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Simulation model of speed–density characteristics for mixed bicycle flow—Comparison between cellular automata model and gas dynamics model

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HIGHLIGHTS

- The mixed bicycle flow refers to the bicycle flow containing electric bicycles.
- The speed–density data of mixed bicycle flow was obtained with the virtual coil method.
- Mixed bicycle flow has unique traffic characteristic compared to pure bicycle flow.
- Cellular automata model can well simulate the mixed bicycle flow in low density.
- Gas dynamics model can well simulate the mixed bicycle flow in high density.

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ABSTRACT

The mixed bicycle flow refers to the bicycle flow containing electric bicycles. The traffic characteristics data of the mixed bicycle flow was collected by the virtual coil method in Nanjing and Ningbo, China. And the speed–density characteristics of the mixed bicycle flow with different proportions of electric bicycles were obtained. The results show that [illegible] the electric bicycles tend to be the same when the density is higher than 0.25 bic/m^2. And when the density reaches 0.58 bic/m^2, the mixed bicycle flow becomes blocked and the speed is zero. The cellular automata model and gas dynamics model were also adopted to simulate the speed–density characteristics of the mixed bicycle flow. The simulation results of the cellular automata model are effectively consistent with the actual survey data when the density is lower than 0.225 bic/m^2; the simulation results of the gas dynamics model are effectively consistent with the actual survey data when the density is higher than 0.300 bic/m^2; but both of the two types of simulation models are inapplicable when the density is between 0.225 and 0.300 bic/m^2. These results will be used in the management of mixed bicycles and the research of vehicle–bicycle conflict and so on.

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(a) Bicycle. (b) Electric bicycle.

Fig. 1. Appearances of bicycle and electric bicycle.

1. Introduction

Electric bicycle is a special kind of bicycle, with a storage battery, lithium battery or other electric energy as an auxiliary power source. It has two wheels and functions such as manual riding, electro motion or power-drive. The bicycle and electric bicycle are greatly different in power, speed, acceleration and deceleration and other characteristics. The appearances of a bicycle and an electric bicycle are shown in Fig. 1. Bicycle is human-driven and has a free flow speed of 14.0 km/h or so, which is depending on the degree of physical strength of the rider and other environmental reasons. The electric bicycle is power-driven and its free flow speed is about 21.0 km/h. In recent years, due to the sharp increase in the number of electric bicycles, it is almost impossible to find pure bicycle flows, and of course, impossible to find pure electric bicycle flows. In this paper, the "mixed bicycle flow" refers to the mixed flow of both electric bicycles and bicycles.

Bicycle travel is the main travel mode adopt by the Chinese urban residents. In large and medium-sized Chinese cities, the average ratio of bicycle and electric bicycle travel is about 28%. At the same time, according to the statistics, as of the end of 2012, the quantity of national bicycle is about 540 million, and the quantity of electric bicycle is about 140 million. Therefore, China is not only a big holding country of bicycles, but also a big travel country of bicycles. Therefore, it becomes essential to research the traffic characteristics related to mixed bicycle flow, especially the speed–density characteristics of the mixed bicycle flow, i.e. the overall speed change situation of mixed bicycle flow when the density changes. The characteristics data serves as an important basic theory in the planning and design of bicycle lanes as well as bicycle and electric bicycle traffic management. Current studies in this field are relatively weak.

