Static and dynamic characteristics of resilient mats for vibration isolation of railway tracks

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Abstract

The paper presents essential static and dynamic properties characterizing resilient mats for vibration isolation of railway tracks. The attention was focused on under-ballast mats and slab-track mats used in the construction of railways. Referring to this type of mats, selected test procedures for determining the values of parameters describing static and dynamic characteristics were described. Basic classification of vibration isolation mats was presented in the paper. Moreover, the essential functional and operation features related to various types of mats were given. Some theoretical aspects concerning viscoelastic dynamic modelling of resilient track elements were also outlined. In order to characterize requirements and methodology of research for resilient mats in railway tracks, German Standards [1-3] were used herein.

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Keywords: railway construction; vibro-acoustic isolators; vibration reduction; under-ballast mats; slab-track mats; structural vibrations

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1. Resilient mats in railway tracks

Resilient mats in railways are used for vibration isolation of railway track by reducing dynamic effects of rail traffic. The reduction relates mainly to vertical and transverse material vibration, but also to structure-borne noise.

Under-ballast mats are used in the track superstructure on engineering structures such as bridges, track troughs and tunnels. They increase resilience between the ballast and the track, and reduce vibrations emitted into the environment.

In ballast-less track system resilient mats have similar purpose as under ballast mats. Nonetheless, a ballast-less track system is always an individual engineering solution for a particular application. In particular, in a slab track system resilient mats are applied horizontally and vertically directly below and from the sides of the concrete track base plate. The resulting floating slab track system is effective in mitigating vibrations, especially in low frequencies. It also can achieve a significant reduction in vibrations and structure-borne noise emissions at excitation frequencies above $\sqrt{2}$ times the tuning frequency.

Resilient mats can be grouped into two categories, because of the range of applications in various types of railways track construction:

- Under-Ballast Mats (UBM) / Sub-Ballast Mats (SBM) or ger. Unterschottermatten (USM) – used in ballast railway track;
- Slab-Track Mats (STM) – used in ballast-less railway track.

Under-Ballast Mats have two varieties with regard to the main purpose of their use:
- used primarily for isolation from vibration,
- used primarily for stress reduction in ballast.

Slab-Track Mats have three varieties with regard to the slab support system:
- elastomeric pads (discrete support),
- strip mats (linear support),
- elastomeric mats (continuous support).

Due to materials used and production technologies, resilient mats can be grouped into mats made of various kinds of elastomers (elastomeric mats) or mineral wool mats. Elastomeric mats can be divided into two groups:
- mats based on polyurethane in versions with closed or open pores;
- rubber mats (composites based on blends of natural rubber and / or synthetic rubber), which resiliency is ensured by shape (channels, grooves or protrusions of different shape) and cross-sectional structure of the mat (density and pore volume).

Resilient mats can have an uniform structure (homogeneous) or a layered structure (with layers of different materials and features).

2. Functional properties of resilient mats

Technical characteristics of resilient mats relates to technical and functional properties of materials, which are the most relevant for fulfillment of its essential functions, such as:

- effective vibration isolation - maximum reduction in the level of impact in form of vibration and structure-borne noise;
- increased technical lifespan - maintaining long-term ability to fulfill functions mentioned above in real operating conditions, with maximum 20% of variability of main parameters of the states during laboratory fatigue tests conducted under extreme operating conditions.

Given the above requirements, the most important parameters for assessing the quality of resilient track mat can be determined due to the function (vibration isolation, working life, material characteristics in accordance with the specifics of operating conditions). In case of vibration isolation it is:

- static and dynamic vertical stiffness ($k_{stat,z}$, $k_{dyn,z}$), static and dynamic vertical ($C_{stat,z}$, $C_{dyn,z}$) bedding modulus,
- horizontal static stiffness ($k_{stat,h}$) and horizontal static bedding modulus ($C_{stat,h}$),
- loss factor $\eta$ defined by the ratio of energy dissipated to energy expended.

