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A light sterile neutrino from Friedberg–Lee symmetry

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A B S T R A C T

Light sterile neutrinos of mass about an eV with mixing $\tilde{U}_{ls}$ of a few percent to active neutrinos may solve some anomalies shown in experimental data related to neutrino oscillation. How to have light sterile neutrinos is one of the theoretical problems which have attracted a lot of attentions. In this article we show that such an eV scale light sterile neutrino candidate can be obtained in a seesaw model in which the right-handed neutrinos satisfy a softly-broken Friedberg–Lee (FL) symmetry. In this model a right-handed neutrino is guaranteed by the FL symmetry to be light comparing with other two heavy right-handed neutrinos. It can be of eV scale when the FL symmetry is softly broken and can play the role of eV scale sterile neutrino needed for explaining the anomalies of experimental data. This model predicts that one of the active neutrino is massless. We find that this model prefers inverted hierarchy mass pattern of active neutrinos than normal hierarchy. An interesting consequence of this model is that realizing relatively large $|\tilde{U}_{es}|$ and relatively small $|\tilde{U}_{\mu s}|$ in this model naturally leads to a relatively small $|\tilde{U}_{\tau s}|$. This interesting prediction can be tested in future atmospheric or solar neutrino experiments.

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1. Introduction

Experiments have confirmed the existence of three active neutrinos, ν_e, ν_μ and ν_τ participating the usual weak interactions and mixing with each other, beyond reasonable doubt. Many of the experimental data on neutrino oscillation can be explained by mixing among these neutrinos and the mixing angles have been determined [1]. Various data, such as the invisible decay width of the Z boson, have excluded the existence of a fourth light active neutrino. However, additional light sterile neutrinos which do not participate the usual weak interactions, but may mix with the active ones, have not been excluded. In fact there are several experimental indications showing that sterile neutrinos may help to solve some problems show in experimental data. These problems include anomalies show in data from the LSND appearance experiment [2], the MiniBooNE neutrino and anti-neutrino appearance experiments [3], the reactor neutrino flux anomaly data [4], and the data from the deficit of neutrino spectrum in Gallium radioactive source experiment [5]. If the sterile neutrinos have masses of order an eV, some or all these problems can be resolved [6–9]. In view of its possible solution to these problems, an eV scale sterile neutrino, although not favored by some other experiments [10,11] and the tension between appearance and disappearance experiments [8], has raised great interests of particle physicists [12] with several experiments proposed to test the existence of sterile neutrinos [13,14]. How to have light sterile neutrinos is one of theoretical problems which have attracted a lot of attentions. In this article we discuss such an eV scale light sterile neutrino candidate in seesaw model [15] in which the right-handed neutrinos satisfy the Friedberg–Lee (FL) symmetry. We show that in this model, one can naturally have a light neutrino. With soft-breaking of the FL symmetry, we find parameter spaces which can explain preferred sterile neutrino mass and mixing.

2. Friedberg–Lee symmetry and neutrino mass pattern

We will work with type I seesaw model with 3 active neutrinos which belong to electroweak doublet $L_{Li} = (\nu_{Li}, e_{Li})^T$ and 3 right-handed neutrinos ν_{Ri} which transform as singlets under the SM gauge group. The Lagrangian responsible to neutrino masses is

$$\mathcal{L} = -\frac{1}{2}\bar{\nu}_R M \nu_R^c - \bar{L}_L Y H \nu_R + \text{h.c.}, \tag{1}$$

where $H = (H^0, H^-)^T$ is the Higgs doublet. ν_R^c is the charge conjugate of ν_R. M and Y are 3×3 matrices. M is the Majorana mass matrix of ν_R and is symmetric.

After the electroweak symmetry breaking, that is, the Higgs develops a non-zero vacuum expectation value $\langle H \rangle = (v, 0)^T$, the neutrino mass matrix in the basis $(\nu_L, \nu_R^c)^T$ is given by