2. Literature review

Several pieces of research have been previously undertaken. John Parkin studied the design speed and acceleration characteristics of bicycles for use in the planning and design of a bicycle traffic system [1]. Chenpeng Shi studied the status of electric bicycles traffic in China [2]. Binjie Dong briefly studied the integrated traffic characteristics of electric bicycles [3]. Hongyi Guan directly presents the speed–density characteristics of bicycle flow, but does not obtain a systematic result [4]. Chunyan Liang studied the speed–density characteristics of bicycle flow based on the motor speed–density characteristics model but the research results only conform to the survey data in a certain density scope [5]; Congkun Zhu studied the relationship between speed and density of mixed bicycle flow, and put forward a linear model and a nonlinear model, but neither reached an ideal conformity result [6]. Navin FPD made series of experimental studies to determine the operating performance of a single bicycle and the traditional traffic flow characteristics of a stream of bicycles, and compare them to survey data. It was found that under certain conditions, bicycle flow can be treated as vehicular flow [7]. Though applying the cellular automata simulation model, Sven Maerivoet systematically explained the application of the cellular automata in the simulation of motor vehicle traffic flow [13]. Xingang Li established the bi-lane traffic flow model of one vehicle overtaking another based on the cellular automata model [8]. Lawrence W. Lan simulated the mixed traffic flow of motorcycles and vehicles with the cellular automata [9]. Rui Jiang took the stochastic randomization into two different multi-value cellular automata models in order to model the bicycle flow. It is shown that with the randomization effect considered, the multiple states in the deterministic multi-value cellular automata models disappear and the unique flow–density relations (fundamental diagrams) exist [10]. B Jia investigated mixed bicycle flow using the multi-value cellular automata model. And the system of mixed bicycles was investigated under both deterministic and stochastic regimes. On this basis, space–time plots were presented to show the evolution of mixed bicycle flow [11]. Jin Zhang established a model and carried out a simulation of bicycle flow with cellular automata, the results achieved a firm conformity between the simulation result and survey data in a certain density scope [12]. With the application of the gas dynamics simulation model, Chunyan Liang studied the stop-wave theories for bicycle flow based on the gas dynamics simulation model and obtained sound data [5], but related research into the speed–density characteristics of bicycle flow was lacking.

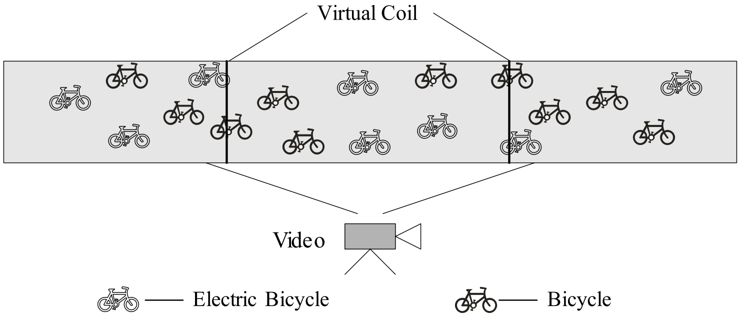

Fig. 2. Survey diagram of the virtual coil method.

From the above, we can see that there are few studies exactly on the speed–density characteristics of mixed bicycle flow. In this paper, we will carry out a systematic study of the speed–density characteristics of mixed bicycle flow, which was based on the survey data and the simulation data of the cellular automata model and gas dynamics model. Besides, we will compare the simulation results and the actual survey data to demonstrate the rationality and suitability of the models.

3. Method

3.1. Density representation of bicycle flow

If bicycles are driven on the rightmost side of bicycle lane, the right side requires 0.25 m lateral clearance. If bicycles are driven on the leftmost side, the left also requires 0.25 m lateral clearance. Therefore, given a 1.5 m bicycle lane, the actual travel space for bicycle traffic is only 1.0 m. And according to practical experience, this is the minimum width for bicycle traveling.

On Chinese urban roads, the width of bicycle lanes is usually 1.5 m, 2.5 m or 3.5 m, corresponding to 1 lane, 2 lanes and 3 lanes respectively. The bicycle lane with 2 or 3 lanes has no strict distinction to mark the lanes, and the bicycles do not drive strictly in the right lane but move to the left and right during riding. Therefore, while studying the density of the bicycle flow, this paper took the number of bicycles per unit lane area at a given time as the density of bicycle flow, and the dimension is bic/m^2. Of course, the lane area is calculated after the lateral clearance area on both sides has been subtracted.