The last coefficient is related to, important for the damping characteristic of the elastomer, dynamic stiffening ratio. By working life (fitness) of resilient mat is meant mechanical fatigue strength including changes of vertical
static modulus of elasticity \( C_{stat,z} \) or vertical static stiffness \( k_{stat,z} \) induced by dynamic load simulation in the laboratory service loads, as well as long-term resistance to shear (for Slab-Track Mats). For Under-Ballast Mats number of load cycles is 12.5 million and for Slab-Track Mats – 3 million.

Research of material characteristics may include the following parameters:
- freeze-thaw resistance,
- ageing resistance,
- water resistance.

3. Viscoelastic dynamic model of resilient track elements

In order to describe rheological and kinetic properties of resilient truck elements a very simple viscoelastic dynamic model is proposed in standard [1]. Our objective in this point is to provide a description of linear properties of an oscillatory system obeying the principle of superposition and presented in [1]. We will explain the terminology used in [1] pointing out some theoretical aspects being important in next chapters dealing with laboratory test procedures for resilient elements used in railway tracks [4-7].

Let us analyze a viscoelastic Kelvin-Voigt dynamic system to be shown in Fig. 1a. It contains one elastic element characterized by a spring coefficient \( k \) [N/m] and one viscous element with damping coefficient \( d \) [Ns/m]. Moreover, \( m_{eff} \) [kg] denotes effective dynamic mass of resilient elements. The system is excited by a force \( F \) [N]. The response of the system is specified by displacement \( u \) [m]. Thus, the equation of motion is as follows
\[
m_{eff} \ddot{u}(t) + d \dot{u}(t) + k u(t) = F(t)
\]  

(1)

where a superimposed dot denotes differentiation with respect to time.

Harmonic excitation of the system and its response, assuming linear properties, can be expressed via following equations (steady-state behavior)
\[
F(t) = F_o \sin \omega t ; \quad u(t;\omega) = u_o(\omega) \sin(\omega t - \zeta(\omega))
\]  

(2)

where \( \omega \) [rad/s] denotes angular frequency of excitation, \( F_o \) and \( u_o \) denote amplitudes of force and displacement respectively and \( \zeta \) is phase angle (loss angle). Angular frequency is related to ordinary frequency \( f \) [Hz] by the relation \( \omega = 2\pi f \).

Let us define the dynamic stiffness modulus of the system as a ratio between amplitudes of excitation and response

\[ k^* = \frac{F_o}{u_o(\omega)} \]  

Fig. 1. Viscoelastic dynamic model of resilient truck elements (a) and complex stiffness modulus representation (b)
The behavior of linear viscoelastic systems excited by harmonic functions can be analyzed using complex variables. Let us assume the following transformation of variables

\[ F(t) = \text{Im} F^*(t), \quad u(t; \omega) = \text{Im} u^*(t; \omega) \]  

(4)

The complex variables defined by Eqs. (4) are given as follows

\[ F^*(t) = F_u \exp[i\omega t], \quad u^*(t; \omega) = u_u(\omega) \exp[i\omega t - i\zeta(\omega)] \]

(5)

where \( i \) denotes imaginary unit.

Thus, it is possible to define the complex stiffness modulus containing both dynamic stiffness modulus as well as phase angle

\[ k^*(i\omega) := \frac{F^*(t)}{u^*(t; \omega)} = k_{dyn}(\omega) \exp(i\zeta(\omega)) \quad \text{where} \quad k_{dyn}(\omega) = |k^*(i\omega)| \]

(6)

The complex stiffness modulus can be decomposed as follows (see Fig. 1b)

\[ k^*(i\omega) = k_{dyn}(\omega) \cos \zeta(\omega) + ik_{dyn}(\omega) \sin \zeta(\omega) = k'(\omega) + ik^*(\omega) \]

(7)

where \( k' \) denotes the storage stiffness modulus representing elastic and kinetic properties of resilient track elements while \( k^* \) is the loss modulus used to represent viscous (damping) properties.