$$\begin{pmatrix} 0 & Y^* v \\ Y^\dagger v & M \end{pmatrix}. \tag{2}$$

The usual seesaw model assumes that M is rank 3 and the eigenvalues are much larger than the electroweak scale to obtain light neutrino masses of order 0.1 eV or smaller. Without additional assumptions, there is no light right-handed sterile neutrinos. A possible scenario of having a light sterile neutrino is to impose a symmetry to the model which leads to a massless right-handed neutrino (or neutrinos) to be identified as the light sterile neutrino and to induce a finite small mass by softly breaking this symmetry. In Ref. [16] it was shown that an exact global Friedberg–Lee (FL) symmetry in the right-handed neutrino sector implies that one right-handed neutrino is massless and decoupled from other neutrinos. In Ref. [17] it was argued that an approximate FL symmetry in the right-handed neutrino sector implies that one right-handed neutrino can be very light comparing with other right-handed neutrinos. Therefore a seesaw model accessed with the FL symmetry may provide a natural way to obtain a light sterile neutrino. In this work, we carry out a detailed analysis to show how to realize an eV scale sterile neutrino in this scenario and discuss possible interesting consequences.

To start with, let us briefly review how a FL symmetry can lead to a massless sterile neutrino. A theory is said to have a FL symmetry when the Lagrangian of this theory is invariant under a transformation on a fermionic field of the form $q \to q + \epsilon$ [18–20], where ϵ is a space–time independent element of the Grassmann algebra, anti-commuting with the fermionic field operators q. Imposition of a FL symmetry for the SM particles which actively participate in electroweak interactions may be too restrictive for the theory to survive known experimental constraints. For right-handed neutrinos they may allow such a possibility. One can have the FL symmetry along a particular direction in right-handed flavor space, $q = \xi_1 \nu_{R1} + \xi_2 \nu_{R2} + \xi_3 \nu_{R3}$ and require the theory to be invariant under $q \to q + \epsilon$ transformation. By making an appropriate transformation in flavour space, relabelling ν_{R1} to be q, and the other orthogonal states to be $\nu_{R2,3}$, the invariance of the Lagrangian in Eq. (1) under a global FL transformation becomes the invariance of the Lagrangian under

$$\nu_{R_1} \to \nu_{R_1} + \epsilon. \tag{3}$$

It is easy to check that the kinetic term $\mathcal{L}_k = \bar{\nu}_R \gamma_\mu (i \partial^\mu \nu_R)$ is invariant under a transformation defined in Eq. (3) up to a total derivative. The invariance of the Yukawa coupling term under the transformation equation (3) gives

$$Y = \begin{pmatrix} 0 & \tilde{Y}_{e2} & \tilde{Y}_{e3} \\ 0 & \tilde{Y}_{\mu2} & \tilde{Y}_{\mu3} \\ 0 & \tilde{Y}_{\tau2} & \tilde{Y}_{\tau3} \end{pmatrix}. \tag{4}$$

The invariance of the Majorana mass term under the transformation equation (3) gives

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \tilde{M}_{22} & \tilde{M}_{23} \\ 0 & \tilde{M}_{23} & \tilde{M}_{33} \end{pmatrix}. \tag{5}$$

Note that the Majorana mass matrix is forced to be a rank two matrix. ν_{R1} is massless. The non-zero eigenvalues of M are heavy to facilitate the seesaw mechanism.

One can integrate out two heavy neutrinos $\nu_{R2,R3}$ and get a mass matrix for $(\nu_e, \nu_\mu, \nu_\tau, \nu_{R1}^c)$:

$$m_\nu = \begin{pmatrix} \tilde{m}_\nu & 0_{3 \times 1} \\ 0_{1 \times 3} & 0 \end{pmatrix}, \tag{6}$$

where $\tilde{m}_\nu$ is a 3×3 matrix:

$$\tilde{m}_\nu = -\tilde{Y}^* \tilde{M}^{-1} \tilde{Y}^\dagger v^2. \tag{7}$$

$\tilde{Y}$ and $\tilde{M}$ are

$$\tilde{Y} = \begin{pmatrix} \tilde{Y}_{e2} & \tilde{Y}_{e3} \\ \tilde{Y}_{\mu2} & \tilde{Y}_{\mu3} \\ \tilde{Y}_{\tau2} & \tilde{Y}_{\tau3} \end{pmatrix}, \tag{8}$$

$$\tilde{M} = \begin{pmatrix} \tilde{M}_{22} & \tilde{M}_{23} \\ \tilde{M}_{23} & \tilde{M}_{33} \end{pmatrix}. \tag{9}$$

$\tilde{m}_\nu$ is a rank two matrix which gives two non-zero neutrino masses. One combination of active neutrinos is massless in this model. In this scenario, masses and mixings of low energy neutrinos are given by seesaw mechanism with two heavy right-handed neutrinos, a scenario called the minimal seesaw [21]. We see that an exact FL symmetry in right-handed neutrino sector reduces the usual seesaw model to the minimal seesaw model [16].

