Mathl Modelling, Vol. 9, No. 7, pp. 547–552, 1987
Printed in Great Britain.

0270-0255/87 $3.00 + 0.00

EFFECTIVE PROGRAM PLANNING FOR MULTIPLE PROJECTS UNDER LIMITED RESOURCES

D. C. BRAUER,[1] G. NAADIMUTHU[2] and E. S. LEE[3]
[1]Jersey Central Power & Light, Morristown, NJ 07960, U.S.A.
[2]Fairleigh Dickinson University, Teaneck, NJ 07666, U.S.A.
[3]Kansas State University, Manhattan, KS 66506, U.S.A.

(Received April 1986)

Communicated by X. J. R. Avula

Abstract—This work deals with an analytical approach to effective planning based upon the relative economics of projects contained in a program. A mathematical model is formulated to maximize the total return over the planning horizon subject to various annual cash flow and resource constraints. Stochastic mixed-integer programming is applied to determine the optimal sequence of project planning activities. To illustrate the use of the proposed procedure, a numerical example is solved using the mathematical programming software package SCICONIC/VM in a VAX-11/750 computer system.

1. INTRODUCTION

Program planning can be defined as determining the integration of a set of pre-defined projects to accomplish over some required period of time with a given objective. Given this definition, several factors must be considered when a planned program needs to be developed. Items such as project independence and dependence, project duration, completion date, resource requirements and responsibilities have to be fully recognized and considered. Until these basic factors are determined and assessed, a realistic program cannot be planned and executed for implementation.

Further consideration in scheduling projects into a unified plan are individual project benefits and costs. The specific timing of a project in the defined planning horizon can result in different benefits or returns. At the same time, cost limitations can restrict implementation to a later period. If certain overall return goals as well as completion times are of importance, the decision maker has the problem of allocating available resources in the best manner. Thus, many conflicting factors have to be simultaneously balanced in preparing an effective planned program. Such conditions require the application of a much more sophisticated approach than traditional project planning techniques such as CPM and PERT. Formal mathematical modelling of the situation may be warranted to effectively aid in the development of a planned program. This paper presents one such model using stochastic mathematical programming. Both the model development and a numerical example are presented.

2. BACKGROUND

Traditional project planning techniques such as CPM and PERT aid in formulating a schedule of sequenced activities in order to determine the critical path of those activities required for timely project completion. This approach typically consists of evaluating and determining individual activity duration, usually in calendar time, and precedence. CPM uses deterministic estimates of activity duration while PERT establishes expected estimates based on stochastic assumptions.

A resultant CPM or PERT activity network will most likely contain both parallel and series events. Undertaking these events irrespective of other considerations and constraints can lead to schedule unattainment with added cost. It may be that project or program completion time is but one requirement surrounded by many others. In this case one major program objective should be identified subject to the relevant constraints and goals.

The major program objective could be in terms of cost or return, scheduled completion, resource allocation etc. Each project in the program may have an associated cost or return. Depending upon the value of the individual projects, it may be desirable to impose scheduling criteria to obtain cost or return benefits based on discounted effects. Alternatively, scheduled completion might be of the greatest concern. In this instance the objective might be to obtain some defined program completion date within or above available resources at some penalty. Another approach might be to allocate available resources, e.g. labor, over the planning horizon limited by an upper bound on completion. This assumes that completion time is not a critical factor.

Many other alternative situations can be envisioned that require consideration of several factors in developing a planned program schedule. Such situations will most likely be unique, and demand analysis beyond CPM and PERT. This paper addresses one such circumstance in which a variety of conditions exist that can affect program planning.

3. LITERATURE SURVEY

There have been many publications in the area of project/program planning and scheduling.

Thomas [1] studied scanning systems and environmental models by proposing a system of long-range scanning using the concept of a strategic environment that is amenable to managerial selection. Shrader *et al.* [2] reviewed and evaluated the empirical relationship between strategic planning and measures of organizational performance. The National Association of Bank Women developed and implemented a long-range plan to deal with the effects of a depressed economy, deregulation and changing technology [3]. Based on the inadequacy of computer-produced economic forecasts, Shell U.K. Ltd. undertook a multiscenario approach to corporate strategic planning [4].

Boschi [5] devised a method based on subjective probabilities and used statistical distributions to estimate success within the process of long-range strategic planning of exploratory research in the pharmaceutical industry. Mito [6] discussed the three important elements of Japanese management, including long-term planning which takes into account long-range trends in the corporation's economic, technological and social environment. White and Apple [7] recommended that planning for a material handling system must be initiated early in the product design cycle for maximum benefits; in addition, the planning horizon must extend as far into the future as possible. According to Probert [8], in the development of a long-range planning model for the British Telecommunications Business, emphasis is on the fact that strategic modelling should be primarily motivated by the goal of producing greater insight into strategic issues, thus reducing uncertainty in corporate policy formulation.

