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Slope stability analysis under seismic load by vector sum analysis method

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Abstract: The vibration characteristics and dynamic responses of rock and soil under seismic load can be estimated with dynamic finite element method (DFEM). Combining with the DFEM, the vector sum analysis method (VSAM) is employed in seismic stability analysis of a slope in this paper. Different from other conventional methods, the VSAM is proposed based on the vector characteristic of force and current stress state of the slope. The dynamic stress state of the slope at any moment under seismic load can be obtained by the DFEM, thus the factor of safety of the slope at any moment during earthquake can be easily obtained with the VSAM in consideration of the DFEM. Then, the global stability of the slope can be estimated on the basis of time-history curve of factor of safety and reliability theory. The VSAM is applied to a homogeneous slope under seismic load. The factor of safety of the slope is 1.30 under gravity only and the dynamic factor of safety under seismic load is 1.21. The calculating results show that the dynamic characteristics and stability state of the slope with input ground motion can be actually analyzed. It is believed that the VSAM is a feasible and practical approach to estimate the dynamic stability of slopes under seismic load.

Key words: slope stability; vector sum analysis method (VSAM); seismic load; dynamic finite element method (DFEM)

1 Introduction

Stability analysis of slopes under dynamic loads is an important issue in geotechnical and earthquake engineering. The dynamic loads include natural and artificial loads. The natural seismic load is mainly deemed as a dynamic load, which plays a significant role in stability analysis of slopes.

The seismic responses of slopes are closely related to the characteristics of input ground motions and the dynamic properties of rock and soil media. Therefore, slope stability analysis under seismic load is more complex than that under static load. At present, there are roughly four methods for seismic stability analysis of slopes: pseudo-static method [1], Newmark sliding block analysis method [2], numerical analysis method [3–5], and testing method. The pseudo-static method is the simplest at the earliest applications [6–9], and so far, it is still widely applied. Actually, the seismic force is regarded as a constant, equivalent to a horizontal force and a vertical force in the pseudo-static method. The center of gravity of the potential sliding body is taken as the acting point of the equivalent forces, and the unstable direction of slope is considered as the direction of the equivalent forces. In most cases, according to the degree of seismic damage and the level of slope design, earthquake acceleration coefficients were determined with practical experiences. Based on the pseudo-static method, the Newmark sliding block analysis method was firstly put forward in 1965 in the fifth Rankin lecture, and the limited sliding displacement of sliding blocks was considered. Different from the pseudo-static method, this method depends on the displacement of sliding blocks, but not the minimum factor of safety. Analyses of dynamic stability of slopes in these two methods are simplified into static stability analysis. For this reason, the assumptions in these two methods have considerable differences in practical applications and it is hard to truly reflect dynamic characteristics and failure mechanism of sliding body under seismic load. Testing method can

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directly reflect weak geological features, damage mechanisms and stability states of slopes, and can be employed for seismic stability analysis. At present, only physical shaking table test [10] is used for slopes, limited to single-frequency input ground motion, simple surface and small scale. For large-scale shaking table and centrifuge tests of large-scale slopes with complex sliding surfaces, this method is rarely considered by using real ground motion. Different from testing method, dynamic responses and damage characteristics of slopes under seismic load can be simulated with numerical methods, such as finite element method (FEM) [3], finite difference method [4], boundary element method and discrete element method [5]. In these methods, the FEM is the earliest one that has been applied to seismic stability analysis of slopes. Due to the limitation of the FEM, some damage phenomena (e.g. liquefaction at large deformation, slipping, rolling, and large strain) existing in the process of ground motion may not be reasonably estimated, but vibration features and dynamic responses of slopes can be analyzed in terms of small deformation with the FEM in detail. Rich experiences have been accumulated in engineering applications. Therefore, the FEM has become one of the most important methods in seismic stability analysis of slopes.

The limit equilibrium method (LEM) and the finite element strength reduction method (FE-SRM) are the most commonly used methods in static stability analysis. The pseudo-static method is modified from the LEM in seismic stability analysis. In fact, the seismic load is equivalent to invariably horizontal and vertical forces, and the problem of seismic stability of slopes is simplified into a static problem in the pseudo-static method. Therefore, the factor of safety of slopes under seismic load can be calculated with the LEM, and that under seismic load can also be obtained with the FE-SRM. Different from the static problem, the DFEM can be used for dynamic stability analysis of slopes, wherein their dynamic characteristics are analyzed using reduction coefficients, and the equilibrium condition is estimated based on some key points' displacements or velocities [11]. Because of the iterative calculation in the strength reduction method, computing the static factor of safety is usually time-consuming.

