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All 4-connected Line Graphs of Claw Free Graphs Are Hamiltonian Connected

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Thomassen conjectured that every 4-connected line graph is hamiltonian. Here we shall see that 4-connected line graphs of claw free graphs are hamiltonian connected. © 2001 Academic Press

1. INTRODUCTION

Thomassen conjectured that every 4-connected line graph is hamiltonian [12]. By [11], this conjecture is equivalent to the conjecture of Matthews and Sumner stating that every 4-connected claw free graph is hamiltonian [8].

So far it is known that every 7-connected line graph is hamiltonian connected [16], and that every line graph of a 4-edge-connected graph is hamiltonian connected [15]. Thomassen's conjecture has also been proved to be true for 4-connected line graphs of planar simple graphs [6].

Here we prove that if G is a graph such that every vertex of degree 3 is on a triangle then $L(G)$ is hamiltonian connected if $L(G)$ is 4-connected. From this it follows that all 4-connected line graphs of claw free graphs are hamiltonian connected. It also implies that every hourglass free 4-connected line graph is hamiltonian connected, which extends a recent result of [2] where it was proved that these graphs are hamiltonian.

All graphs considered here are supposed to be finite, undirected, and may contain multiple edges but no loops. We call a graph *simple* if it contains no multiple edges. If we want to emphasize that multiple edges may occur we use the term *multigraph*. The set of edges between two vertices x,

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y of a multigraph $G=(V, E)$ will be denoted by $[x, y]_G$. If $|[x, y]_G|=1$ then we use the symbol $[x, y]_G$ for the element of $[x, y]_G$ as well. We say that G' is obtained from G by *adding a new edge e* between two distinct vertices x and y if $e\in[x, y]_{G'}$ and $G'-e=G$ holds (where $G'-e:=(V(G'), E(G')-\{e\}))$. For $e\in[x, y]_G$ let $V(e):=\{x, y\}$. The *degree* of a vertex x in a multigraph G is the number of edges in G incident with x; the *parity* of x in G is the parity of its degree in G. If G is connected and all vertices of G have even degree then G is called an Euler graph. For any non-negative integer k, we define $V_k(G):=\{x\in V(G): x \text{ has degree } k \text{ in } G\}$ and $V_{\geqslant k}(G):=\bigcup_{i=k}^{\infty} V_i(G)$. We say that four vertices form a *claw* in G if they induce a $K_{1,3}$, and we say that five vertices form an *hourglass* if they induce a graph $K_{1,4}+e+f$, where the edges e, f match the vertices of degree 1 in the $K_{1,4}$.

Let G be a connected graph. A set $T\subseteq E(G)$ is called an *edge cut* if $G-T$ is not connected; we call T *trivial*, if at least one component of $G-T$ consists of a single vertex, and we call T *nontrivial* otherwise. We call G *k-edge-connected* if it has no edge cut with less than k edges. In particular, a graph K_1 is k-edge-connected for every k.

To *contract* an induced subgraph H in a multigraph G means to delete $V(H)$ and all edges incident with it from G, and then add a new vertex u, and, for each edge e in G between a vertex in $V(G)-V(H)$ and a vertex in $V(H)$, add precisely one *corresponding* edge $\phi(e)$. If G' denotes the graph obtained in that way then $E(G-V(H))=E(G'-\{u\})$, and ϕ can be extended to a bijective map between $E(G)-E(H)$ and $E(G')$ by setting $\phi(e):=e$ for all $e\in E(G-V(H))$. We say that an edge set $E\subseteq E(G-E(H))$ and an edge set $E'\subseteq E(G')$ *correspond* to each other if $\phi(E)=E'$. In almost all cases it is convenient to consider corresponding edges as equal. To *contract* and edge e means to contract $G(V(e))$.

For terminology not defined here we refer to [1] or [3].

2. PRELIMINARY RESULTS

An ancestor of Lemma 2 below has first been proved by Thomassen. It is based on the well known criterion of Nash-Williams [9] and Tutte [14] for the existence of k edge-disjoint spanning trees: A graph G contains a system of k edge-disjoint spanning trees if and only if for each partition $\mathscr{P}$ of $V(G)$ the number $\|G/\mathscr{P}\|$ of edges that join vertices in different sets of $\mathscr{P}$ is at least $k\cdot(|\mathscr{P}|-1)$.