It is easy to see in Eq. (6) that this scenario gives a massless right-handed neutrino which decouples from all other neutrinos. It cannot provide a low energy sterile neutrino which mixes with active neutrinos. Deviation from or breaking of FL symmetry introduced in Eq. (3) is needed to accommodate an eV scale sterile neutrino which mixes with active light neutrinos to solve some of the problems mentioned earlier.

3. Low energy sterile neutrino with soft-breaking FL

In this section we discuss how soft FL symmetry breaking can help to make a realistic model. Soft breaking of FL symmetry can only occur in the Majorana mass sector M. With soft-breaking terms of FL symmetry, the Majorana mass matrix can be written as

$$M = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{12} & \tilde{M}_{22} & \tilde{M}_{23} \\ x_{13} & \tilde{M}_{23} & \tilde{M}_{33} \end{pmatrix}. \tag{10}$$

Non-zero values of x_{1i} softly break the FL symmetry. Since these terms break the FL symmetry, they are naturally much smaller than the eigenvalues of $\tilde{M}_{ij}$ according to 't Hooft naturalness condition. The actual values of x_{1i} are not known. We will take them as free parameters to be determined or constrained by experimental data.

After integrating out two heavy neutrinos we get a mass matrix for $(\nu_e, \nu_\mu, \nu_\tau, \nu_{R1}^c)$:

$$m_\nu = \begin{pmatrix} \tilde{m}_\nu & -\tilde{Y}^* v \tilde{M}^{-1} X^T \\ -X \tilde{M}^{-1} \tilde{Y}^\dagger v & x_1 \end{pmatrix}, \tag{11}$$

where

$$x_1 = x_{11} - X \tilde{M}^{-1} X^T, \tag{12}$$

and $X = (x_{12}, x_{13})$. $\tilde{m}_\nu$ and $\tilde{M}$ have been given in Eqs. (7) and (9).

In the limit that $x_{12,13}$ are zero, x_{11} is the sterile neutrino mass m_{ν_s} which we assume to be of order eV. In this case, only active neutrinos mix with each other and the light neutrino mass matrix in Eq. (11) is diagonalized by

$$\tilde{U} = \begin{pmatrix} U & 0 \\ 0 & 1 \end{pmatrix}, \tag{13}$$

where U is the usual PMNS mixing for active neutrinos defined by $\tilde{m}_\nu = U^* \tilde{m}_\nu' U^\dagger$. Here $\tilde{m}_\nu' = \mathrm{diag}\{0, m_2, m_3\}$ for normal hierarchy (NH) of light active neutrino masses and $\tilde{m}_\nu' = \mathrm{diag}\{m_1, m_2, 0\}$ for inverted hierarchy (IH). We will work with the convention that the Majorana phases are kept in the mass eigenvalues, and therefore, U does not contain any Majorana phases.

In this case a general expression for $\tilde{Y}$ which can produce the desired NH neutrino mass pattern can be written as follows [17]

$$\tilde{Y}v = iU(\tilde{m}_\nu'^*)^{1/2} \begin{pmatrix} 0 & 0 \\ \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} (\tilde{M}'^*)^{1/2}, \quad (14)$$

where θ is a complex number, and $\tilde{M}' = \mathrm{diag}\{M_2, M_3\}$ is a diagonalized mass matrix for heavy neutrinos. Without loss of generality we can diagonalize $\tilde{M}$ and make discussion in this base. Using (7) and $\tilde{M}'$ one can easily check that (14) reproduces the NH neutrino mass matrix.