Different from conventional methods, the VSAM can be adopted based on the current stress state, and it is clear in physical definition in terms of factor of safety. If the current stress state of slopes under external loads is known, the factor of safety of slopes can be quickly derived with the VSAM. Combining with the FEM, the VSAM can be used in seismic stability analysis of slopes. The time-history curve of the factors of safety of slopes can be easily obtained under seismic load with the VSAM, and the global stability of the slope can be evaluated by reliability theory.

2 The VSAM under seismic load

2.1 The VSAM

Different from the LEM and the FE-SRM, the VSAM is proposed based on current stress state and globally potential sliding direction. The factor of safety therein is defined as the ratio of projection of vector sum of ultimate resistance forces to that of current driving forces in the potential sliding direction.

Figure 1 shows a computational model for plane problems. In the figure, θ is the potential sliding direction of the slope; $\boldsymbol{\sigma}_{\mathrm{s}}$, $\boldsymbol{\sigma}_{\tau}$ and $\boldsymbol{\sigma}_{\mathrm{n}}$ are the resultant stress vector, shear stress vector and normal stress vector, respectively, provided by the sliding mass at point A on the potential slip surface.

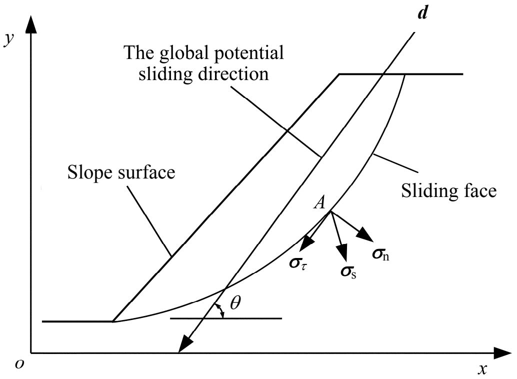


Fig.1 Sketch of potential sliding direction of whole slope.

The factor of safety can be deduced by using the VSAM as follows:

$$\boldsymbol{\sigma}_{\mathrm{s}} = \boldsymbol{\sigma}\boldsymbol{n} \tag{1}$$

$$\boldsymbol{\sigma}_{\mathrm{n}} = (\boldsymbol{\sigma}_{\mathrm{s}}\,\boldsymbol{n})\,\boldsymbol{n} \tag{2}$$

$$\boldsymbol{\sigma}_{\tau} = \boldsymbol{\sigma}_{\mathrm{s}} - \boldsymbol{\sigma}_{\mathrm{n}} \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor at point A on the potential sliding face s, and $\boldsymbol{n}$ is the unit normal vector.

The normal stress at the corresponding point A' applied by the bedrock can be expressed as

$$\boldsymbol{\sigma}'_{\mathrm{n}} = -\boldsymbol{\sigma}_{\mathrm{n}} \tag{4}$$

For simplicity, the tensile stress is considered positive and the compressive stress is negative.

The factor of safety can be obtained with the VSAM:

$$K = \frac{R}{T} \tag{5}$$

$$T = \int_{s} (\boldsymbol{\sigma}_{\mathrm{s}} \boldsymbol{d}) \mathrm{d}s \tag{6}$$

$$R = \int_{s} \boldsymbol{\sigma}'_{\mathrm{s}} (-\boldsymbol{d}) \ \mathrm{d}s \tag{7}$$

where R and T are the total anti-sliding force and the total sliding force in the potential sliding direction, respectively; and $\boldsymbol{d}$ is the global potential sliding direction (Fig.1).

The maximum shear stress can be expressed as

$$\boldsymbol{\sigma}'_{\tau} = -(c - \boldsymbol{\sigma}_{\mathrm{n}} \tan\varphi) \boldsymbol{d}_{\mathrm{r}} \tag{8}$$

where c is the cohesion of the soil, φ is the friction angle, and $\boldsymbol{d}_{\mathrm{r}}$ is the direction of unit shear stress at any point on the sliding face.

