids system are identified in proposed in figure 1 scheme of generalized information flows.

In the shown scheme \( q_i \) denotes a generalized coordinate, \( \dot{q}_i \) denotes the generalized velocity, \( T \) denotes the kinetic energy, \( I \) represents the potential energy, \( \dot{\varphi} \) represents the Rayleigh dissipation function. Generalized force \( I_Q \) describes the effect of forces that are outside \( I \) or \( \dot{\varphi} \). The scheme can be a basis for the implementation of the object-oriented approach for modeling of complex mechanical systems.

For planar mechanical system corresponding to the standard tasks of the study discipline "Theoretical (classical) mechanics" [4], there are three possible types of motion, namely: translational, rotational and plane-parallel, which in turn is decomposed into translational motion together with the center of mass and rotation around its center of mass. The type of movement and type of solid body (set of its parameters) allow only one way to determine the expression of the kinetic energy of each body and of the complete mechanical system. The number and type of generalized coordinates are determined by the type of the relationship and interaction between bodies. Within the system (subsystem) with one degree of freedom the number of ways of connection between bodies is limited and definitely sets the speed ratio expressed through a generalized speed. The use of these ratios in the expression of the total kinetic energy of the complete system allow to find the the expressions of "reduced mass" of the system to each particular generalized coordinate. The presence of the elastic connection between the bodies defines an additional degree of freedom and its associated generalized coordinate and generalized velocity. An additional degree of freedom and generalized coordinate, of course, is also determined in the case of independent motion of one body relative to another moving body in one system.

The type of external constraints in a flat system, first, determines the type of motion of the solid and, second, the external force reactions including external forces of elasticity or friction (which can be considered as "dry" or "viscous"). The right side of the conventional Lagrange equations describes the impact of potential forces, with the power function, dissipative forces, depending on speed, and also forces which aren't entering the described categories. Each expression in the right part corresponds to the some generalized coordinate. The expression should be constructed in depending on external forces that are reaction forces and active forces.
Formalization of data flows and interrelation of objects easily leads to object-oriented [14] modeling and program realization: to encapsulation of data, formal definition of classes and establishment of communications between them. Similar methods of application of object-oriented technology are currently widely used to solve mathematical and specialized tasks [16, 17], including by multi-agent approach as a development of object-oriented programming technology [18, 19].

Design of equations of dynamics of mechanical systems with many degrees of freedom can be a complicated procedure as it discussed in [15]. This is due to the growing complexity of the expressions for the kinematic variables that determine the position, velocity and acceleration of solids included in the system, by increasing the length of kinematic chains. Object-oriented approach is very convenient to two-step mathematical dynamics macro-modeling of mechanical systems [20] with many degrees of freedom, similar to the "transport type" system (railway or automotive vehicle), presented in figure 2. From the point of view of theoretical mechanics separation of macro-elements means splitting system into the interconnected subsystems for such task, each of which has one degree of
freedom and is characterized by own generalized coordinate. The complete system of macro-elements can be considered as translational moving system.

Fig. 2. System with display of sampling, active forces and velocities.

In figure 2 are denoted the gravity forces \( \mathbf{G} \), force moment \( M \) applied to the axis of the driving wheel pair, the angular velocity \( \omega \) of the wheelsets and velocities \( \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \) of centers-of-mass of macro-elements, where total (absolute) velocity

\[
\mathbf{v}_2 = \mathbf{v}_1^k + \mathbf{v}_1^r
\]

is the sum of velocity of transporting body and the relative velocity.

As a result of preparing [15] the conventional Lagrange equations the mathematical model is obtained in the form of a system of strongly coupled differential equations of the second order, as:

\[
\begin{align*}
\ddot{q}_1 &= \frac{1}{m_1} \left( F - \left( \mathbf{b}_{13} \cdot \ddot{q}_3 \right) - \mathbf{b}_{21} \cdot \left( \ddot{q}_1 - \ddot{q}_2 \right) \right) - \left( \mathbf{c}_{12} \cdot (q_1 - q_2) + \mathbf{c}_{13} \cdot (q_1 - q_3) \right) ; \\
\ddot{q}_2 &= \frac{1}{m_2} \left( \mathbf{b}_{21} \cdot (\ddot{q}_1 - \ddot{q}_2) + \mathbf{c}_{12} \cdot (q_1 - q_2) \right) ; \\
\ddot{q}_3 &= \frac{1}{m_3} \left( - (\mathbf{b}_{31} \cdot \ddot{q}_3) + \mathbf{c}_{13} \cdot (q_1 - q_3) \right) .
\end{align*}
\]