For IH, $\tilde{m}_\nu' = \mathrm{diag}\{m_1, m_2, 0\}$ and we have [17]

$$\tilde{Y}v = iU(\tilde{m}_\nu'^*)^{1/2} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \\ 0 & 0 \end{pmatrix} (\tilde{M}'^*)^{1/2}. \quad (15)$$

When $x_{12,23}$ become non-zero, mixing between active and the light sterile neutrino will happen. One can approximate, in general, the mixing matrix for small active-sterile neutrino mixing as follows

$$\tilde{U} = \begin{pmatrix} 1 & R \\ -R^\dagger & 1 \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & 1 \end{pmatrix}, \quad (16)$$

where R is a 3×1 matrix representing the mixing of active neutrinos and sterile neutrino. The above expression is valid as long as R_{ls} ($l = e, \mu, \tau$), the element of R, satisfies $|R_{ls}|^2 \ll 1$. Diagonalizing the fourth row and the fourth column in Eq. (11) using the first matrix in Eq. (16) we find that R is solved as

$$R^* \approx -\frac{1}{x_1} \tilde{Y}^* v \tilde{M}^{-1} X^T, \quad (17)$$

and the neutrino mass matrix becomes

$$\begin{pmatrix} \hat{m}_\nu = \tilde{m}_\nu - R^* x_1 R^\dagger & 0 \\ 0 & x_1 \end{pmatrix}, \quad (18)$$

where order $R^\dagger R$ correction to x_1 has been neglected. $\hat{m}_\nu$ in Eq. (18) is further diagonalized using U in the second matrix in Eq. (16) with $U^T \hat{m}_\nu U = \mathrm{diag}\{m_1, m_2, m_3\}$. The order $R^\dagger R$ correction to the sterile neutrino mass from active and sterile neutrino mixing can be neglected for $|R_{ls}|^2 \ll 1$ and we have

$$m_s \approx x_1 = x_{11} - X\tilde{M}^{-1}X^T. \quad (19)$$

If $x_{12,13}$ are of the order as x_{11}, i.e. of order eV or tens eV, they cannot provide any explanation for the anomalies mentioned earlier since the mixing of the sterile neutrino with the active ones will be very small, as can be seen in Eq. (17). To have the mixing to be of order of interests, say, about 0.1, $x_{12,13}$ should satisfy $\tilde{Y}v\tilde{M}^{*-1}X^\dagger/x_1 \approx 0.1$. With x_1 of order in the eV range, elements in X should be an order of magnitude larger than elements in $\tilde{Y}v$. In this case the contribution to sterile neutrino mass from $x_{12,13}$ may not be neglected. In order that there is no fine-tuning of two terms in Eq. (19) greater than 1% level, we get that $x_{12,13} \lesssim 10\sqrt{\mathrm{eV}\, M_{2,3}}$. For this range of the magnitude of X, it's sufficient to get a mixing of active-sterile neutrinos of $\gtrsim 0.1$. The hierarchy for various quantities are therefore: $\tilde{Y}v < X \ll \tilde{M}$.

This estimate of the order of magnitude of $x_{12,13}$ may also come from considerations of how large the soft-breaking terms

should be. Since the soft-breaking terms are all related to right-handed neutrino mass matrix, a reasonable criteria for the size of the soft-breaking terms is that the smallest eigenvalue should be much smaller than two large eigen-masses already existed when the soft-breaking terms are absent. The lightest eigenvalue, phenomenologically, should be m_s, the sterile neutrino mass, which is about an eV or so to be of interests. We therefore take that as a requirement. In the case that this requirement is satisfied the lightest eigenvalue of M can be computed as

$$m_s = \det(M)/\det(\tilde{M}) = x_{11} - X\tilde{M}^{-1}X^T, \quad (20)$$

which is consistent with Eq. (19). One can see in Eq. (20) that this requirement only limit m_s to be of order eV, but still allow $x_{12,13}$ to be larger since its contribution to the mass eigenvalues are of order $X\tilde{M}^{-1}X^T$. On the other hand, the order of magnitude of $x_{12,13}$ should be restricted to much smaller than large non-zero masses in $\tilde{M}$. When these conditions are satisfied, the soft-breaking scale of FL symmetry can be considered to be natural although $x_{12,13}$ can be orders of magnitude different from x_{11}. We will work with the approximation conditions described above and turn to discuss realizing active neutrino mixing in this scenario.

