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Simulation and Analysis of Integral LQR Controller for Inner Control Loop Design of a Fixed Wing Micro Aerial Vehicle (MAV)

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Abstract

The focus of this paper is on the autopilot control loop design of fixed wing Micro Aerial Vehicles (MAVs).The control methodologies used to design the lateral and longitudinal control are based on Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) with integral action control techniques. The design of these controllers is based on the assumption that the system dynamics can be decoupled to longitudinal and lateral dynamics. A nominal model is chosen among many linear models linearized under various operating conditions. The resulting controllers are simulated in MATLAB® SIMULINK® workspace and results are studied. The simulation results show that both the controllers gives satisfactory performances with or without disturbances, but the LQR controller provides better disturbance rejection and exhibits better overall performance.

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Keywords: Micro Aerial Vehicles (MAVs);PID Controller;Linear Quadratic Regulator (LQR);Integral Action;lateral stability;longitudinal stability

1. Introduction

Micro Aerial Vehicles (MAVs) are typically a class of unmanned aerial vehicles (UAVs) with wing span less than 450mm and are capable of operating at speeds of 30mph or less. These vehicles provide inexpensive and expandable platforms for surveillance and data collection. Some of the major applications of these aerial systems are monitoring the disaster areas, localization of victims, infrastructure inspection, tasks of surveillance and photography. MAVs operate at low velocities in low Reynolds number aerodynamic regimes, have small mass and moments of inertia, exhibit complex nonlinear dynamics and are very susceptible to winds and gusts. Hence, innovative methods are

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needed for the control of MAVs. The PID controller, which has proportional, integral and derivative elements, is widely applied in feedback control of industrial processes. These controllers are described with their simple structure and principle providing good performance for various systems. They require tuning for each MAV, and quickly loose performance in the presence of actuator failures or changes in MAV dynamics [1]

In this paper, an advanced control strategy, Integral Linear-Quadratic Regulator (LQI) optimal control is introduced to overcome the problems that are faced by PID controller. Linear Quadratic Regulator (LQR) is a control scheme that gives the best possible performance with respect to some given measure of performance [2]. The performance measure is a quadratic function composed of state vector and control input. As the considered control problem is tracking problem Linear Quadratic Regulator with Integral action (LQI) is proposed.

Nomenclature

θ	pitch angle
Ψ	yaw angle
Φ	roll angle
u, v, w	body axes velocities
p, q. r	angular rates
δe	elevator deflection
δa	aileron deflection
m	aircraft mass

2. Flight Dynamics of Micro Aerial Vehicle (MAV)

A nonlinear model of the fixed wing MAV is generated from first-principles modeling approach [3]. The MAV models are developed with medium-complexity based on only basic flight dynamics. Satisfactory accuracy of the model can be achieved and the obtained model is adequate for flight simulations over a large portion of the flight envelope.

2.1 Rigid Body Equations of Motions

The rigid body equations for the Fixed Wing MAV obtained from Newton's Second Law along with Euler's rotational equations of motion is applied to establish the six degree of freedom (6Dof) rigid body dynamics of the MAV [4].The aircraft dynamics are simulated using the following math models describing the aircraft 6DoF equations of motion

$$\dot{u} = \frac{F_x}{m} - qw + rv - g\sin(\theta) \tag{1}$$

$$\dot{v} = \frac{F_y}{m} - ru + pw. + g\cos(\theta)\sin(\phi)$$

$$\dot{w} = \frac{F_z}{m}) - pv + qu + g\cos(\theta)\cos(\phi$$

$$\dot{p} = \frac{L}{I_x} - qr\frac{I_z - I_y}{I_x} + \frac{I_{xz}}{I_x}pq + +\frac{I_{xz}}{I_x}\dot{r}$$

$$\dot{q} = \frac{M}{I_y} - qr\frac{I_x - I_z}{I_y} + \frac{I_{xz}}{I_y}(p^2 - r^2)I_y$$

$$\dot{r} = \frac{N}{I_z} - qr\frac{I_{xz}}{I_x} - \frac{I_y - I_x}{I_x}pq + I_{xz}\dot{r}$$

$$\dot{\phi} = p + (q\sin(\phi)\tan(\theta) + r\cos(\phi))$$

$$\dot{\theta} = q\cos(\phi) - r\sin(\phi)$$

$$\dot{\psi} = (q\sin(\phi) + r\cos(\phi))\sec(\theta)$$

Here F_x, F_y and F_z are the aerodynamic forces along the different body axes and L, M and N are the aerodynamic moments about the centre of gravity. Φ, θ and Ψ are the 3 Euler angles and p, q and r are the 3 body angular rates. The linear models of the aircraft is deduced using these rigid body equations using proper assumptions.