3.2. Survey method of speed–density characteristics

Traffic survey on the speed–density characteristics of the bicycle flow was carried out by the virtual coil method, and the virtual coils are the two straight lines drawn on the lanes and perpendicular to the direction of the traffic flow, as shown in Fig. 2. In the survey, we recorded the number of passing bicycles and electric bicycles within the virtual coil by video, and then obtained the number of vehicles and the proportion of bicycles versus electric bicycles within the coil at any given time. Finally, the density of mixed bicycle flow and driving speed of each bicycle and electric bicycle can be measured by this method.

3.3. Cellular automata simulation model

The cellular automata model has been widely used to simulate the speed–density characteristics of motor flow [12–14], so in this paper, we also tried to use this model to simulate the speed–density characteristic of mixed bicycle flow. But it needs to assume that the number of bicycle lanes should be specific and bicycles drive in proper lanes.

The body length of most bicycles and electric bicycles is 1.7–1.9 m, the body width is 0.5–0.6 m, and the occupied length of the road while vehicles are ridden normally is about 2.0 m and the width is about 1.0 m. Therefore, the size of the bicycle and electric bicycle "cell" is a rectangular 2.0 m long and 1.0 m wide. According to the survey result, the free-riding speed of bicycles is about 14 km/h, that is about two cells which is taken as the maximum speed of bicycles, and the free-riding speed of electric bicycles is about 21 km/h, that is about three cells which is taken as the maximum speed of electric bicycles.

Bicycles and electric bicycles are expressed by i and j respectively. Suppose the longitudinal speed of bicycles and electric bicycles is respectively v_i and v_j and the lateral speed is respectively v'_i and v'_j, the maximum speed of bicycles and electric bicycles is respectively $v_{1\max} = 2$ and $v_{2\max} = 3$. d_1, d_2 and d_3 are the respective empty cellular number between the current cellular and the non-empty cellular in its front, left front and right front. a_1 and a_2 are the empty cellular number between the current cellular and the non-empty cellular in its left and right respectively. In the process of $t \to t+1$, the model evolves according to the following rules.

Step 1: longitudinal acceleration. $v_i \to \min(v_i+1, v_{1\max})$, $v_j \to \min(v_j+1, v_{2\max})$. This corresponds to cyclists' realistic characteristics of traveling at the maximum speed.

Table 1
Value taking rules of d_i.

Judgment conditions	Value of d_i
$d_{i1} \neq d_{i2} \neq d_{i3}$	$d_i = \max\{d_{i1}, d_{i2}, d_{i3}\}$
$d_{i1} = d_{i2} = d_{i3}$	$p_1 = 1/4, p_2 = 1/2, p_3 = 1/4$
$d_{i1} = d_{i2} > d_{i3}$	$p_1 = 2/3, p_2 = 1/3, p_3 = 0$
$d_{i1} = d_{i3} > d_{i2}$	$p_1 = 2/3, p_2 = 0, p_3 = 1/3$
$d_{i2} = d_{i3} > d_{i1}$	$p_1 = 1/2, p_2 = 0, p_3 = 1/2$

Table 2
Value taking rules of v_i'.

Judgment conditions	Value of v_i'
$a_{i1} > a_{i2}$	$v_i' = -1$
$a_{i1} < a_{i2}$	$v_i' = 1$
$a_{i1} = a_{i2}$	$p_1 = 1/2, p_2 = 1/2$

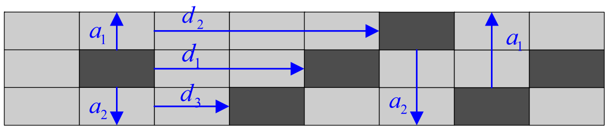

Fig. 3. Lane-crossing rule.

Step 2: longitudinal deceleration. $v_i \to \min(v_i, d_i)$, $v_j \to \min(v_j, d_j)$. In this equation, the value rules of d_i are as in Table 1.

In this table, p_1, p_2, p_3 are the probabilities of d_{i1}, d_{i2}, d_{i3} respectively. The value taking method of d_j is the same with that of d_i.