In case of linear viscoelastic systems, it is possible to define the loss factor being equal to the tangent of the loss angle. Thus, we obtain (see Fig. 1b)

\[ \eta(\omega) = \tan \zeta(\omega) = \frac{k^*}{k'} \]

(8)

Summarizing, we can express a steady-state harmonic response of the system exited by sinusoidal force as follows

\[ u(t; \omega) = F_u \text{Im} \left[ \frac{\exp(i\omega t)}{k'(i\omega)} \right] = F_u N(\omega) \sin(\omega t - \zeta(\omega)) \]

(9)

where \( N(\omega) \) denotes dynamic compliance defined as an inverse of dynamic stiffness (see [1]). Consequently, \( N(\omega) \) can also be interpreted as a magnitude of the complex compliance \( N^*(i\omega) \):

\[ N(\omega) = |N^*(i\omega)| = \frac{1}{k_{dyn}(\omega)} \]

(10)

Let us move back to the analysis of viscoelastic dynamic system shown in Fig. 1a. In this case, the complex stiffness modulus may be expressed by the following equation
\[ k^s(i\omega) = k - m_{\text{eff}}\omega^2 + i\omega d \]  

(11)

while the dynamic stiffness modulus and the loss factor are as follows

\[ k_{\text{dyn}}(\omega) = \sqrt{(k - m_{\text{eff}}\omega^2)^2 + (\omega d)^2} \quad \text{and} \quad \eta(\omega) = \frac{\omega d}{k - m_{\text{eff}}\omega^2} \]  

(12)

The above Eq. (12) can be substituted into Eq. (10) and Eq. (9) in order to obtain the solution of equation of motion (1), under sinusoidal force excitation (steady-state response) taking additionally \( \zeta(\omega) = \tan(\eta(\omega)) \).

It should be noted, that the standard [1] introduces more precise definitions of the spring coefficient \( k \) and dynamic stiffness modulus \( k_{\text{dyn}} \). For example, in case of the stiffness modulus, the following definitions are used in [1]: \( k_{\text{stat}} \) – static stiffness under a quasi-static force, \( k_{\text{stat0}} \) – at-rest static stiffness, \( k_{\text{kin}} \) – kinetic or lower-frequency stiffness under a harmonically varying force with frequencies up to 1 Hz. Moreover, in case of the dynamic stiffness modulus, applied for the frequency range above 1 Hz, the standard [1] distinguishes between \( k_{\text{dyn1}} \) – lower-frequency dynamic stiffness and \( k_{\text{dyn2}} \) – higher-frequency dynamic stiffness.

4. Regulation identification of parameters characterizing the static and dynamic properties of resilient mats

As there are no European standards, which would define the requirements and methodology of research for resilient mats in railway tracks, it is substantiated to use German Standards: DIN 45673-1 [1], DIN 45673-5 [2] and DIN 45673-7 [3]. Document DIN 45673-7 [3] covers parameters related to the effectiveness of a track structure in mitigating vibrations and parameters needed for the static analysis and for the verification of track safety in floating slab track system. However, it is not possible to specify generally applicable load ranges for the resilient elements in a floating slab track as in case of under-ballast mats (DIN 45673-5 [2]). This is due to the fact that a floating slab track system is always an individual engineering solution for a particular application. Therefore, said standard sets out procedures for testing fitness for purpose, but does not contain requirements pertaining to the properties of resilient elements.

Below are described selected parameters characterizing only dynamic properties of resilient mats. Whereas all the substantial characteristics of resilient mats and procedures for their research are presented in studies [8] and [9].

4.1. Static and dynamic bedding modulus, static and dynamic stiffness

Static and dynamic bedding modulus and static and dynamic stiffness determine effectiveness of dumping vibrations transmission to the environment. It should be noted that the mats with very low value of static bedding modulus or stiffness give in greater vertical deflection of rail and railway track. That results in greater amplitudes and stresses, reduces fatigue strength and working life of the railway track construction. The value of the static and dynamic bedding modulus is defined for resilient mats for linear and continuous support. It includes a wide range of values from \( \sim 0.003 \) to \( \sim 0.33 \) [N/mm\(^3\)], depending on type and structure of the material, thickness of the mat, load value in which bedding modulus is defined and load frequency for dynamic modulus. For small elements (discrete support) static and dynamic stiffness is determined.