The control objective includes the stability of the system over complete flight envelope, disturbance rejection and to meet the performance specification like rise time, settling time with minimum overshoots.

2.2. *Aerodynamic data*

The aircraft data used in the study and analysis corresponds to that of a fixed wing MAV developed at IIT Bombay [6], India is given in Table 1. The aerodynamic force and moment coefficients are given in Table 2 .

Table 1. Geometric, mass and inertial data

Parameter	Symbol	Values
Mass	m	0.290kg
Wing Area	S	0.0612m^2
Mean Chord	c	0.25m
Span	b	0.3m
Moments of inertia:	I_{xx}	0.0030kgm^2
	I_{yy}	0.0030kgm^2
	I_{zz}	0.0030kgm^2

Table 2. List of Aerodynamic Force and Moment Coefficients

Aerodynamic Force Coefficients	Values	Aerodynamic Moment Coefficients	Values
CL_0	0.0530	Cm_0	-0.0580
CL_α	2.6358	Cm_α	-0.9741
$CL_{\delta e}$	0.6143	$Cm_{\delta e}$	-0.3862
CL_{mind}	0.1651	Cm_q	-0.8757
CL_q	2.6818	Cn_β	0.4383
CD_{min}	0.3170	$Cn_{\delta a}$	-0.0068
$CD_{\delta e}$	0.0894	Cn_p	-0.1337
$CD_{\delta a}$	-0.0190	Cn_r	-0.1859
CY_β	-0.9099	Cl_p	-0.1387
CY_p	0.4113	Cl_r	0.1491
CY_r	0.3382	$Cl_{\delta a}$	-0.0977
$CY_{\delta a}$	0.0211	Cl_β	-0.5900

2.2. Linear Models

The nonlinear dynamic model of the MAV is to be linearized at certain operating points before applying the optimal control techniques. The aircraft equations of motion are trimmed for wings level flight conditions and were linearized at various operating conditions. The position and yaw of the MAV becomes significant only in the design of a higher level of control when performing navigation. Thus, the inputs are defined as $[\delta e\ \delta a]^T$ and the states are $[u, v, w, p, q, r, \phi, \theta]^T$. The operating conditions considered were velocities ranging from 10m/s to 20m/s. A typical nominal model corresponding to V=15m/s is chosen for analysis. Assuming the system dynamics can be decoupled along the lateral and longitudinal axes, the linearized nominal model is as given below

Longitudinal Axis

Input vector $u^T = [\delta_e]$
State vector $x^T = [u\ w\ q\ \theta]$

$$A = \begin{bmatrix} -1.29 & 0.883 & -2.246 & -9.672 \\ -0.411 & -5.758 & 14.179 & 0 \\ 0.732 & -4.718 & -0.464 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4.037 \\ -17.238 \\ -140.395 \\ 0 \end{bmatrix} \tag{2}$$

Lateral-Directional Axis

Input vector $u^T = [\delta_a]$
State vector $x^T = [v\ p\ r\ \Phi]$

$$A = \begin{bmatrix} -1.746 & 2.418 & -14.725 & 9.672 \\ -32.65 & -1.154 & 1.236 & 0 \\ 135.353 & -3.073 & -9.735 & 0 \\ 0 & 1 & 0.155 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.60 \\ -81.231 \\ -15.159 \\ 0 \end{bmatrix} \tag{3}$$

Units: - u, v, w --> m/sec p, q, r--> rad/sec; θ, Φ --> rad

3. Flight Control System Design

The architecture of flight control system of the fixed wing MAV is illustrated in Fig. 1.

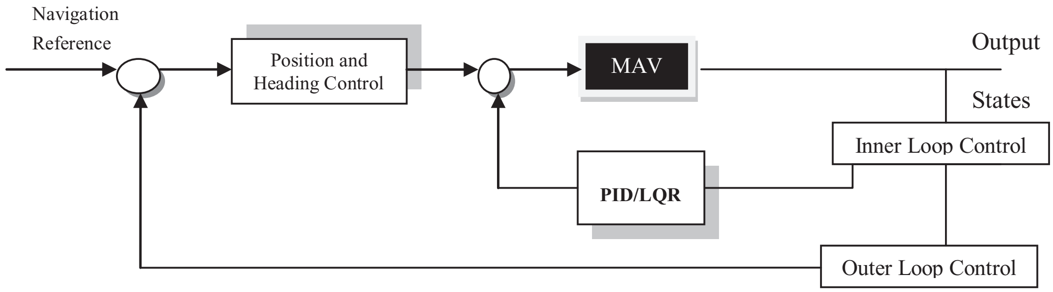


Fig.1 Architecture of Flight Control System

The inner loop control applies PID/LQR techniques to internally stabilize the MAV and the outer loop performs the position and heading control in order to navigate the helicopter along the predefined trajectory with specified yawing angles [7]. In this section, we describe the PID and LQR controllers that have been designed for the control the fixed wing MAV.