Vincent [9] analyzed the three critical aspects of effective long-range corporate planning in the paper industry—an information input of appropriate depth, critical assessment of the firm's relative position and application of advanced evaluation methods. Sethi [10] indicated that there are many factors, external to the environment and internal to the company, that must be assessed before any long-range planning can be conceived or successfully implemented within a multinational corporation. Horwitz [11] dealt with long-range corporate planning, concentrating on the two major issues of capital budgeting—resource allocation and risk taking. In recent surveys [12] done to assess the current practice of strategic planning in U.S. industry: environmental surveillance seems to have become a common element in strategic planning processes due to changes in economic, social, legal and technological environments; competitive factors and technical developments seem to be the two areas in which firms have the most interest.

Fontela [13] found cross-impact analysis to be an efficient tool in establishing the most probable scenario of upcoming events, which is helpful in long-range planning and anticipating opportunities or problems. Kurtulus and Davis [14] applied heuristic solution procedures, based on the categorization process, to the practical problem of project scheduling. Mehrez and Sinuary-Stern [15] presented an overall modelling procedure for the problem of resource allocation to indivisible projects with uncertain outcomes, using a multiattribute utility function. Liberatore and Titus [16] conducted an empirical study on the practice of management science in R&D project management and suggested that R&D managers enlist the support of management scientists in the development of decision support systems for R&D project management, especially for multiproject planning and control.

The above articles highlight the importance of long-range planning in industrial and business projects. Some papers present the application of project planning techniques such as PERT/CPM to scheduling a sequence of planned activities. Other publications deal with scheduling of activities under limited resources and undertaking crash projects based upon cost considerations. However, none of the above works presents an analytical approach to project or program planning based upon the relative economics of a whole integrated plan. This article attempts to determine a sequence of activities that will maximize return subject to economic constraints over a planning horizon.

4. MODEL DEVELOPMENT

Let us assume that a planned program must be formulated and implemented within some established planning horizon. The objective in this particular case is for the program plan, consisting of individual projects to be accomplished, to achieve maximum return subject to other various conditions. Based on this objective it would be beneficial to schedule the projects in such a manner as to yield the highest aggregate present worth return. In doing so, several items arise that need to be addressed. Given that all the projects will be undertaken over a period of time, both inflationary and discounting effects are present. Additionally, individual project precedence is another characteristic that needs to be determined. Project dependence will also establish earliest start and latest finish times. Related to these attributes is the upper limit or desired completion of the entire program. Taking into account uncertainty, a confidence level for program completion is an advisable element for inclusion. In this instance a stochastic relationship can be included limiting latest finish time.

The other item that impacts the scheduling and timing of individual projects is resources. Both the capital and labor required to accomplish each project must be considered. In any one period there are assumed to be ceilings on these resources. Therefore, they should be allocated in such a way as to achieve the objective under timing constraints. It may also be possible to utilize additional capital and labor above initialized limits. Specifically, incremental capital may be obtained through other sources of funding while added labor can be acquired through overtime or contract services. Any of these actions will result in added cost. This added cost would need to be reflected as a penalty against program return. Depending on the significance of any such cost, it may be still beneficial to advance projects if the discounting effects and timing offset these costs.

The scope of the model requirements, as outlined above, consists of maximizing the program return of multiple projects on a present worth basis. This objective is subject to project precedence, completion uncertainty and resource limitations. Additional labor and capital can be provided at a cost penalty to the overall return. The decision to be made is which projects in what period should be implemented under the impending constraints and conditions.

Based on the criteria discussed above, a mathematical model is presented here. This model is a stochastic binary programming formulation that maximizes the total return of multiple projects subject to the various annual cash flow and resource constraints. The one major assumption associated with the model is that all resources are allocated and projects assigned on a periodic basis. If resources are not fully utilized they cannot be carried over to future periods. The model is stated as follows:

maximize

$$Z = \sum_{j=1}^{n} \sum_{t=p_j}^{m_j} \left\{ X_{jt}(1+i)^{-t} \left[\prod_{q=1}^{t} (1+b_q)(f_j c_j) \right] - \alpha_t O_t - g_t A_t \right\}$$

subject to:

$$\prod_{q=1}^{t} (1+b_q)\left(\sum_{j=1}^{n} c_j x_j t \right) - D_t = A_t - E_t, \quad \text{for } t = 1, 2, \ldots, T; \tag{1}$$