Step 3: horizontal speed. If $v_i \neq 0$, then $v_i' = 0$, $v_i' = -1$, $v_i' = 1$ is respectively corresponding to $d = d_{i1}$, $d = d_{i2}$ and $d = d_{i3}$. If $v_i = 0$, then the value rules of v_i' are as in Table 2.

In this table, p_1 and p_2 are respectively the probabilities of -1 and 1. The value taking method of v_j' is the same with that of v_i'.

Step 4: randomization deceleration. The probability p, $v_{1i} \to \max(v_{1i} - 1, 0)$, $v_{2j} \to \max(v_{2j} - 1, 0)$ are used to express the vehicle deceleration of cyclists caused by various uncertain factors.

Step 5: motion. $s_i \to s_i + v_i$, $s_i' \to s_i' + v_i'$, $s_j \to s_j + v_j$, $s_j' \to s_j' + v_j'$. Bicycles and electric bicycles move forward at an adjusted speed.

Here, s_i and s_i' are the vertical and horizontal positions of No. i bicycle respectively; s_j and s_j' are the vertical and horizontal positions of No. j electric bicycle respectively.

The lane-crossing rule is showed as Fig. 3.

3.4. Gas dynamics simulation model

3.4.1. Basic concept of gas dynamics

Compare to the motor traffic, it is not have specific lanes compartmentation in mixed bicycle traffic, and the bicycles can travel as gas moving. Gas dynamics is a basic theory to study the motion law of compressible gas, so this paper used the gas dynamics model to simulate the speed–density characteristic of mixed bicycle flow. When the state equation of compressible gas is studied, its pressure and temperature can only adopt absolute pressure and Kelvin temperature. Its equation of state can be written as:

$$\frac{p}{\rho} = RT \tag{1}$$

In the equation, p is the gas pressure, ρ is the gas density, T is Kelvin temperature, and R is the gas constant (air: 287 J/kg · K).

Based on the above state equation and in accordance with the Law of Conservation of Mass in hydromechanics, we get the following motion equation of the one-dimensional constant flow of ideal gas:

$$\frac{\mathrm{d}p}{\rho} + v\,\mathrm{d}v = 0. \tag{2}$$

In this equation, v is the speed of gas. And this equation is also known as the Euler equation which determines the relationship between p, ρ and v of the one-dimensional motion of gas.

Table 3
Corresponding relationships between relevant parameters of compressible gas and mixed bicycle flow.

Gas parameters	Parameters of mixed bicycle flow
Density ρ	Road occupancy density of mixed bicycles ρ (veh/m^2)
Speed v	Average speed of mixed bicycle flow v (km/h)
Pressure p	Spatial exclusion among mixed bicycles p (1/m^2)*

* Note: The spatial exclusion among bicycles listed in the table is associated with the clearance size around the bicycles. The greater the clearance is, the more flexible the motion of the bicycles will be, and the smaller the exclusion will be; otherwise, the greater the motion of bicycles is affected by surrounding bicycles, the greater the exclusion will be.

During the common thermodynamic process, there are three main types of process: the isometric process, isothermal process and adiabatic process. The isometric process refers to the fact that the gas density remains unchanged during the motion process; the isothermal process refers to the fact that the gas temperature remains unchanged during the motion process while the gas density and pressure show a kind of univariate linear relationship and the gas density and speed change according to the motion equation; the adiabatic process refers to the motion pattern without energy loss and with no heat exchange with the outside world.

3.4.2. Mixed bicycle flow simulation model

In considering the speed–density characteristics of bicycle flow, its density keeps changing. Meanwhile, bicycles have self-drive capability so the overall energy is not constant. Therefore, neither the isometric process nor adiabatic process is suitable to describe the speed–density characteristics of bicycles. On the other hand, in the isothermal process of the gas, there exists a simple linear relationship between gas density and speed. If the relevant parameters are compared with the parameters of bicycle flow, the isothermal process of the gas can effectively match the speed–density characteristics of bicycle flow. Furthermore, in the case of mixed bicycle flow, the characteristics of bicycles are not distinguished from those of electric bicycles. Instead, they are both taken as the same characteristics. (See Table 3.)

