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ORIGINAL ARTICLE

An effective modification of the homotopy perturbation method for MHD viscous flow over a stretching sheet

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Abstract In this paper, we propose a powerful modification of the homotopy perturbation method that will accelerate the rapid convergence of series solution. The modified method is employed to solve the MHD boundary-layer equations. The viscous fluid is electrically conducting in the presence of a uniform applied magnetic field and the induced magnetic field is neglected for small magnetic Reynolds number. Similarity solutions of ordinary differential equation resulting from the momentum equation are obtained. Finally, some numerical comparisons among the new modified homotopy perturbation method, the standard homotopy perturbation, the Exact Solution and the Shooting method have been made, which manifest that the modified method is a very accurate and effective algorithm to solve the two-dimensional MHD viscous flow over a stretching sheet.

1. Introduction

The steady two-dimensional laminar flow of an incompressible, viscous fluid past a stretching sheet has become a classical problem in fluid dynamics as it admits an unusually simple closed form solution, first discovered by Crane (1970). Gupta and Gupta (1977) added surface suction (and injection) which models condensation (and evaporation), a uniform transverse magnetic field, when the fluid is electrically conducting, by Andersson (1992). The uniqueness of Crane's solution is also shown (Mcleod and Rajagopal, 1987; Troy et al., 1987). For general values of the parameter the solution was derived by Ariel (1995), though these solutions have been shown to be unstable. The joint effect of viscoelasticity and magnetic field on Crane's problem has been investigated by Ariel (1994). The flow past a stretching sheet need not be necessarily

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two-dimensional because the stretching of the sheet can take place in a variety of ways. It can be three-dimensional, axisymmetric when the stretching is radial and for this problem there is no closed form exact analytical solution. The three-dimensional case was solved by Wang (1984). The effects of viscoelasticity on the axisymmetric flow past a stretching sheet have been analyzed by Ariel for an elastico-viscous fluid (Ariel, 1992) and a second grade fluid (Ariel, 2001). A non-iterative solution for the MHD flow has been given by Ariel (2004) using the technique of Samuel and Hall (1973), Ariel et al. (2006) and Ariel (2009) applied HPM and extended HPM to derive the analytical solution to the axisymmetric flow past a stretching sheet.

Magnetohydrodynamics (MHD) is the study of the interaction of conducting fluids with electromagnetic phenomena. The flow of an electrically conducting fluid in the presence of a magnetic field is of importance in various areas of technology and engineering such as MHD power generation, MHD flow meters, MHD pumps, etc. Most boundary-layer models can be reduced to systems of nonlinear ordinary differential equations which are usually solved by numerical methods. It is however interesting to find approximate analytical solutions to boundary layer problems. Analytical methods have significant advantages over numerical methods in providing analytic, verifiable, rapidly convergent approximation. Various powerful analytical techniques such as Similarity solutions (Banks, 1983; Chaim, 1995), Adomian decomposition method (Wazwaz, 1997, 2006; Awang Kechil and Hashim, 2007, 2008; Hayat et al., 2009; Noor et al., 2010; Ganji and Ganji, 2010), Laplace decomposition method (Khan, 2009; Khan and Faraz, 2010, 2011; Khan and Austin, 2010), homotopy analysis method (Sajid and Hayat, 2009; Sajid et al., 2008), differential transform method (Rashidi, 2009) and variational iteration method (Mirgolbabaei et al., 2009; Noor and Mohyud-Din, 2009; Mohyud-Din et al., 2010; Ganji et al., 2010b) have been proposed for obtaining exact and approximate analytic solutions.. Most of these techniques encounter the inbuilt deficiencies and involve huge computational work. He (He, 1999, 2000, 2004, 2006a, 2008; Yıldırım and Berberler, 2010; Mustafa Inc., 2010) developed and formulated homotopy perturbation method (HPM) by merging the standard homotopy and perturbation. The He's homotopy perturbation method (HPM) proved to be compatible with the versatile nature of the physical problems and has been to a wide class of functional equations; see (Ganji et al., 2009, 2010a; Fathizadeh and Rashidi, 2009; Kelleci and Yıldırım, 2011; Raftari and Yıldırım, 2010; Ates and Yıldırım, 2010; Yıldırım and Sezer, 2010; Xu, 2007; Madani and Fathizadeh, 2010) and the references therein. It is worth mentioning that the HPM is applied without any discretization, restrictive assumption or transformation and is free from round off errors. Unlike the method of separation of variables that requires initial and boundary conditions, the homotopy perturbation method (HPM) provides an analytical solution by using the initial conditions only. The fact that HPM solves nonlinear problems without using Adomian's polynomials can be considered as a clear advantage of this technique over the decomposition method.