$$\sum_{t=p_j}^{m_j} X_{jt} = 1, \quad \text{for } j = 1, 2, \ldots, n; \tag{2}$$

$$P\left(\sum_{t=p_j}^{m_j} tX_{jt} \leqslant Y\right) \geqslant 1 - \gamma, \quad \text{for } j = 1,2,\ldots,n; \tag{3}$$

$$\sum_{j=1}^{n} r_j X_{jt} - H_t = O_t - U_t, \quad \text{for } t = 1,2,\ldots,T; \tag{4}$$

$$O_t - U_t \leqslant \theta H_t, \quad \text{for } t = 1,2,\ldots,T; \tag{5}$$

$$X_{jt} = 0 \text{ or } 1, \quad \text{for } t = 1, 2,\ldots,T, j = 1,2,\ldots,n; \tag{6}$$

$$A_t, E_t, O_t, U_t \geqslant 0, \quad \text{for } t = 1,2,\ldots,T. \tag{7}$$

The notation used in the above model is as follows:

j = project or program designation,
n = total number of projects,
t = year,
T = total number of years in planning horizon,
i = cost of capital,
b_q = escalation rate in year q,
f_j = fractional return on project j,
c_j = cost of project j,
α_t = discounted overtime cost rate for year t,
O_t = overtime hours in year t,
g_t = discounted interest rate on borrowed capital for year t,
A_t = capital borrowed in year t,
D_t = available capital for year t,
E_t = unused capital in year t,
p_j = earliest starting year for project j,
m_j = latest completion year for project j,
Y = latest completion year for the entire program,
γ = probability of not completing the entire program,
r_j = manhours required for project j,
H_t = total manhours available in year t,
U_t = undertime manhours in year t,
θ = overtime as a fraction of regular time

and

X_{jt} = integer decision variable taking the value 1 or 0 (binary) depending on whether project j will be undertaken in year t or not.

The objective function of the model maximizes the present value of total return on all projects based upon the sequence of individual projects as determined by the associated constraints. All of the constraints in the model limit and set criteria which must be met over the planning horizon to accomplish each of the projects.

Constraint (1) limits the available capital in year t and allows other sources of funds to be obtained to meet any shortfalls. However, other sources of funding used to finance projects are penalized against the return in the objective function as interest on the amount used. This condition permits projects to be undertaken sooner provided that the return outweighs the cost of borrowing on a present value basis. Constraint (2) places a timing restriction on the projects, recognizing the precedence relationships among the projects, in terms of earliest starting times and latest completion times. This will permit the timing of the projects with the greatest return, on a present value basis, to be undertaken in the most economical period. Constraint (3) imposes an overall program completion date for all the projects in the plan under stochastic conditions. This provides for a confidence level of completion of all the projects by a specified deadline Y. Constraint (4) limits the internal manhour resources in year t and allows for overtime at the expense of penalizing the return. Additional manhours can be made available subject to the economics of the entire program. Constraint (5) restricts the total amount of overtime available in year t. This limitation is set as a fraction of regular time. Constraint (6) is simply the definition of the primary decision variable, being either 1 or 0, depending on whether a particular project is undertaken in a specified time period or not. Constraint (7) defines the noninteger decision variables as nonnegative.

Numerical example

Suppose seven projects, all part of one major overall program are to be initiated as part of a business plan. The total cost of the program in current dollars is $27.6m. The goal is to implement these seven projects so that the present value of the total return is maximized. In addition, this program must be accomplished under certain internal resource limitations, capable of being extended at extra cost, penalizing the return. The planning horizon for the program, as desired by management, is to be no more than 7 years, if at all possible. Specific project costing, return, manhours, earliest starting and latest completion years are given in Table 1; in addition, the available resources in each year and other associated factors are also displayed.

As qualified in the model previously presented, projects can also be undertaken by utilizing overtime and borrowing from external sources. This will permit the program elements to be implemented sooner. However, the incremental costs of overtime and capital borrowing are to be subsidized totally by the program return in any particular year. Depending upon the financial position of a firm, outside sources of capital will be made available under certain conditions and terms. The capital borrowing rate for each year must be ascertained and forecast, typically based upon future projections of the economy and the firm's financial ratings. For this example a projection of the prime rate was used to represent the capital borrowing rate. Other criteria assigned in the example are 12% assumed cost of capital, 25% overtime capacity to regular time and a 90% probability of completing the entire program no later than the 7th year. Expressing the stochastic constraint as an equivalent certainty constraint resulted in an overall program completion bound occurring in year 6.