The basic equation for the isothermal process of one-dimensional motion of gas is as follows:

$$\frac{1}{\rho} = \frac{RT}{p} \tag{3}$$

$$RT \ln p + \frac{v^2}{2} = c. \tag{4}$$

Put Eq. (3) into Eq. (4), we can get

$$v = \sqrt{2c - 2RT \ln RT - 2RT \ln \rho}. \tag{5}$$

The above equation is the speed–density equation of the isothermal process of one-dimensional motion of gas.

For mixed bicycle flow, replace to a certain extent relevant parameters in the above equation, and we will get the following basic equation of bicycle flow:

$$p = k\rho \tag{6}$$

$$k \ln p + \frac{v^2}{2} = c. \tag{7}$$

Putting Eq. (6) into Eq. (7), we get

$$v = \sqrt{a + b \ln \rho}. \tag{8}$$

The above equation is the speed–density equation of mixed bicycle flow on the basis of gas dynamics, where a and b are undetermined parameters.

4. Results

4.1. Survey results of speed–density characteristics

The virtual coil method was used to collect the speed–density data of mixed bicycle flow with different proportion of electric bicycle. In the survey, 38 sections whose bicycle lanes have well condition in Nanjing and Ningbo (China) were selected to be the survey sections. According to the virtual coil method, the videos were set up on the adjacent high building. Meanwhile, the videos of mixed bicycle flow at different period were shot and brought back to the laboratory. At last, 112 groups of speed–density data meeting the requirements were obtained. In the sample, the proportion of electric bicycle is generally from 0.3 to 0.5. Therefore, the survey data was divided into two categories: data where the proportion of

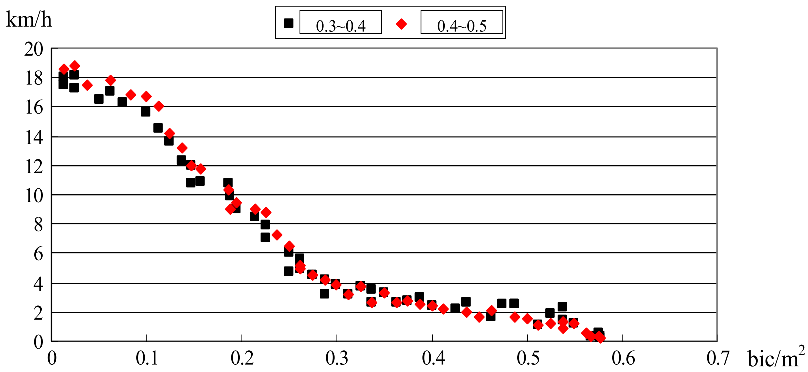

Fig. 4. Graph for speed–density characteristics of mixed bicycle flow.

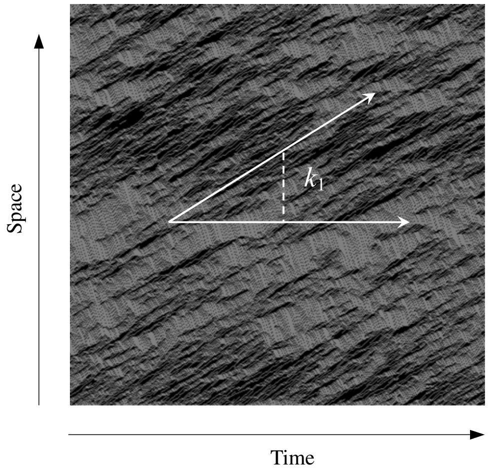

Fig. 5. Graph for space–time changes of pure bicycles.

electric bicycles is 0.3–0.4 and data where the proportion of electric bicycles is 0.4–0.5. In addition, we drew the graph of the speed–density characteristics of bicycles under different proportions of bicycles and electric bicycles, as shown in Fig. 4.