The basic motivation of this paper is to propose a new modification of this reliable technique (HPM) for solving MHD boundary-layer equations for stretching sheet problem. Although the modified technique needs only a slight variation from the standard homotopy perturbation method, but the proposed modification will accelerate the rapid convergence of the series solution if compared with the standard HPM, and therefore provides a major progress. While this slight variation is rather simple, it does demonstrate the reliability and the power of the proposed modification. It is important to note that the modified technique works effectively independent of other phenomena in some cases. To the best of our knowledge no attempt has been made to exploit this method to solve MHD boundary layer equation. Also our aim in this article is to compare the results with solutions to the standard HPM, exact and shooting method.

2. A new modified homotopy perturbation method

In order to elucidate the solution procedure of the homotopy perturbation method, we consider the following nonlinear differential equation:

$$L(u) + N(u) = f(r), \quad r \in \Omega \tag{1}$$

with boundary condition

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma, \tag{2}$$

where L is any linear integral or differential operator while N is non-linear differential operator B, Γ is the boundary of the domain Ω and $f(r)$ is a known analytic function. In view of HPM (He, 1999, 2000, 2004, 2006a, 2008; Yıldırım and Berberler, 2010; Mustafa Inc., 2010) was introduced by He, we can construct a homotopy for Eq. (1) as follows:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(r)] = 0, \tag{3}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{4}$$

where $p \in [0, 1]$ is an embedding parameter. If $p = 0$ Eqs. (3) and (4) become

$$L(v) - L(u_0) = 0,$$

and when $p = 1$ both Eqs. (3) and (4) turn out to be the original nonlinear differential equation (1). The Homotopy perturbation method (He, 1999, 2000, 2004, 2006a, 2008; Yıldırım and Berberler, 2010; Mustafa Inc., 2010) admits a solution in the form

$$v(x,t) = p^0 v_0(t) + p^1 v_1(t) + p^2 v_2(t) + \cdots \tag{5}$$

The convergence of the above series is discussed in He (2006b) and the asymptotic behavior of the series is illustrated in He (2006c). Setting $p = 1$ results in the solution of Eq. (5) we get

$$v(x,t) = v_0(t) + v_1(t) + v_2(t) + \cdots \tag{6}$$

For the nonlinear term in (1), let us set $N(u) = h(u)$. Invoking Eq. (5) into Eq. (4) and collecting the terms with the same powers of p, we can obtain a series of equations of the following form:

$$\begin{aligned} p^0 &: \quad L(v_0) = L(u_0), \\ p^1 &: \quad L(v_1) = h_1(u_0) - f(r), \\ p^2 &: \quad L(v_2) = h_1(u_0, u_1), \\ p^3 &: \quad L(v_2) = h_1(u_0, u_1), \\ &\vdots \end{aligned} \tag{7}$$

and so on, where the function $u_1, u_2, u_3, \ldots$, satisfies the following equation:

$$h(u_0 + pu_1 + p^2 u_2 + \cdots) = h_1(u_0) + ph_2(u_0, u_1) + p^2 h_3(u_0, u_1, u_2) + \cdots$$

A new modified form of the HPM can be established based on the assumption that we decompose u_0 into two parts:

$$u_0 = H_0 + H_1, \tag{8}$$

Instead of the iteration procedure, Eq. (7), we suggest the following modification

$$\begin{aligned} p^0 &: \quad v_0 = H_0, \\ p^1 &: \quad v_1 = H_1 + L^{-1}[h_1(u_0) - f(r)], \\ p^2 &: \quad L(v_2) = h_1(u_0, u_1), \\ p^3 &: \quad L(v_2) = h_1(u_0, u_1, u_2), \\ &\vdots \end{aligned} \tag{9}$$

The solution through the new modified homotopy perturbation method highly depends upon the choice of $H_0(x, t)$ and $H_1(x, t)$. We will show how to suitably choose $H_0(x, t)$ and $H_1(x, t)$ by example. This suggestion will facilitate the calculations of the terms $u_0, u_1, u_2, \ldots$ and hence accelerate the rapid convergence of the series solution.