The maximum stress vector is

$$\boldsymbol{\sigma}'_{\mathrm{s}} = \boldsymbol{\sigma}'_{\tau} + \boldsymbol{\sigma}'_{\mathrm{n}} \tag{9}$$

According to the Mohr-Coulomb yield criterion, the global potential sliding direction can be written as

$$\boldsymbol{d} = \frac{-\int_{s} (c - \sigma_{\mathrm{n}} \tan\varphi) \boldsymbol{d}_{\mathrm{r}} \mathrm{d}s}{\left\| \int_{s} (c - \sigma_{\mathrm{n}} \tan\varphi) \boldsymbol{d}_{\mathrm{r}} \mathrm{d}s \right\|} \tag{10}$$

It can be seen from the above equations that the VSAM is similar to the conventional methods. Each of these methods has advantages and disadvantages. The LEM has been widely adopted mainly due to its simplicity and applicability. However, this method does not take into account the formation of slopes and the initial stress state before excavation or filling, and also suffers from the assumption limitation of an interslice shear force. Although the FE-SRM can locate critical failure surface automatically without trial and error, the SRM suffers from time-consuming and instability criterion, which is sensitive to the nonlinear solution algorithm/flow rule for some special cases. The VSAM proposed in this paper considers the vector characteristics of forces based on the current stress state of slopes, instead of their critical states. Also, it is applicable to the shear failure of slopes.

2.2 Procedure of seismic stability analysis with the VSAM

Based on the dynamic stress state of slopes, the factor of safety and the global potential sliding direction of sliding body at any moment under seismic load can be obtained with the VSAM. The procedure of seismic stability analysis is illustrated in Fig.2, which is similar to that of static stability analysis [12].

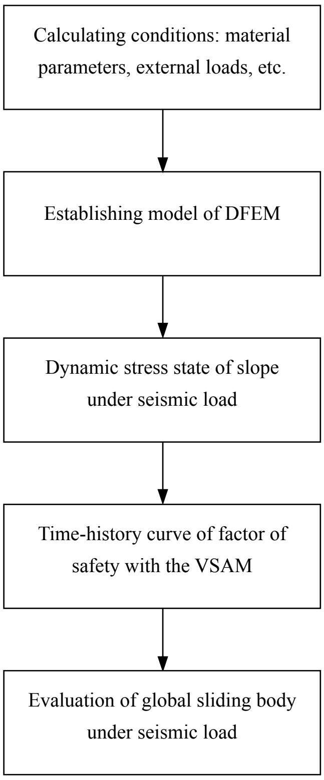


Fig.2 Procedure of seismic stability analysis of a slope with the VSAM.

3 Case study

The VSAM is employed to examine the dynamic response of a homogeneous slope with specified slip surface under seismic load. The geometry and the critical slip surface of the slope are illustrated in Fig.3, in which the critical slip surface is searched with the Morgenstern-Price method.

3.1 Computational conditions

The material parameters of the slope are shown in Table 1. Given the fact that the VSAM is not sensitive

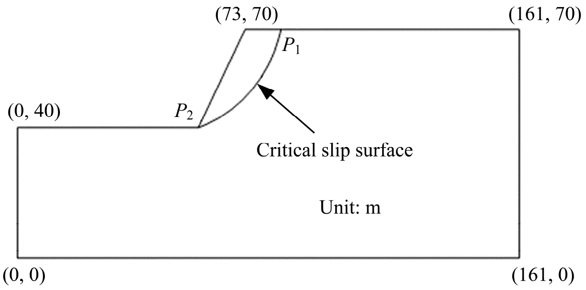


Fig.3 Geometry and critical slip surface of a homogeneous slope.

Table 1 Material parameters of a homogeneous slope.

c (kPa)	φ(°)	γ (kN/m^3)	E (GPa)	ν
80	30	22.0	0.138	0.35

to elastic stress state or elastoplastic stress state of the slope [13], only the linear elastic model is used for the dynamic stability analysis of the homogeneous slope.

For the DFEM and static FEM, the numerical procedures and the boundary conditions are quite different. Unless the domain of problems is large enough, the stress wave will be reflected back to the model upon reaching the solid boundaries if the solid boundary is used in dynamic calculation. In order to overcome such an unreasonable disturbance, the infinite element boundary is commonly used in the DFEM. The actual circumstance of seismic wave energy absorbed in far field can be simulated with the infinite element boundary. The zero displacement at infinity can also be reasonably simulated. Furthermore, it can decrease the number of elements and save calculating time. The dynamic analysis model with plane strain quadrilateral finite-infinite elements is illustrated in Fig.4. The numbers of elements and nodes are 1 566 and 1 715, respectively.