It is easy to see that if G is $2k$-edge-connected then G satisfies also the latter condition. It is even possible to remove any k edges of a $2k$-edge-connected graph, and still the remainder will satisfy it. This has been used first by Zhan in [15] in order to prove that a line graph of a 4-edge-connected graph is hamiltonian connected. For the sake of completeness, we add the proofs here.

LEMMA 1. *For any three distinct edges e, f, g of a 4-edge-connected graph G there exist two edge-disjoint spanning trees S, T such that $e, f \notin E(S) \cup E(T)$ and $g \in E(S)$.*

Proof. For each partition of $V(G-\{e, f\}) = V(G)$ we have $\|(G-\{e, f\})/\mathscr{P}\| \geqslant \|G/\mathscr{P}\| - 2 \geqslant 2 \cdot |\mathscr{P}| - 2$. Due to Tutte's [14] and Nash-Williams's [9] theorem, this is a necessary and sufficient condition for the existence of two edge-disjoint spanning trees S', T in $G-\{e, f\}$. Without loss of generality, $g \notin E(T)$. If $g \notin E(S')$ then let $S := (S' - g') + g$, where g' is any nonseparating edge of $S' + g$ distinct from g, otherwise let $S := S'$. Clearly, S, T is a pair of trees as required. ∎

LEMMA 2. *For any three distinct edges e, f, g of a 4-edge-connected graph G there exists a spanning Euler subgraph F such that $e, f \notin E(F)$ and $g \in E(F)$.*

Proof. Let G be 4-edge-connected and $e, f \in E(G)$. By Lemma 1 there exist two edge-disjoint spanning trees such that e, $f \notin E(S) \cup E(T)$ and $g \in E(S)$. There exists a system of paths in T such that each vertex having odd degree in S occurs exactly once as an endvertex. A vertex in the symmetric difference U of this system has odd degree if and only if it has odd degree in S, and thus $T + U$ is a spanning Euler subgraph of G with the required properties. ∎

COROLLARY 1 [15]. *Every line graph of a 4-edge-connected graph is hamiltonian connected.*

Proof. Let e, f be distinct edges in a 4-edge-connected graph G. Let $G+g$ be obtained from G by adding a new edge g between a vertex of e and (a different) one of f. Then $G+g$ is 4-edge-connected, too. By Lemma 2, $G+g$ contains a spanning Euler subgraph F with $g \in E(F) \subseteq E(G+g) - \{e, f\}$. So there exists an edge dominating Euler trail in $G-\{e, f\}$ between the endvertices of g. Consequently, there exists an edge dominating Euler trail starting with edge e and ending with edge f. From this we easily obtain a hamiltonian path with endvertices e, f in $L(G)$. ∎

According to [7], we call a multigraph $(k+\frac{1}{2})$-*edge-connected* if it is k-edge-connected and every edge cut of size k is trivial. As for k-edge-connectivity, the contraction of an edge also preserves $(k+\frac{1}{2})$-edge-connectivity:

LEMMA 3. *Let G be a* $(k+\frac{1}{2})$*-edge-connected graph. Then the contraction of any edge yields also a* $(k+\frac{1}{2})$*-edge-connected graph.*

Proof. Let G' be obtained from G by contracting an edge. Clearly, G' is k-edge-connected. Since any nontrivial edge cut of G' corresponds to a nontrivial edge cut of G it follows that G' contains no nontrivial edge cut of size k as well. ∎

We now proceed to prove our main theorem.

3. THE MAIN THEOREM

THEOREM 1. *Let G be a* $(3+\frac{1}{2})$*-edge-connected graph such that every vertex of degree* 3 *is on an edge of multiplicity at least* 2 *or on a triangle.*

Then for each pair e, f of distinct edges of G there exists a connected subgraph F with precisely two vertices a, b of odd degree such that

1. $a \in V(e)$, $b \in V(f)$, $e, f \notin E(F)$,
2. $V(G) - (V(e) \cup V(f)) \subseteq V(F)$,
3. $V(e) \subseteq V(F)$ *if* $V(e) \nsubseteq V_3(G)$,
4. $V(f) \subseteq V(F)$ *if* $V(f) \nsubseteq V_3(G)$, *and*
5. $V(e) \subseteq V(F)$ *or* $V(f) \subseteq V(F)$.