A computer solution of the model using SCICONIC/VM software resulted in a feasible solution of the data in Table 1. All seven projects were scheduled to be undertaken over 6 years, yielding a total present value of $1,976,200. Table 2 displays the solution on an annual basis showing the use of resources. Overtime hours were scheduled for the 4th year with capital borrowing occurring in years 3 and 6.

Table 1. Data for the numerical example

Project j	Cost (c_j), $m	Return (f_j)	Manhours (r_j)	Earliest start (p_j)	Latest finish (m_j)
1	3.5	0.15	3250	1	5
2	1.2	0.22	4180	2	6
3	4.8	0.12	2375	1	6
4	2.5	0.28	5515	3	7
5	5.5	0.09	3210	2	7
6	3.0	0.17	4315	3	7
7	7.1	0.11	2150	4	8
	$27.6m		24,995		

Year t	Available capital (D_t), $m	Available manhours (H_t)	Discounted capital borrowing rate (g_t), %	Discounted overtime rate (α_t), $	Escalation rate (b_q), %
1	4.0	4500	12.9	134	3.9
2	4.0	4500	10.6	132	4.5
3	4.0	4500	9.5	128	5.4
4	4.0	5000	8.2	124	5.7
5	5.0	5000	6.2	119	5.9
6	5.0	5000	5.4	114	6.1
7	5.0	5500	4.7	109	6.2
8	5.0	5500	4.1	103	6.3

Table 2. Results for the numerical example

Item	Year 1	2	3	4	5	6
Project j	1	2	5	4	6	3,7
Earliest start (p_j)	1	2	2	3	3	1,4
Latest finish (m_j)	5	6	7	7	7	6,8
Required manhours	3250	4180	3210	5515	4315	4525
Available manhours	4500	4500	4500	5000	5000	5000
Net manhours	(1250)	(320)	(1290)	515	(685)	(475)
Required capital (000), $m	3637	1303	6294	3024	3843	16,174
Available capital (000), $m	4000	4000	4000	4000	5000	5000
Net capital (000), $m	(363)	(2697)	2294	(976)	(1157)	11,174

CONCLUSION

This paper has discussed and presented an integrative approach to program planning utilizing mathematical programming. The model presented consists of realistic characteristics associated with the determination of an optimal sequence of actions. The proposed procedure is not impeded by computational limitations, given sufficient hardware capacity and software efficiency, and is applicable to general planning problems.

This work may be extended by considering additional internal and external attributes. Such items as external short-term investment with unused internal capital, minimal project implementation criteria and project risk can be added.

REFERENCES

1. P. S. Thomas, Scanning strategy: formulation and implementation. *Managl Plann.* **33**(1), 14–20 (1984).
2. C. B. Shrader *et al.*, Strategic planning and organizational performance: a critical appraisal. *J. Mgmt* **10**(2), 149–171 (1984).
3. A. L. Bryant, Manage change with a long-range plan. *Ass. Mgmt* **36**(6), 113, 117 (1984).
4. P. W. Beck, Corporate planning for an uncertain future. *Long Range Plann.* **15**(4), 12–21 (1982).
5. R. A. A. Boschi, Modelling exploratory research. *Eur. J. Opns Res.* **10**(3), 250–259 (1982).
6. T. Mito, The internationalization of Japanese management. *Plann. Rev.* **10**(2), 30–34, 46 (1982).
7. J. White and J. M. Apple, Long-range view, better systems integration needed in designs for material handling. *Ind. Engng* **14**(3), 50–58 (1982).
8. D. E. Probert, The development of a long-range planning model for the British telecommunications business: from initiation to implementation. *J. opl Res. Soc.* **32**(8), 695–719 (1981).
9. J. D. Vincent, Long range planning of paper and board supplies. *Long Range Plann.* **13**(2), 60–66 (1980).
10. N. K. Sethi, Multinational corporate planning. *Ind. Mgmt* **21**(3), 5–9 (1979).
11. R. Horwitz, Corporate planning—a conceptual critique. *Long Range Plann.* **12**(1), 62–66 (1979).
12. R. J. Kudla, The components of strategic planning. *Long Range Plann.* **11**(6), 48–52 (1978).
13. E. Fontela, Industrial applications of cross-impact analysis. *Long Range Plann.* **9**(4), 29–33 (1976).
14. I. Kurtulus and E. W. Davis, Multi-project scheduling: categorization of heuristic rules performance. *Mgmt Sci.* **28**(2), 161–172 (1982).
15. A. Mehrez and Z. Sinuary-Stern, Resource allocation to interrelated risky projects using a multiattribute utility function. *Mgmt Sci.* **29**(4), 430–439 (1983).
16. M. J. Liberatore and G. J. Titus, The practice of management science in R&D project management. *Mgmt Sci.* **29**(8), 962–974 (1983).