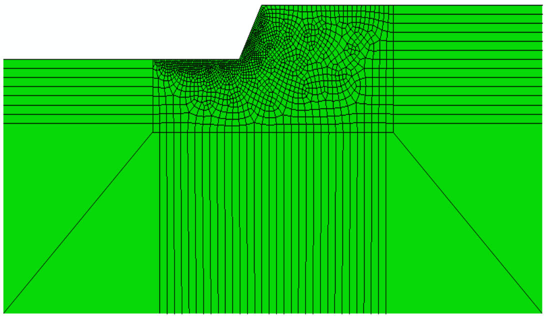


Fig.4 Dynamic analysis model.

Due to lack of earthquake data, the time-history acceleration of the El-Centro earthquake occurring in California, USA, in 1940, is adopted as the input ground motion, as shown in Fig.5. The direction of this earthquake is SN, the magnitude, M, is 6.7, the epicentral distance is 9.3 km, the maximum acceleration is 2.49 m/s^2, and the duration is 25 s. The most dangerous situation of seismic load is considered, and the acceleration time-history of the El-Centro earthquake can be simultaneously employed to simulate the horizontal shear wave and the disturbance of vertical wave at the bottom of the model. According to the analytical results of modes at vibration, the damping coefficients α and β are determined by two natural frequencies, with damping ratio 0.05 of the slope soil.

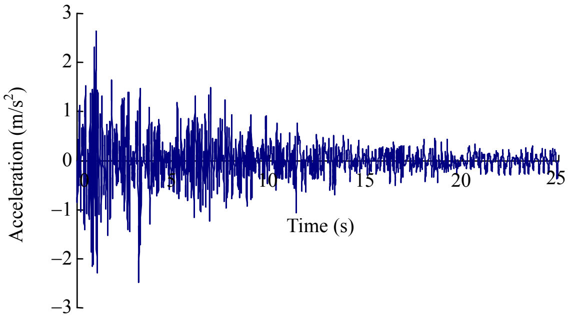


Fig.5 Time-history curve of acceleration in SN direction of El-Centro earthquake.

The commercial software ABAQUS is used in dynamic stability analysis in this paper, and the direct integral method is used to solve the dynamic balance equation. The dynamic balance equation in the DFEM can be expressed as

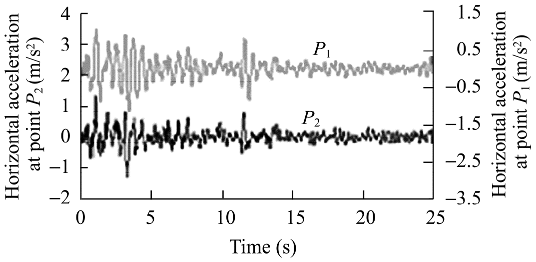
$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{M}\ddot{\boldsymbol{x}}_g \tag{11}$$

where $\boldsymbol{M}$ is the global mass matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{K}$ is the total stiffness matrix, $\boldsymbol{u}$ is the displacement vector at each node, and $\ddot{\boldsymbol{x}}_g$ is the acceleration vector at each node.

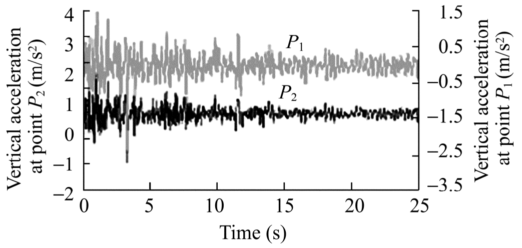
3.2 Results of the DFEM

According to the computing results of the DFEM, the time-history curves of accelerations at points P_1 and P_2 on the critical slip surface (Fig.3) are shown in Figs.6(a) and (b), respectively, and the time-history curves of the maximum and minor principal stresses are shown in Figs.6(c) and (d), respectively.