Proof. Throughout the proof, corresponding edges are considered to be equal. Let F be a connected subgraph as in the assertion. For brevity, we call (F, a, b) an (e, f)-*etrail*.

First note that the contraction of an edge and thus the contraction of any connected subgraph H yields again a graph that fulfills the conditions of the theorem.

The assertion is true for $|G| \leqslant 4$. Let us assume that G is a minimal counterexample to the assertion. We shall see that several configurations cannot occur in G. By excluding them we shall end up in a situation that allows us to argue similarly to the proof of Corollary 1.

If G' arises from contracting some connected subgraph H, $|H| \in \{2, 3\}$, to a single vertex u, and if e, f are edges in G' then G' has an (e, f)-etrail (F, a', b') by choice of G. If $a' \neq u$ let $a = a'$, otherwise let a be the vertex in $V(e) \cap V(H)$. If $b' \neq u$ then let $b = b'$, otherwise let b be the vertex in $V(f) \cap V(H)$. We call (F, a, b) an (e, f)-*pretrail* with respect to H. Note that $a = a'$ or $b = b'$ holds, since $a' \neq u$ or $b' \neq u$. Moreover, if e is incident with some vertex of H then $V(e) \nsubseteq V_3(G')$, since u has degree at least 4 in G' (by $|G| \geqslant 5$). Hence $V(e) \subseteq V(F)$ in this case. Similarly it follows $V(f) \subseteq V(F)$ if $V(f) \cap V(H) \neq \varnothing$.

CLAIM 1. *Two vertices x, y are not linked by more than one edge unless they are linked by e or f.*

Otherwise let (F, a, b) be an (e, f)-pretrail with respect to $G(\{x, y\})$. If $\{a, b\} \cap \{x, y\} = \emptyset$ then x, y have the same parity in F. So we may add one or two edges of $[x, y]$ to F in order to link x, y and to get even parity at x, y. If $a = x$ then x, y have distinct parities. So we may add one or two edges of $[x, y]$ to F in order to link x, y and to get odd parity at x and even parity at y. Denoting by F^+ the subgraph obtained in that way, (F^+, a, b) is an (e, f)-etrail. This proves Claim 1.

CLAIM 2. *Three vertices x, y, z do not induce a triangle unless two of them are linked by e or f.*

Otherwise let (F, a, b) be an (e, f)-pretrail with respect to $G(\{x, y, z\})$.

Case 1. If $\{a, b\} \cap \{x, y, z\} = \emptyset$ then either all vertices x, y, z have even parity, or exactly two have odd parity in F.

Case 2. If $a = x$ then either a has odd parity and $\{y, z\}$ have the same parity, or a has even parity and the others have distinct parities in F.

In each case we may add two or three edges of the triangle x, y, z in order to link x, y, z and in order to make x, y, z even (Case 1), or to make x odd, y, z even (Case 2). Denoting the subgraph obtained in that way by F^+, (F^+, a, b) is an (e, f)-etrail.

This proves Claim 2.

In both Claims 3 and 4, we consider the following situation. Let x be a vertex of degree 3 with three distinct neighbors u, y, z. By assumption, x is on a triangle Δ, whose vertices are, without loss of generality, x, y, and z. Let $g = [u, x]$. By Claim 2 we may assume that Δ contains at least one of the edges e, f.

CLAIM 3. *If $e \in [y, z]$ then $f \notin [y, z]$.*

Suppose that $e, f \in [y, z]$. Consider an $([x, y], [x, z])$-pretrail (F, a, b) with respect to $G(\{y, z\})$. Since $x \in \{a, b\}$ and $\{a, b\} - \{x\} \subseteq \{y, z\}$, x and one of y, z is odd in F. Without loss of generality, y has odd degree in F, and so $F + [x, z]$ is connected. Consequently, either $(F + [x, z], y, z)$ or $(F + [x, z], z, y)$ is an (e, f)-etrail. This proves Claim 3.