Using Eq. (17), one can see in Eq. (18) that the mass matrix giving rise to the PMNS matrix is no longer Eq. (7). It is

$$\hat{m}_\nu = \tilde{m}_\nu - R^* x_1 R^\dagger = -\tilde{Y}^* v \tilde{M}^{-1/2} S \tilde{M}^{-1/2} \tilde{Y}^\dagger v, \quad (21)$$

where

$$S = 1 + \frac{1}{x_1} \tilde{M}^{-1/2} X^T X \tilde{M}^{-1/2}. \quad (22)$$

Since x_1 is of order eV and $|R_{ls}| \sim 0.1$, the correction to $\tilde{m}_\nu$ due to mixing R can not be neglected. Although the mass matrix Eq. (21) is more complicated than Eq. (7), it is still rank two and has one zero eigenvalue. This can be clearly seen in Eq. (21) by noting that $\tilde{Y}$ is rank two. One can also see this point in Eq. (11) by noting that the first to third rows are proportional to $\tilde{Y}^*$ which is rank two and the total matrix is rank three.

Similar to Eq. (14) we can obtain an expression of $\tilde{Y}$ for NH

$$\tilde{Y}v = iU(\tilde{m}_\nu'^*)^{1/2} \begin{pmatrix} 0 & 0 \\ \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} (\hat{S}^*)^{-1/2} \Lambda^\dagger (\tilde{M}'^*)^{1/2}, \quad (23)$$

where $\hat{S}$ is a diagonalized matrix and Λ is a unitary matrix which diagonalizes matrix S:

$$\Lambda^T S \Lambda = \hat{S}. \quad (24)$$

The mixing of active neutrinos with sterile neutrino R is expressed as

$$R^* \approx \frac{i}{x_1} U^*(\tilde{m}_\nu')^{1/2} \begin{pmatrix} 0 & 0 \\ (\cos\theta)^* & (\sin\theta)^* \\ -(\sin\theta)^* & (\cos\theta)^* \end{pmatrix} \\ \times \hat{S}^{-1/2} \Lambda^T (\tilde{M}')^{-1/2} X^T. \quad (25)$$

For IH an expression similar to Eq. (25) can be obtained:

$$R^* \approx \frac{i}{x_1} U^*(\tilde{m}_\nu')^{1/2} \begin{pmatrix} (\cos\theta)^* & (\sin\theta)^* \\ -(\sin\theta)^* & (\cos\theta)^* \\ 0 & 0 \end{pmatrix} \\ \times \hat{S}^{-1/2} \Lambda^T (\tilde{M}')^{-1/2} X^T. \quad (26)$$

Introducing $\hat{X}^T = \frac{1}{\sqrt{x_1}} \tilde{M}^{-1/2} X^T$, S can be re-expressed as $S = 1 + \hat{X}^T \hat{X}$. A simple scenario is when $\hat{X}$ is real. In this case, a Λ diagonalizing S is

$$\Lambda = \hat{X}_1^T \, (1 \;\; 0) + \hat{X}_2^T \, (0 \;\; 1), \tag{27}$$

where $\hat{X}_{1,2}$ are two normalized 1×2 real matrices and they satisfy: $\hat{X}_i \hat{X}_j^T = \delta_{ij}$ $(i, j = 1, 2)$ and $\hat{X}_2 \hat{X}^T = 0$. It is easy to check that $\Lambda^T S \Lambda = \mathrm{diag}\{1 + \hat{X}\hat{X}^T, 1\}$,

$$R^* \approx \frac{i}{\sqrt{x_1}} U^* (\tilde{m}'_\nu)^{1/2} \begin{pmatrix} 0 & 0 \\ (\cos\theta)^* & (\sin\theta)^* \\ -(\sin\theta)^* & (\cos\theta)^* \end{pmatrix}$$
$$\times \begin{pmatrix} \sqrt{\hat{X}\hat{X}^T/(1 + \hat{X}\hat{X}^T)} \\ 0 \end{pmatrix} \tag{28}$$

for NH, and

$$R^* \approx \frac{i}{\sqrt{x_1}} U^* (\tilde{m}'_\nu)^{1/2} \begin{pmatrix} (\cos\theta)^* & (\sin\theta)^* \\ -(\sin\theta)^* & (\cos\theta)^* \\ 0 & 0 \end{pmatrix}$$
$$\times \begin{pmatrix} \sqrt{\hat{X}\hat{X}^T/(1 + \hat{X}\hat{X}^T)} \\ 0 \end{pmatrix} \tag{29}$$

for IH.