3.1. Proportional Integral Derivative (PID) Controller

The Proportional-Integral-Derivative (PID) method is a type of feedback controller which is generally based on the error (e) between desired set point and actual value. The error is then used to adjust a chosen input to the plant in order to track its defined set point. Three parameters must be designed in the PID controller and each parameter has an effect on the error. They provide control signals that are proportional to the error between the reference signal and the actual output (proportional action), to the integral of the error (integral action), and to the derivative of the error (derivative action). The transfer function of the PID controller is written as:

$$u(t) = K_p\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{d}{dt}e(t)\right] \tag{4}$$

Where, K_p is the controller gain, T_i is the integral time and T_d is the derivative time. It is important to determine appropriate parameters to guarantee stability and system performance. There are several methods for tuning PID parameters [8]. However, in this study the three parameters of PID controller values are computed by Zeigler-Nicholas Method of tuning.

3.2. Linear Quadratic Regulator with Integral Action (LQR)

LQR is an optimal control technique that provides the best possible performance with respect to some given performance measure. The LQR design problem is to design a state feedback controller K such that the objective function J is minimized. In this technique a feedback gain matrix is designed which minimizes the objective function in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response guaranteeing a stable system.

For a continuous-time linear system described as

$$\dot{x} = Ax + Bu \tag{5}$$

With a cost function defined as

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{6}$$

where Q and R are the weight matrices, Q is required to be positive definite or positive semi-definite symmetry matrix; R is required to be positive definite symmetry matrix. One practical method is to Q and R to be diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function J. The feedback control law that minimizes the value of the cost function is given by

$$u = -Kx \tag{7}$$

where K is obtained as

$$K = R^{-1} B^T P \tag{8}$$

and P is found by solving the continuous time Algebraic Riccati Equation (ARE)

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \tag{9}$$

The design procedure for finding the feedback gain K for LQR can be formulated as given steps:

• Selection of the design parameter matrices Q and R [12].

In this paper trial and error method on weights is used to obtain tracking and to achieve less control effort.

• Find P by solving ARE.

• Find the state feedback matrix K using $K= R^{-1} B^T P$.

In order to obtain zero steady state error an integral action is included in the LQR Control. The basic approach in integral feedback is to create a state within the controller that computes the integral of the error signal, which is then used as a feedback term. It is done by augmenting the description of the system with a new state z:

$$\frac{d}{dt}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ y - r \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ Cx - r \end{bmatrix} \tag{10}$$

The final compensator is given by

$$u = -K(x - x_e) - K_i z + u_d \tag{11}$$
$$\dot{z} = y - r$$

where we have now included the dynamics of the integrator as part of the specification of the controller

4. Simulation Results

Simulations were performed using decoupled linearized models of MAV in MATLAB/SIMULINK. Separate controllers were employed for longitudinal and lateral motions. Disturbances were injected to the system through the input channel and the performances of the controllers were evaluated. For the given MAV system for a flight condition with roll input equal to 20 degrees and pitch input equal to 11 degrees, the inner loop control required performance specifications are, the settling time less than 6secs, peak overshoot less than 10 percent and steady state error equal to zero**.** A 2 degree of elevator and aileron disturbances were injected at 73 seconds. The control constraint is fixed to be less than 30 degrees for aileron deflection and less than 20 degrees for elevator deflection.

The inner loop control should have good disturbance rejection capability.