CLAIM 4. *If $e \in [y, z]$ then $f \in \Delta$.*

Suppose that $e \in [y, z]$ and $f \notin \Delta$. Let (F, a, b) be an (g, f)-pretrail with respect to Δ. Let $i = [x, y]$, $j = [x, z]$.

Case 1. If u is even in F then $a = x$ and $b \notin \Delta$ follows. So y, z have distinct parities in $F + i + j$, which implies that either $(F + i + j, y, b)$ or $(F + i + j, z, b)$ is an (e, f)-etrail.

Case 2. If u is odd in F then $a = u$. Without loss of generality, we may assume that y is in the same component as u (otherwise z must be, and we swap the roles of y, z). So $F + g + j$ is connected. If $b \notin \Delta$ then y, z have the same parity in F and thus distinct parities in $F + g + j$—hence either $(F + g + j, y, b)$ or $(F + g + j, z, b)$ is an (e, f)-etrail of G. If $b \in \Delta$ then y, z have distinct parities in F and thus the same parity in $F + g + j$. If they have both odd parity, then either $(F + g + j, y, z)$ or $(F + g + j, z, y)$ is an (e, f)-etrail, otherwise $F + g$ is connected (since all vertices of $F + g + j$ are even), and so either $(F + g + i, y, z)$ or $(F + g + i, z, y)$ is an (e, f)-etrail.

This proves Claim 4.

Claim 5. *If x is a vertex of degree* 3 *in G then every edge of multiplicity at least* 2 *and every triangle that contains x has to be incident in x with e or f.*

If x is on an edge of multiplicity at least 2 then this follows from Claim 1. Otherwise, x is on a triangle Δ with vertices x, y, z. By Claim 2, we may assume that e connects two of these vertices. If $x \in V(e)$, we are done. So $e \in [y, z]$. By Claim 4 we know $f \in \Delta$. Again we may assume $x \notin V(f)$. But then $e, f \in [y, z]$, contradicting Claim 3. This proves Claim 5.

By Claim 5 it follows that $V_3(G) \subseteq V(e) \cup V(f)$, and so G has at most four vertices of degree 3.

Case 1. $V(e) \nsubseteq V_3(G)$ and $V(f) \nsubseteq V_3(G)$. We may choose two distinct vertices $a \in V(e)$, $b \in V(f)$, such that the graph G^+ obtained from G by adding a new edge g between a, b has no vertices of degree 3. Since G^+ is 4-edge-connected, there exists a spanning Euler subgraph F of G^+ containing g and neither e nor f by Lemma 2. So $(F - g, a, b)$ is an (e, f)-etrail of G.

Case 2. $V(e) \subseteq V_3(G)$ and $|V(e) \cap V(f)| = 0$. Let G^+ be obtained from G by adding two independent new edges between $V(e)$ and $V(f)$, say $[a, b]$ and $[a', b']$. Since G^+ is 4-edge-connected, there exists a spanning Euler subgraph F of G^+ containing $[a, b]$ but neither $[a', b']$ nor f by Lemma 2.

Case 2a. If $e \notin F$, too, then $(F - [a, b], a, b)$ is an (e, f)-etrail of G.

Case 2b. If $e \in F$ and e is not a bridge in $F - [a, b]$ then $(F - [a, b] - e, c, b)$ is an (e, f)-etrail of G, where c is the vertex in $V(e) - \{a\}$.

Case 2c. If $e \in F$ and e is a bridge in $F - [a, b]$ then let C be the component of $F - [a, b] - e$ containing the two vertices of odd degree in $F - [a, b] - e$, namely c and b, where c denotes the vertex in $V(e) - \{a\}$, and let C' be the component containing a. Since a has degree 3 in G, e must be either on an edge of multiplicity at least 2 or on a triangle by Claim 5. The first case may not occur since $|G| \geqslant 4$. For the second case, let z be a common neighbor of a, c. If a has degree 0 in $F - [a, b] - e$ then $C' = \{a\}$ and $(F - a, c, b)$ will serve as an (e, f)-etrail. If a has degree 2 in $F - [a, b] - e$ then $[a, z] \in F$ follows, and since e is a bridge in $F - [a, b]$, $[c, z] \notin F$ holds; but then $(F - [a, b] - e - [a, z] + [c, z], a, b)$ is an (e, f)-etrail.