4. Numerical analysis

In this section, we will study some implications of the light neutrino mass matrix discussed in the previous section resulting from soft-breaking FL symmetry with sterile neutrino mass of order eV and sterile-active neutrino mixing of order 10%. For an illustration of our scenarios we will try to obtain the best fit of the sterile neutrino mass and mixing [6]:

$$m_s = 1.27 \text{ eV}, \quad |\tilde{U}_{es}|^2 = 0.035, \quad |\tilde{U}_{\mu s}|^2 = 0.0086, \tag{30}$$

where $|\tilde{U}_{\mu s}|$ is considerably smaller than $|\tilde{U}_{es}|$ given by the null result of short-baseline ν_μ disappearance experiment. $\tilde{U}_{\tau s}$ is constrained by atmospheric and solar neutrino data [8]:

$$|\tilde{U}_{\tau s}|^2 < 0.2(2\sigma). \tag{31}$$

In our numerical analysis, we use the neutrino mass squared differences and mixing of active neutrinos as the following [22]

$$\Delta m_{21}^2 = 7.62 \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{31}^2| = 2.50 \times 10^{-3} \text{ eV}^2, \tag{32}$$

$$\sin^2\theta_{12} = 0.32, \quad \sin^2\theta_{23} = 0.60, \quad \sin^2\theta_{13} = 0.025. \tag{33}$$

Since mass squared differences and mixing angles for NH and IH are almost the same [22] we neglect the differences for these two mass patterns and use Eqs. (32) and (33) for both cases.

Since one of the active neutrino mass is zero in both NH and IH cases, the neutrino mass are all known. We find solutions for our scenarios:

NH: $\quad m_1 = 0, \quad |m_2| = \sqrt{\Delta m_{21}^2} \approx 0.873 \times 10^{-2} \text{ eV}$,

$$|m_3| \approx \sqrt{|\Delta m_{31}^2|} \approx 0.05 \text{ eV}, \tag{34}$$

IH: $\quad |m_1| = \sqrt{|\Delta m_{31}^2|} \approx 0.05 \text{ eV}$,

$$|m_2| \approx \sqrt{|\Delta m_{31}^2| - \Delta m_{21}^2} \approx 0.05 \text{ eV}, \quad m_3 = 0. \tag{35}$$

The mixing matrix U is expressed using θ_{ij} as follows

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{36}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ is a CP violating phase.

For NH we can read in Eq. (28)

$$R \approx \frac{-i}{\sqrt{m_s}} \sqrt{\frac{x_{11}-x_1}{x_{11}}} \begin{pmatrix} U_{e2}(m_2^*)^{1/2}\cos\theta - U_{e3}(m_3^*)^{1/2}\sin\theta \\ U_{\mu 2}(m_2^*)^{1/2}\cos\theta - U_{\mu 3}(m_3^*)^{1/2}\sin\theta \\ U_{\tau 2}(m_2^*)^{1/2}\cos\theta - U_{\tau 3}(m_3^*)^{1/2}\sin\theta \end{pmatrix}, \tag{37}$$

where $\hat{X}\hat{X}^T/(1 + \hat{X}\hat{X}^T) = (x_{11} - x_1)/x_{11}$ has been used. In this scenario it's difficult to have larger $|\tilde{U}_{es}|^2$ ($|R_{es}|^2$) and smaller $|\tilde{U}_{\mu s}|^2$ ($|R_{\mu s}|^2$) as shown in Eq. (30). One can see this by noting that $|m_3/m_s| \approx 0.0394$ and $|U_{e3}| < |U_{\mu 3}|$ contrary to the associated hierarchy of $|R_{es}|$ and $|R_{\mu s}|$. So suppression of contributions of $\sqrt{m_3^*/m_s}$ in $|R_{es}|$ and $|R_{\mu s}|$ is needed. This can be achieved by taking $|\sin\theta| < 1$. Unfortunately we can find that $|m_2/m_s| \approx 0.00685$ and for $|R_{es}|^2$ to reach 0.035 we need $|\cos\theta|^2 \gg 1$. These two requirements on $\cos\theta$ and $\sin\theta$ are hard to reconcile even allowing complex θ.