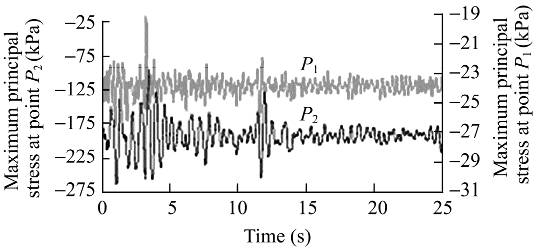
As shown in Fig.6, the stress responses at any position of the sliding body in a homogeneous slope are not the same under seismic load. In fact, this phenomenon will occur similarly in a heterogeneous



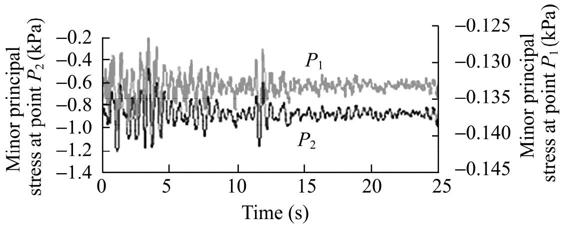
(a) Horizontal acceleration.



(b) Vertical acceleration.



(c) The maximum principal stress.



(d) The minor principal stress.

Fig.6 Time-history curves at points P_1 and P_2 on the critical slip surface.

slope. Therefore, the pseudo-static method is not suitable for dynamic stability analysis of the slope.

3.3 Results and analysis with the VSAM

Figure 7 shows the resultant sliding force, $\int_s \sigma_s \, ds$, and the resistance force, $\int_s \sigma_s' \, ds$, of the sliding body. The time-history curves of the factor of safety and the global potential sliding direction are shown in Fig.8.

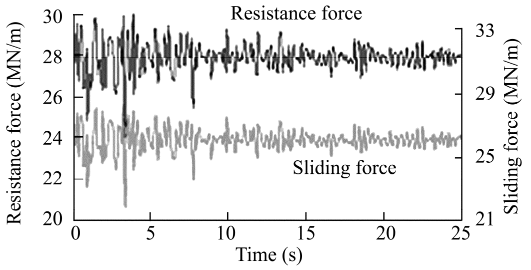


Fig.7 The seismic responses of whole sliding body.

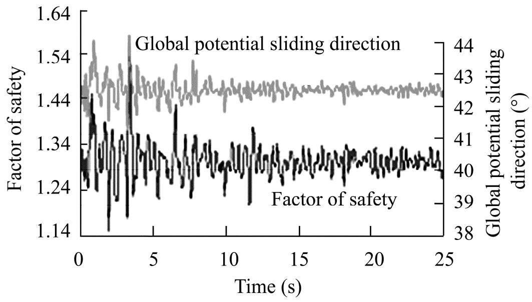


Fig.8 Calculating results with the VSAM under seismic load.

It can be observed from Fig.7 that the dynamic responses of the resistance force and the sliding force of the sliding body are basically the same. According to the theory of the VSAM, the resistance force and the sliding force of the entire sliding body are calculated with the same normal force, but with different shear forces. This is the reason that they have similar dynamic responses under seismic load. As shown in Fig.8, the predicted maximum factor of safety by the VSAM is 1.54, the angle between the sliding direction and the horizontal direction is 44.2°, and the appearing time is 3.3 s. On the other hand, the minimum predicted factor of safety is 1.15, and the sliding angle and its appearing time are 41.6° and 1.9 s, respectively.

4 Estimation of dynamic global stability

Up to now, there is no common sense at home and abroad for evaluations of dynamic stability of slopes. In practices, there are mainly two estimation indices, i.e. the factor of safety and the cumulative deformation. The factor of safety has been widely used to estimate the dynamic stability of slopes. Based on the time-history curve of the factor of safety, methods were proposed for dynamic stability assessment, including minimum mean factor of safety [14], minimum dynamic factor of safety [15], average factor of safety [16], and dynamic factor of safety based on reliability theory [17], etc. In minimum mean factor of safety method, 0.65 times of the maximum factor of safety is taken as the mean amplitude of vibration to evaluate the change in factor of safety with the ground motion time-history, and the minimum average factor of safety is defined as the static factor of safety minus the mean amplitude of vibration. In minimum dynamic factor of safety method, the minimum factor of safety is used to estimate the dynamic stability of slopes. In average factor of safety method, the factor of safety obtained by the evaluation criterion of equal areas of time-history curve, is taken as the estimation index of dynamic stability. These three estimation indices are all obtained by artificial mathematical process of time-history factor of safety, absent from sound theoretical basis and mechanical meaning. Therefore, these methods have clear shortages in estimating the dynamic stability of slopes.