From the graph we can see, when the density of bicycle flow is low (bicycles are in a free-driving state), the overall speed of the mixed bicycle flow containing electric bicycles is higher than that of pure bicycles (the free-driving speed of pure bicycle flow is about 14 km/h). And the greater the proportion of electric bicycles, the higher the bicycle flow speed is. It is shown in Fig. 4, where the red points are basically higher on the graph than corresponding black points. When the density of mixed bicycle flow is higher than 0.08 bic/m^2, the speed of mixed bicycle flow tends to decrease with the increase of density. It indicates that the overall free-driving space of bicycle flow decreases and the moving speed fails to reach the maximum free-driving speed when the density increases, but the red points are still higher than corresponding black points. When the density is higher than 0.25 bic/m^2, the speed–density characteristics of mixed bicycle flow with different proportions of electric bicycles tend to be the same (that is, the red points basically coincide with the corresponding black spots). And when the density of bicycle flow is higher than 0.58 bic/m^2, the mixed bicycle flow will be blocked and the speed will be 0 km/h.

4.2. Simulation results of cellular automata model

By simulating pure bicycle flow and mixed bicycle flow containing electric bicycles, we obtained the space–time changes graph as shown in Figs. 5 and 6. The black areas in the figures represent cumulative driving situations. The space–time slope coefficient represents the speed of vehicles in the traffic flow. It can be seen from the figures that the mixed bicycle flow is more disorderly than the bicycle flow, but its speed is greater than that of pure bicycle flow. This is consistent with the actual survey results.

In addition, the speed–density characteristics of mixed bicycle flow are simulated when the proportion of electric bicycles is 0.35, 0.40, 0.45 and 0.50, we get the following speed–density characteristics graph. (See Fig. 7.)

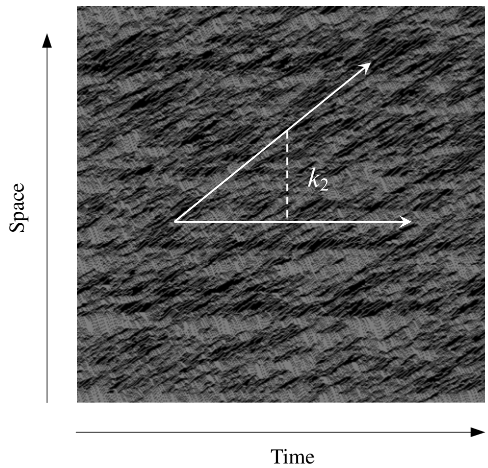

Fig. 6. Graph for space–time changes of bicycles containing electric bicycles.

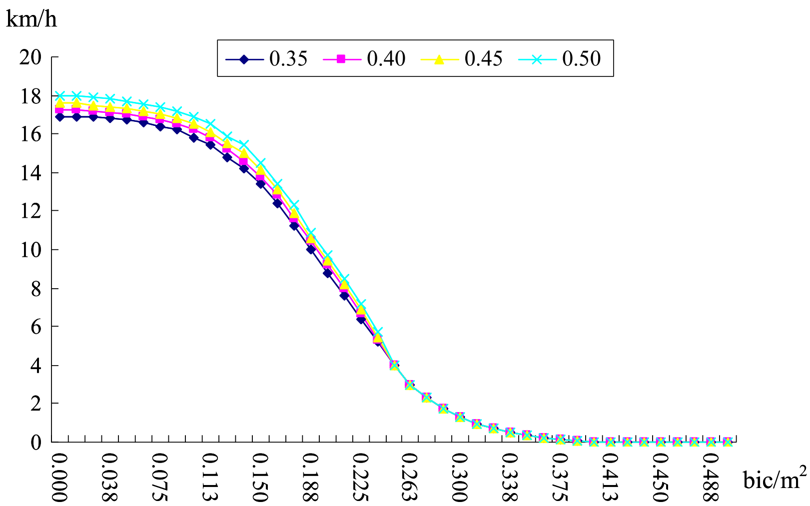

Fig. 7. Graph for speed–density relationships under different proportions of electric bicycles.