For IH we can read in Eq. (29)

$$R \approx \frac{-i}{\sqrt{m_s}} \sqrt{\frac{x_{11}-x_1}{x_{11}}} \begin{pmatrix} U_{e1}(m_1^*)^{1/2}\cos\theta - U_{e2}(m_2^*)^{1/2}\sin\theta \\ U_{\mu 1}(m_1^*)^{1/2}\cos\theta - U_{\mu 2}(m_2^*)^{1/2}\sin\theta \\ U_{\tau 1}(m_1^*)^{1/2}\cos\theta - U_{\tau 2}(m_2^*)^{1/2}\sin\theta \end{pmatrix}. \tag{38}$$

Since $|m_1/m_s| \approx |m_2/m_s| \approx 0.0394$ for IH, their contributions to R_{ls} are equally important. Suppression of $R_{\mu s}$ can be achieved by making two terms proportional to $U_{\mu 1}$ and $U_{\mu 2}$ in $R_{\mu s}$ are of opposite signs and cancel with each other while two terms proportional to U_{e1} and U_{e2} in R_{es} are of the same sign. This is possible because in our convention $U_{e1}U_{e2} = \cos^2\theta_{13}\sin\theta_{12}\cos\theta_{12}$ and $U_{\mu 1}U_{\mu 2} \approx -\cos^2\theta_{23}\sin\theta_{12}\cos\theta_{12}$ which is exactly the case we want. An example to realize Eq. (30) is $\delta = \pi$, $(m_2^*/m_1^*)^{1/2} \approx -i$, $\cos\theta = \sqrt{0.3}i$, $\sin\theta = \sqrt{1.3}$ and $(x_{11}-x_1)/x_{11} = 1/1.3$. Using these parameters we can find that $|\tilde{U}_{\mu s}|^2 \approx 0.0087$ and $|\tilde{U}_{es}|^2 \approx 0.036$. A prediction of this scenario is that $|\tilde{U}_{\tau s}|$ is suppressed together with $|\tilde{U}_{\mu s}|$. Since $U_{\tau 1}U_{\tau 2} \approx -\sin^2\theta_{23}\sin\theta_{12}\cos\theta_{12}$ and $U_{\mu 1}U_{\mu 2} \approx -\cos^2\theta_{23}\sin\theta_{12}\cos\theta_{12}$, one can see in Eq. (38) that two terms contributing to $R_{\tau s}$ will cancel with each other when two terms contributing to $R_{\mu s}$ cancel with each other. For parameters shown above we have $|\tilde{U}_{\tau s}|^2 \approx 0.0044$.

We see in the above example that realizing $|\tilde{U}_{es}| > |\tilde{U}_{\mu s}|$ in our model, to be consistent with the evidences of sterile neutrino, leads to a preference of IH than NH. In more general case, one can find that this preference of IH is also true. One can check that it's always difficult to suppress the contributions proportional to U_{e3} and $U_{\mu 3}$ in Eq. (37) while making $|\tilde{U}_{es}|$ and $|\tilde{U}_{\mu s}|$ of the order of magnitude of interests. For IH there is no such a problem.

5. Conclusions

In summary we have shown that seesaw mechanism plus FL symmetry provide a natural mechanism for having a light sterile neutrino. A FL symmetry in right-handed neutrino sector requires that one of the three right-handed neutrinos is massless and decoupled from all other neutrinos. With soft-breaking of FL symmetry in Majorana mass sector, an eV scale right-handed neutrino coupled to other light neutrinos can emerge and it can play the role of eV scale sterile neutrino required for explaining experiments such as LSND, MiniBooNE, reactor flux anomaly and Gallium radioactive source experiment. We solve the Yukawa coupling terms for the case with soft-breaking of FL symmetry and find that the mass squared differences and mixing angles of active neutrinos can be easily accommodated in this framework.

We find that one light neutrino has to be massless and the mass pattern of active neutrinos is either NH or IH. Mixing of active neutrinos with sterile neutrino can be computed using the Yukawa couplings solved for explaining the mass squared differences and mixings of active neutrinos. Interestingly, we find that the evidences of sterile neutrino prefer to have IH of active neutrinos in our model. We find that for NH it is difficult have $|\tilde{U}_{es}| > |\tilde{U}_{\mu s}|$ which is preferred by the evidences of sterile neutrino. For IH we have shown it is not hard to accommodate this hierarchy in our model of sterile neutrino.

We give an explicit example which gives a nice explanation of the best fit of the sterile neutrino mass and the mixing with active neutrinos. We find that realizing relatively large $|\tilde{U}_{es}|$ and relatively small $|\tilde{U}_{\mu s}|$ in our model naturally leads to relatively small $|\tilde{U}_{\tau s}|$. This interesting prediction can be tested in future atmospheric or solar neutrino experiments.

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