In the dynamic factor of safety method based on the reliability theory, according to the mean factor of safety, standard deviation and reliability index, the dynamic factor of safety can be expressed as

$$F_{\mathrm{R}} = \mu_{\mathrm{F}} - \beta_{\mathrm{r}}\sigma_{\mathrm{F}} \tag{12}$$

$$\mu_{\mathrm{F}} = \frac{1}{N}\sum_{i=1}^{N} F(t_i) \tag{13}$$

$$\sigma_{\mathrm{F}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(F(t_i) - \mu_{\mathrm{F}})^2} \tag{14}$$

where N is the total number of discrete time points under seismic load, $F(t_i)$ is the factor of safety at time t_i, and β_{r} is the reliability index corresponding to failure probability of slopes. The derivation process of dynamic factor of safety (Eq.(11)) can be found in Ref.[18].

Different from other methods, the conventionally quantitative analysis and risk analysis are considered in the dynamic factor of safety method, and the dynamic factor of safety can be obtained by the reliability theory. Risk and economic effects of slopes are comprehensively taken into account. As long as the probability of failure of slope is small and less than the allowable value, the dynamic factor of safety can be taken as reliable. If the factor of safety follows the normal distribution, the reliability index β_{r} and probability of failure P_{f} satisfy the following relation:

$$P_{\mathrm{f}} = 1 - \Phi(\beta_{\mathrm{r}}) \tag{15}$$

where $\Phi(\cdot)$ is the standard normal distribution function. For the sake of safety, the probability of failure determined in this paper is 0.01, and the corresponding reliability index is 2.33. Accordingly, the dynamic factor of safety can be obtained based on Eq.(12).

The factor of safety of the homogeneous slope under gravity only is 1.30 with the VSAM, and the angle between the global sliding direction and the horizontal direction is 42.5°. The factor of safety is 1.33 with the Morgenstern-Price method. According to the time-history curve of the factor of safety with the VSAM, different estimation indices for the dynamic stability of the slope under seismic load are listed in Table 2.

Table 2 Results of dynamic stability analysis.

Minimum dynamic factor of safety	Minimum mean factor of safety	Average factor of safety	Dynamic factor of safety
1.15	1.20	1.30	1.21

As shown in Table 2, it is basically the same for the minimum mean factor of safety and the dynamic factor of safety, but the minimum dynamic factor of safety is the smallest. It is unreasonable that the average factor of safety is equal to the factor of safety under gravity only. There is a certain theoretical basis for the calculation of dynamic factor of safety based on the reliability theory because risk analysis and probability of failure are connected. The dynamic factor of safety seems to be more reasonable and reliable than other estimation indices. Therefore, the factor of safety of the homogeneous slope in this paper can be considered to be about 1.21 under horizontal and vertical seismic loads.

5 Discussion

(1) Different from the conventional methods, the VSAM is based on the current stress state of slopes. Water pressure, seismic load and other external loads can be considered in the DFEM, and the stability of slope under water or other loads can be analyzed with the VSAM.

(2) The VSAM is applied to evaluating the stability of a homogeneous soil slope in this paper. Actually, this method can also be used in practical engineering, such as rock slopes. In general, the slip surface of a rock slope can be mainly determined by weak structures and no search of the slip surface in the domain is required for rock slopes. Given the purpose of this study, the critical slip surface is assumed to be known. However, the critical slip surface of slope under seismic load may vary with time and this issue will be addressed in the further studies.

(3) The probability of failure of the slope is considered to be 0.01 in this paper. However, it is not easy to determine the failure probability of slopes in practical engineering, which is related to the reliability index of slopes. So, the probability of failure or the reliability index β_{r} can be discussed and determined based on specific circumstance of the slope.

(4) The VSAM has been applied to practical engineering under static load. According to the above mentioned analyses, it is not difficult for the VSAM method to be applied in the slope engineering under seismic load, though further studies are needed.

6 Conclusions

(1) A new method, the VSAM, is employed for seismic stability analysis of slopes. This method is based on current stress state of slopes, and the factor

of safety obtained with this method has a clear physico-mechanical meaning.

(2) Combining with the DFEM, the procedure of seismic stability analysis with the VSAM is given, and then this method is applied to a homogeneous slope. The calculated results show that the dynamic characteristics and stability state of the slope can be actually simulated.

(3) The global stability of the homogeneous slope is analyzed under seismic load. The results show that the dynamic factor of safety based on reliability theory is more reasonable and reliable than other estimation approaches.

(4) The VSAM is a feasible and practical approach to estimate seismic stability of slopes. It is reasonable to believe that the application of this new method has a good prospect.

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