Case 3. $V(e) \subseteq V_3(G)$ and $|V(e) \cap V(f)| = 1$. We may choose a new edge $[a, b]$ between the two vertices not in $V(e) \cap V(f)$, choose a new edge $[a', b']$ with the same endpoints as f, and proceed as in Case 2.

Case 4. $V(e) = V(f) \subseteq V_3(G)$. Let $G + g$ be obtained from G by adding a new edge g between the endpoints a, b of e and f. Since $G + g$ is 4-edge-connected, there exists a spanning Euler subgraph F containing g but containing neither e nor f by Lemma 2. Consequently, $(F - g, a, b)$ is an (e, f)-etrail of G. ▮

Before continuing with applications to line graphs let us consider the following class of examples, which show that in general, given a graph G which satisfies the conditions of Theorem 1, we can not expect two edge-disjoint spanning trees in G. If there were such trees then we could use the technique of the proofs of Lemma 2 and Corollary 1 in order to give a more elegant proof for Theorem 1.

Let $m \geqslant 2$ and let C be a cycle of length m, D be a cycle of length $2m$ vertex-disjoint to C, and F be a 1-factor of D. For each vertex x in C choose an edge $f_x \in F$ such that $f_x \neq f_y$ for $x \neq y$ in $V(C)$. Let G_m be the graph obtained from C and D by connecting each x to the vertices of $V(f_x)$. Let T be a minimal edge cut of G_m. If T separates two vertices x, y in C then $|T \cap E(C)| \geqslant 2$ and $|T - E(C)| \geqslant 2$, since C and $G_m - E(C)$ are 2-edge-connected. If T separates two edges f, g in $E(D)$, then similarly $|T \cap E(D)| \geqslant 2$ and $|T - E(D)| \geqslant 2$. Therefore, if T has size at most 3, then C is contained in one of the two components H, H' of $G - T$, say in H, and also H or H' does not contain an edge of D. Since C has at least $2m \geqslant 4$ neighbors, there exists an $x \in H \cap D$. If H contains no edge of D then all four edges between the neighbors of x in D and $\{x\} \cup C$ would be in T, a contradiction. So $H' \subseteq D$ contains no edge of D and so H' consists of a single vertex of degree 3. It follows that G is $(3 + \frac{1}{2})$-edge-connected. Furthermore, every vertex of degree 3 is on a triangle, so G_m satisfies the conditions of Theorem 1.

Since $\|G_m\| = \frac{(3\cdot 2\cdot m+4\cdot m)}{2} = 5\cdot m \not\geqslant 2\cdot(|G_m|-1) = 6\cdot m-2$ for $m\geqslant 3$, it follows that for $m\geqslant 3$ we may not expect a system of two edge-disjoint spanning trees in G_m.

4. APPLICATIONS

In order to apply Theorem 1 to line graphs, we need the following.

LEMMA 4. *Let G be a graph such that $L(G)$ is noncomplete and* 4*-connected.*

Then the graph G' obtained from G by deleting all vertices of degree at most three with precisely one neighbor and then replacing each vertex of degree 2 *with a new edge between its neighbors is $(3+\frac{1}{2})$-edge-connected.*

Furthermore, every vertex of degree 3 *in G' has to have degree* 3 *in G as well, and is in G' on an edge of multiplicity at least* 2 *or on a triangle if it is in G.*

Proof. Since $L(G)$ is 3-connected, the set of vertices of degree at least 3 is 3-edge-connected in G. Therefore, G' is 3-edge-connected. Since a nontrivial edge cut of G' always yields a nontrivial edge cut of G, which induces a vertex cut in $L(G)$, there can not be a nontrivial edge cut of G' of size 3.