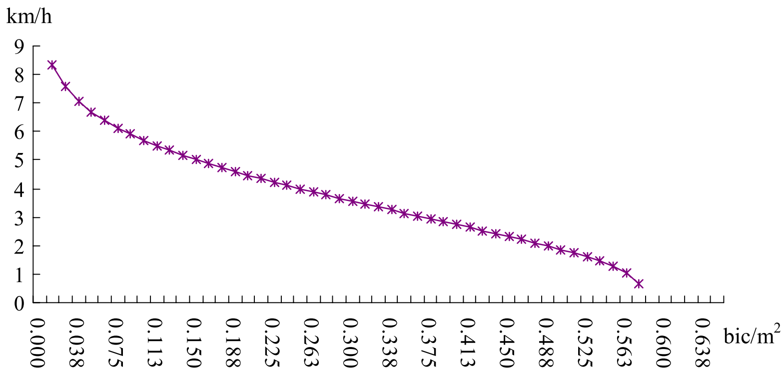

Fig. 8. Graph showing the speed–density characteristics of bicycle flow.

4.3. Simulation results of gas dynamics model

Using the survey data and Eq. (8) we get $a = -9.02$ and $b = -18.21$. The subsequent graph about the speed–density characteristics of mixed bicycle flow is shown in Fig. 8.

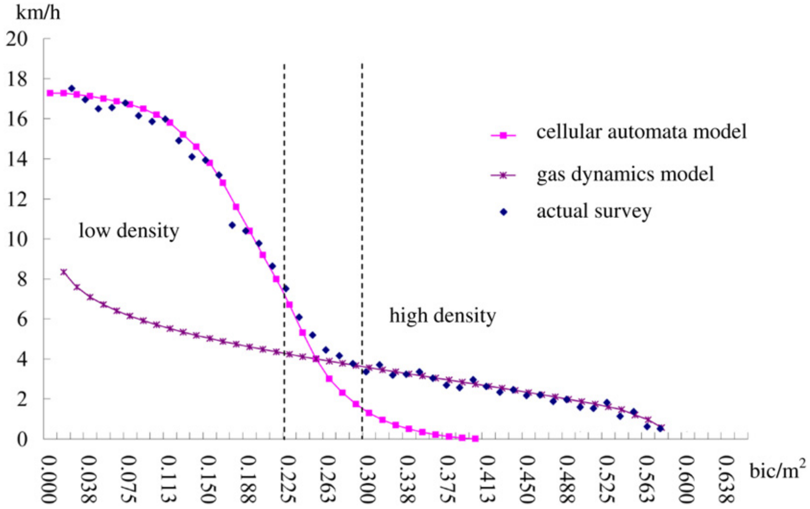

Fig. 9. Analysis of simulation data and survey data.

5. Discussion and conclusions

The simulation results of the cellular automata model and the gas dynamic model and the actual survey data are plotted on the same graph shown in Fig. 9. In this graph the proportion of electric bicycles is 0.40.

It can be seen from the graph that when the density is lower than 0.225 bic/m^2, the simulation results of the cellular automata model are effectively consistent with the actual survey data. It means that it is safe to think that bicycles and electric bicycles run in the proper lanes when the density is low, and that the overall speed of mixed bicycle flow increases because it contains electric bicycles, which is also consistent with the actual survey results. When the density is higher than 0.300 bic/m^2, the simulation results of gas dynamics are effectively consistent with the actual survey data which means that when it is in a state of high density, there is not much difference between the characteristics of bicycles and electric bicycles, i.e. the driving characteristics of bicycles and electric bicycles are basically identical when ridden in a crowded state. And in that state, the relevant parameters of gas dynamics can be used to give comparative simulation explanation in Physics to relevant parameters of mixed bicycle flow. And when the density of mixed bicycle flow is between 0.225 bic/m^2 and 0.300 bic/m^2, the simulation results of the two models are different from the survey data, which shows that bicycles and electric bicycles drive irregularly instead of driving in proper lanes. Therefore, the cellular automata model is inapplicable. Meanwhile, in this density range, electric bicycles and bicycles have some free driving space and their characteristics still have some differences. Therefore, the gas dynamics model is inapplicable in this state.