The force in (2) have [15] expression

\[
F = m_c \cdot g \cdot \sin \alpha + F_d,
\]

where \( m_c \) is the total mass of all system elements, and the driving force \( F_d \) generated by the traction of the supporting surface and depends on the moments acting on the axis of the driving wheelsets. The mathematical model [15, 20] can be easily transformed into a representation of the standard form Cauchy problem to further solving by numerical methods. The quality is not quite evident at first glance of the results of mathematical modeling in [15], it is possible to note the fact that when driving on an inclined plane in the equations for bodies 2 and 3 connected elements of elasticity with the first body, the resulting removal of the force of gravity due to the initial elastic deformation of compounds that persist in the law of motion of these bodies relative to the first body during the whole time of motion of a system. Significantly, the sliding friction "on ground" present only at the stage of braking and it is only for items with driving wheel pairs, which is reflected in the inheritance of the options classes in the software implementation. The relevant components of the equations (2) are marked by asterisks.

Rational way consists in the complex [20] application of methods that use an object-oriented approach to define the inertial characteristics of the macro-elements and to describe the relationship between subsystems. To do this
should first identify the subsystems (macro-elements) with one degree of freedom and then to do some stage of simplified modeling by method of the conventional Lagrange equations for each subsystem. Only those actions which will allow obtain correctly parameters of system for application of a method of the generalized energy phase variables (GEPV) have to be executed [20].

Described in [9] method GEPV is based on the principles of the existence of analogies mathematical description of the energy processes occurring in the systems of different physical nature. This method uses the equations in the form of dependences of the energy variables of type “I”–“stream” and type “U”–“potential” corresponding to the standard VHDL-AMS [8, 12] "global" variables "through quantity" and "quantity across".

Further formal modeling of the translational motion of a system of macro-elements sufficient to carry out the GEPV method as discussed in [15, 20]. A mathematical model (2) of the method of conventional Lagrange equations can easily be brought to an absolute coincidence [15] GEPV constructed mathematical model by adding the equations the equations arising from the mathematical representation of the law of elastic interaction.

The formal method GEPV is useful in educational process. In the algorithm of this method not only the principles of Electromechanical analogies and circuitry are used clearly, but also representation of mechanical systems in form of oriented graphs is used with the further constructing the topological equations by formalism of discrete mathematics. Formalization of the description of the basic "energy elements" by types of energy storage (C and L) and by energy dissipation type R is an independent source to use the principles of object-oriented approach to designing a mathematical model of the mechanical system and its computer realization.

3. Computer simulation

With the help of the described approaches in laboratory practical course on "Computer Modeling" by languages object-oriented programming designed a number of software products, performing simulation of mechanical systems as composed vehicle with different layout of macro-elements. Computer simulation using the created software allows the student to get a clear understanding of the impact of system parameters on the consistency of the laws of motion subsystems and the nature of the changing force of the elastic interaction between the subsystems.

As an example figure 3 shows the concerted velocity graphs of five macro-elements at the stages of "speed up", "inertia" and "braking". The figure 3 presents the results of computer simulation "vehicle" system, which is represented in figure 4. For clarity, the use of the software product in the interface (fig. 4) also includes a display of the simulated phase of the "movement" of the system using the display "traffic light" (the token "green-yellow-red").

The software product also enables to get graphics accelerations, displacements and forces changes in the elastic elements of the connection of subsystems and make changes to the system parameters for the analysis of the motion coherence of its elements. Thus, in Fig. 3 clearly displayed a gradual damping of the velocities of subordinate elements of the kinematic chain.

![Fig. 3. Graphs of velocity of macro-elements.](image-url)
4. Conclusions

Discussed in this article ways to use object-oriented approach to construction of mathematical models of mechanical systems from the point of view of the authors are useful for use in the educational process to establish interdisciplinary connections, as well as when creating e-learning resources [21] for students applied mathematics, information technology and engineering directions, in-depth learning of mathematical and computer modeling of mechanical systems. In other works of authors of this article the various approaches [20, 21] to the organization of the corresponding material and opportunities of use of interactive multimedia tutorials at realization of the similar electronic training resources are considered in more detail.

References