4.1. PID Controller Response

The response of the system with PID controller is shown in Figure 2. Figure 2(a) and Figure2 (b) gives the longitudinal and lateral response respectively. The control effort using PID is as shown in Figure2(c) and Figure2 (d)

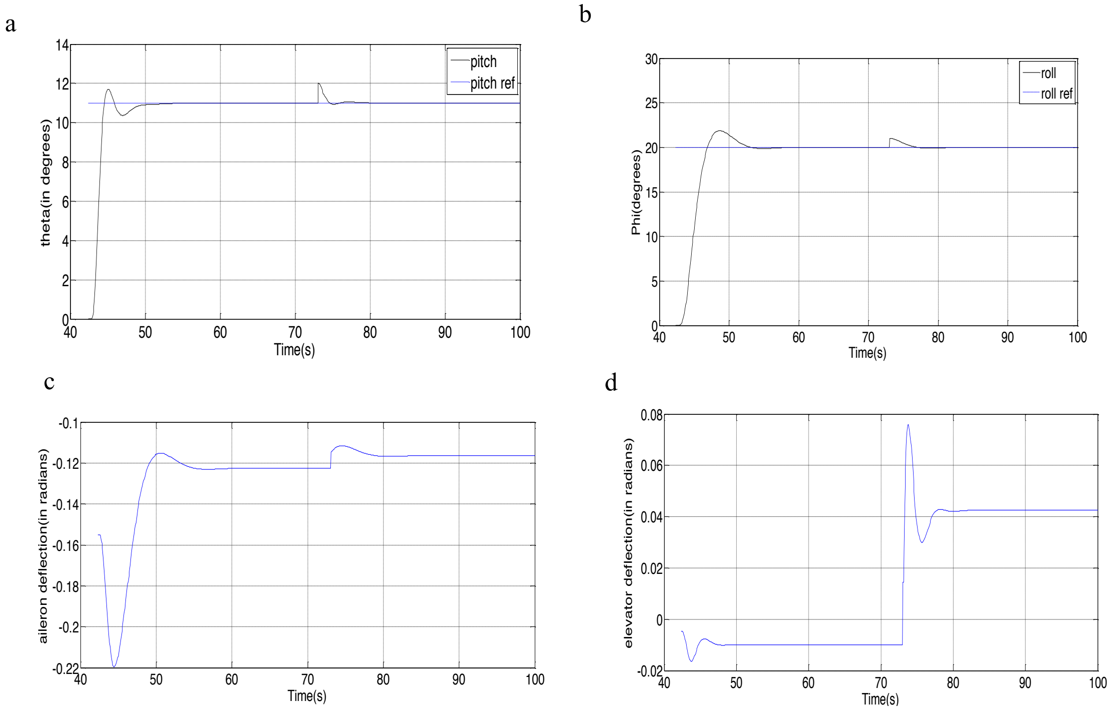


Fig 2. (a) Longitudinal Response (in degrees) with PID; (b) Lateral Response (in degrees) with PID; (c) Elevator deflection (in radians); (d) Aileron deflection (in radians)

Figure 2 shows a reasonably good response obtained by tuning the PID controller. The tuning of the controller was done using Zeigler-Nicholas Method. The PID controller gives satisfactory responses with the linearized aircraft models [8]. The aileron and elevator deflections are within limits. However, the overshoot and settling time of both lateral and longitudinal responses are quite higher than the required limits.

4.2 Integral –LQR Controller Response

The Integral LQR controller gives a much stable and robust response for the system [10]. The response of the system with LQR controller is given in Figure 3. Figure 3(a) shows the longitudinal response and Figure 3(b) shows the lateral response. The SVFB gain K for the system is found using lqr command in Matlab and the gain was given in the Simulink model to obtain the output.

(i)The value of Q, R and K_{long} matrices which gave the best pole placement for longitudinal was

$$Q_{long} = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}, \; R=100 \text{ and } K_{long} = [-0.0053 \quad 0.0299 \quad -0.1941 \quad -3.1618 \quad 0.0000] \quad (12)$$

(ii) The value of Q, R and K_{lat} matrices which gave the best pole placement for lateral was

$$Q_{lat} = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}, \; R= 100 \text{ and } K_{lat} = [0 \; 0.0295 \quad -0.2484 \quad -0.0581 \quad -3.1450 \quad 0.0000] \quad (13)$$