In reality, it takes a lot to get the speed–density characteristics of the mixed bicycle flow through survey, and the number of electric bicycles in cities is relative fixed at a certain period. Therefore, the survey method can only get the speed–density characteristics of the mixed bicycle flow with certain proportion ranges of electric bicycles. However, the cellular automata model proposed in this paper can simulate the speed–density characteristics of the mixed bicycle flow under different proportions of electric bicycles in the state of low density, and thus get the corresponding value of speed and density. The gas dynamics model, on the other hand, can get the speed–density equation of the mixed bicycle flow in the state of high density.

Some studies require to take the speed–density characteristics of the mixed bicycle flow as theoretical foundation, such as studies on the conversion coefficient of electric bicycles conversing into bicycles in the mixed bicycle flow, capacity calculations of bicycle lanes in the mixed bicycle flow situations, the planning and design method of bicycle lanes, studies on vehicle–bicycle conflict of non-isolated road, studies on the setting standards of vehicle–bicycle isolated facilities and so on. Of course, studies on the management and control of bicycles and electric bicycles at intersections also need to take the speed–density characteristics of the mixed bicycle flow as theoretical foundation.

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References

[1] J. Parkin, J. Rotheram, Design speeds and acceleration characteristics of bicycle traffic for use in planning, design and appraisal, Transport Policy 17 (5) (2010) 335–341.
[2] Chenpeng Shi, Analysis on electric bicycles' current traffic situation and counter measures, Journal of Chongqing Jiaotong University (Natural Science) 27 (5) (2008) 772–775 (in Chinese).

[3] Binjie Dong, The study of characteristics of electric bicycle, Doctoral Thesis, Tongji University, Shanghai, China, 2008.
[4] Hongyi Guan, Study on characteristics and application of bicycle traffic flow, Doctoral Thesis, Southwest Jiaotong University, Chengdu, China, 2004.
[5] Chunyan Liang, Study on urban bicycle traffic system, Doctoral Thesis, Jilin University, Changchun, China, 2007.
[6] Congkun Zhu, Yupin Chi, Relationship between speed and density in mixed non-motorized vehicle, Journal of Suzhou University of Science and Technology (Engineering and Technology) 22 (3) (2009) 26–30 (in Chinese).
[7] F.P.D. Navin, Bicycle traffic flow characteristics: experimental results and comparisons, Institute of Transportation Engineers 64 (3) (1994) 31–37.
[8] Xingang Li, et al., A realistic two-lane cellular automata traffic model considering aggressive lane-changing behavior of fast vehicle, Physica A: Statistical Mechanics and its Applications 367 (15) (2006) 479–486.
[9] Lawrence W. Lan, et al., Cellular automaton simulations for mixed traffic with erratic motorcycles' behaviours, Physica A: Statistical Mechanics and its Applications 389 (10) (2010) 2077–2089.
[10] Rui Jiang, Bin Jia, Qing-Song Wu, Stochastic multi-value cellular automata models for bicycle flow, Journal of Physics A: Mathematical and General 37 (6) (2004) 36–42.
[11] B. Jia, X.-G. Li, R. Jiang, Z.-Y. Gao, Multi-value cellular automata model for mixed bicycle flow, The European Physical Journal B 56 (3) (2007) 247–252.
[12] Jin Zhang, Hui Wang, Ping Li, Bicycle flow modeling and simulation based on cellular automaton, Journal of Highway and Transportation Research and Development 23 (1) (2006) 125–129.
[13] Sven Maerivoet, Bart De Moor, Cellular automata models of road traffic, Physics Reports 419 (1) (2005) 1–64.
[14] Yo-Sub Han, Sang-Ki Ko, Analysis of a cellular automaton model for car traffic with a junction, Theoretical Computer Science 450 (7) (2012) 54–67.