Vertical static bedding modulus characterizes conditions of rail deflection under the pressure of not moving rolling stock. It also affects the vertical deflection of railway track in floating slab track system. The relationship between static bedding modulus and the applied force is non-linear. In such cases, the static bedding modulus is determined to different ranges of loads, depending on the application purpose in question resilient mat. Based on displacements \( s_1 \) and \( s_2 \) (see Fig. 2a) the static bedding modulus can be evaluated in N/mm\(^3\) as a secant modulus with respect to the range of stresses to be assessed \( \sigma_1 \) and \( \sigma_2 \) as it was expressed via Eq. (13). Similarly, the static stiffness \( k_{\text{stat}} \) [N/mm] is determined applying Eq. (14) (see Fig. 2b).
Dynamic bedding modulus characterizes mat’s work under the pressure of moving rolling stock, therefore, it
determines limitation of vibrations transmission mat provides. The value of dynamic bedding modulus is related not
only to loading force, as in static bedding modulus, but also to frequencies. Therefore it should be determined under
the standard pressure and frequency ranges.

\[
C_{\text{stat}} = \frac{\sigma_2 - \sigma_1}{s_2 - s_1} \quad \text{[N/mm]} \quad (13)
\]

\[
k_{\text{stat}} = \frac{F_2 - F_1}{s_2 - s_1} \quad \text{[N/mm]} \quad (14)
\]

DIN 45673-5 standard [2] has divided test procedure for two groups. First one is for dynamic bedding modulus
for lower-frequencies \( C_{\text{dyn1}(f)} \) (5, 10, 20 and 30 Hz), second one for higher-frequencies \( C_{\text{dyn2}(f)} \). The parameter
\( C_{\text{dyn1}(f)} \) (see Fig. 3) can be used to determine the lower-frequency bending deformation of the rail under the
influence of the rolling wheel, as a result of the interplay of the bending elasticity of railway track structure. The
parameter \( C_{\text{dyn2}(f)} \) refers to under-ballast mats and can be used to determine level of reduction of structure-borne
noise. It influences the natural frequency \( f_0 \) of the elastically supported track as an oscillatory system and thus the
insertion loss \( D_e \).

Fig. 3. Determination of the dynamic bedding modulus of resilient mat – sequence’s harmonic loads diagram. Key: \( \sigma_m \) – mean value of the applied load, \( \sigma \) – load amplitude, \( t \) – time, 1 – evaluation range covering ten periods [2]
4.2. Dynamic stiffening ratio

Dynamic stiffening ratio $r(f)$ is a proportion between dynamic and static bedding modulus (or dynamic to static stiffness) – Eq. (15). It defines the possibility of achieving effective vibration damping for resilient mats. Because of the known property of elastomeric materials being rate-dependent both bedding modulus as well as stiffness modulus are determined for various frequencies of excitation.

$$r(f) = \frac{C_{\text{dyn}}(f)}{C_{\text{stat}}(f)}$$

(15)

The closer to 1 the value of dynamic stiffening ratio is (for $r(f) > 1$), the better vibration isolation properties has the test piece (e.g. resilient mat).

4.3. Loss factor

Loss (dumping) factor $\eta$ can be defined as tangent of phase shift angle $\varsigma$ between the applied force and the resulting deformation (see Eq. 8). The value of the loss factor is one measure of damping energy emitted from an object excited to vibrations (e.g. rails) to its environment. It also device the ratio of energy dissipated to energy expended. The bigger the loss factor value is, the better vibro-acoustic isolator mat is.

Insertion loss $D_I$ is also damping measurement and can be defined as difference in vibration between system without resilient element and with such element (e.g. resilient mat). The bigger value of insertion loss is, the more effective damping system is [10-12].

5. Conclusions

Requirements for products and systems used for vibration and acoustic isolation should correspond with their functions in railway track. Effectiveness of vibration and secondary noise isolation, as well as the working life of railway track should be taken into account in that matter. In order to accomplish these needs European standards containing requirements to be met by the resilient mats and including the national German standards from DIN 45673 series should be developed.

References

[3] DIN 45673-7:2010-08 Mechanical vibration - Resilient elements used in railway tracks - Part 7: Laboratory test procedures for resilient elements of floating slab track systems.