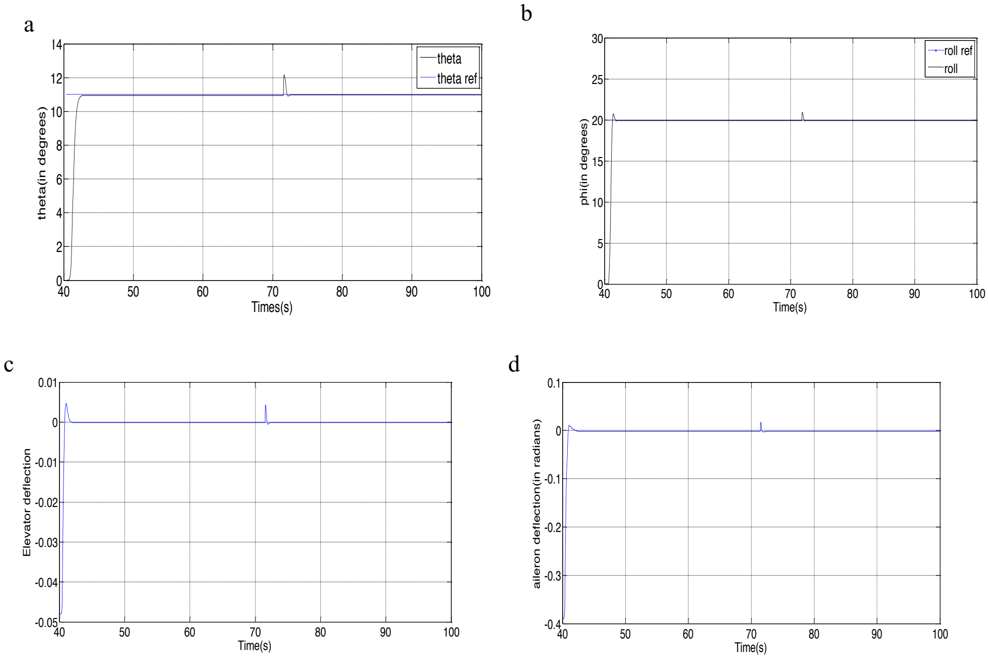


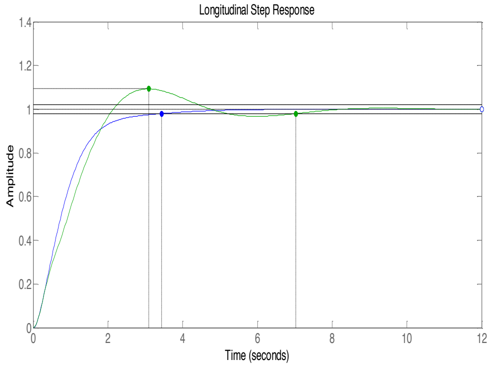
Fig 3. (a) Longitudinal Response(in degrees) with Integral LQR; (b) Lateral Response(in degrees) with Integral LQR ; (c) Elevator deflection(in radians); (d) Aileron deflection (in radians)

There is a considerable reduction in overshoot and settling time with the LQR controller. The response is more stable and robust. The control effort using Integral LQR is satisfactory and within limits.

4.3 Stability and Performance

The stability and performance of both controllers can be analysed using step response plots. Figure 4 shows the step response plots with PID and LQR Controllers. Figure 4(a) shows longitudinal step response of the system and Figure 4(b) shows the lateral step response of the system. In the case of PID Controller the overshoot (O S) and settling time(Ts) is greater than 5 percent and 6 seconds respectively for both lateral (O S=6.86%,Ts=6.18s) and longitudinal (O S=9.33%,Ts=7.03s). In the case of LQR Controller, the overshoot and settling time is less than 5 percent and 6 seconds respectively for both longitudinal(O S=0%,Ts=3.44s) and lateral(O S=1.92%,Ts=2.04s) as seen in Fig 4 (a) and Fig 4 (b)

a



b

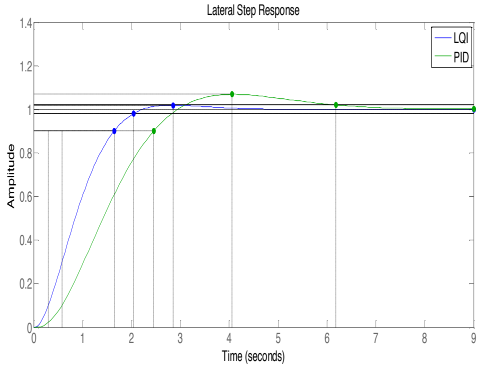


Fig 4. Step response plot (a) Longitudinal; (b) Lateral

5. Conclusions and Future Works

The Integral LQR control methodology is investigated and its control performance is compared with that of the traditional PID Controller in the dynamic system. PID controller for the proposed system model is showing only a very narrow region of stability. When the gains are increased, the system is settled fast but the overshoot is very high. Here, adding integral action to the LQR inner loop controller is proposed. The simulation results validate the proposed LQR methodology and display a better dynamic performance in terms of transition time and speed overshoot and also stronger robustness of LQR control methodology than that of traditional PID controller. This approach provided the desired results and it should be extended to the other controllers as this should result in better performance in the case of uncertainties. Future works include proposing an optimal sliding mode control against uncertainties.

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