From the noncompleteness of $L(G)$ it follows that if x has only one neighbor y in G then y has degree at least 4 in $G-x$. The second operation mentioned above preserves the degrees at all vertices and transforms an edge of multiplicity at least 2 or a triangle into an edge of multiplicity at least 2 or a triangle. Furthermore, the neighbors of a vertex of degree 2 have degree at least 4, since $L(G)$ is 4-edge-connected and noncomplete. The second part of the assertion follows from this. ∎

Now we can apply Theorem 1 to a superclass of the class of 4-connected line graphs of claw free graphs.

COROLLARY 2. *Let G be a graph such that $L(G)$ is* 4*-connected and every vertex of degree* 3 *in G is on an edge of multiplicity at least* 2 *or on a triangle of G.*

Then $L(G)$ is hamiltonian connected.

Proof. Let e, f be two arbitrary edges in G. Let G' be as in Lemma 4. We may choose distinct e', f' such that the following conditions are fulfilled:

1. $e' = e$ if $e \in E(G')$.

2. $f' = f$ if $f \in E(G')$,

3. if a vertex x of degree 2 with two neighbors in G is incident with e or f then e' or f' is the edge x has been replaced with, $V(e) \cap V(e') \neq \emptyset$, $V(f) \cap V(f') \neq \emptyset$.

By Theorem 1, there exists a connected edge dominating subgraph F' of G' not containing e', f' with precisely two odd vertices $a \in V(e')$, $b \in V(f')$, and $V_{\geqslant 4}(G') \subseteq V(F)$. Since every vertex of degree at most 2 in G must be adjacent to some vertex of degree at least 4, there exists a connected edge dominating subgraph F of G not containing e, f, e', f', and with precisely two odd vertices, namely a, b. Consequently, there must be a hamiltonian path between the vertices induced by e, f in $L(G)$. ∎

Clearly, Corollary 2 implies Corollary 1. Since every claw free graph satisfies the conditions of Corollary 2, we obtain the following Corollary.

COROLLARY 2. *Every 4-connected line graph of a claw free graph is hamiltonian connected.*

In [2] the class of 4-connected hourglass free claw free graphs has been considered. An important step in the proof for hamiltonicity of such graphs was the following.

COROLLARY 4. *Every 4-connected hourglass free line graph is hamiltonian connected.*

Proof. Let G be a graph such that $L(G)$ is noncomplete, 4-connected, and hourglass free.

Suppose that there is a vertex x of degree 3 which is neither on an edge of multiplicity at least 2 nor on a triangle. Let $e = [x, y], f, g$ be the edges incident with x. Since $L(G)$ is noncomplete, y has degree at least 3. Let $f' \neq g'$ be edges distinct from e incident with y. Since x is not on a triangle, f' and g' are not incident with f or g, and so e, f, g, f', g' form an hourglass in $L(G)$.

So every vertex of degree 3 is either on an edge of multiplicity at least 2 or on a triangle, and applying Corollary 2 accomplishes the proof. ∎

Let us finish by reformulating Theorem 1 without referring to the properties of the graph G from which the line graph considered there has been constructed.

A triangle Δ of a graph G is called *odd*, if there exists a vertex adjacent to precisely one or to all vertices of Δ. Odd triangles have been used in [10] in order to characterize the line graphs of simple graphs among the claw free graphs. (It is also possible to give a characterization of line graphs of multigraphs using this terminology, see [5].) We formulate the following corollary only for line graphs of simple graphs, although there is a variant for line graphs of multigraphs.

COROLLARY 5. *Suppose that G is a* 4*-connected line graph of a simple graph such that every odd triangle has precisely one edge that lies in some other triangle. Then G is hamiltonian connected.*

Proof. Let H be a simple graph with $L(H)=G$, suppose that $L(H)$ is non-complete, and that H is nonisomorphic to a K_4. Let x be a vertex of degree 3 in H. The edge neighborhood E' of x in H induces an odd triangle in $L(H)$ (since $N(N(x))-\{x\}\neq\emptyset$). There exists an edge in $E(H)-E'$ which is incident with at least two of the edges in E'; clearly, the endvertices of such an edge must be contained in $N(x)$, so x is on a triangle. Applying Corollary 2 accomplishes the proof. ∎

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