3. Governing equations

The MHD boundary layer flow over a flat plate is governed by the continuity and the Navier–Stokes equations for an incompressible viscous fluid. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the stretching sheet. The induced magnetic field is neglected. The resulting boundary-layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B^2(x)}{\rho}u, \tag{11}$$

where u and v are the velocity components in the x and y directions, respectively. Also ν, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid. There is $B(x)$ equal to $B(x) = B_0 x^{n-1/2}$.

The boundary conditions are given below:

$$u(x, 0) = cx^n, \quad v(x, 0) = 0, \quad \text{and} \quad u(x, \infty) = 0. \tag{12}$$

To solve the problem, momentum and energy equations are firstly nondimensionalized by introducing the following dimensionless variables:

$$\eta = \sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y, \tag{13}$$

$$u = cx^n f'(\eta), \tag{14}$$

$$v = -\sqrt{\frac{c\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta) + \frac{\eta - 1}{\eta + 1}\eta f'(\eta)\right]. \tag{15}$$

Using Eqs. (13)–(15), the governing equations can be reduced to non-linear differential equation where f is a function of the similarity variable (η):

$$f''' + ff'' - \beta f'^2 - Mf' = 0, \quad f(0) = 0, \quad f'(0) = 1 \quad \text{and} \quad f'(\infty) = 0, \tag{16}$$

where

$$\beta = \frac{2n}{n+1}, \quad M = \frac{2\sigma\beta_0^2}{\rho c(1+n)}. \tag{17}$$

4. Application

To demonstrate the effectiveness of the proposed modification and to compare the new modification of the HPM with the standard HPM, we have chosen MHD boundary layer equation (16). In view of the homotopy (3), we construct the homotopy

$$\begin{aligned} &(1-p)(f''' - f_0''') + p(f''' + ff'' - \beta f'^2 - Mf') = 0, \\ &f(0) = 0, \quad f'(0) = 1, \quad f''(0) = \alpha. \end{aligned} \tag{18}$$

Substituting (5) and the initial conditions into the homotopy (18) and equating the terms with identical powers of p we obtain the following set of linear differential equations:

$$\begin{aligned} &p^0: \quad f_0''' = 0, \\ &f_0(0) = 0, \quad f_0'(0) = 1, \quad f_0''(0) = \alpha, \end{aligned} \tag{19}$$

$$\begin{aligned} &p^1: \quad f_1''' + f_0''' - Mf_0' + f_0 f_0'' - \beta f_0'^2 = 0, \\ &f_1(0) = 0, \quad f_1'(0) = 0, \quad f_1''(0) = 0, \end{aligned} \tag{20}$$

$$\begin{aligned} &p^2: \quad f_2''' - Mf_1' - 2\beta f_0' f_1' + f_1 f_0'' + f_0 f_1'' = 0, \\ &f_2(0) = 0, \quad f_2'(0) = 0, \quad f_2''(0) = 0, \end{aligned} \tag{21}$$

$$\begin{aligned} &p^3: \quad f_3''' - Mf_2' - \beta(f_1'^2 + 2f_0' f_2') + f_1 f_1'' + f_0 f_2'' + f_2 f_0'' = 0, \\ &f_3(0) = 0, \quad f_3'(0) = 0, \quad f_3''(0) = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} &p^4: \quad f_4''' - Mf_3' - 2\beta(f_3' f_0' + f_1' f_2') + f_3 f_0'' + f_0 f_3'' + f_1 f_2'' + f_2 f_1'' = 0, \\ &f_4(0) = 0, \quad f_4'(0) = 0, \quad f_4''(0) = 0, \end{aligned} \tag{23}$$

by solving Eqs. (19)–(23):

$$f_0 = \frac{1}{2}\alpha\eta^2 + \eta, \tag{24}$$

$$f_1 = \frac{(2\beta - 1)}{120}\alpha^2\eta^5 + \frac{(M - 1 + 2\beta)}{24}\alpha\eta^4 + \frac{(M + \beta)}{6}\eta^3, \tag{25}$$

$$\begin{aligned} f_2 = {} & \frac{(20\beta^2 - 40\beta - 11)}{40320}\alpha^3\eta^8 \\ & + \frac{(11 + 10M\beta + 20\beta^2 - 8M - 32\beta)}{5040}\alpha^2\eta^7 \\ & + \frac{(3 - 12\beta - 8M + 10\beta^2 + 10M\beta + M^2)}{720}\alpha\eta^6 \\ & + \frac{(M^2 + 2\beta^2 - 2\beta - 2M + 3M\beta)}{120}\eta^2, \end{aligned} \tag{26}$$

$$\begin{aligned} f_3 = {} & \frac{(600\beta^3 - 1596\beta^2 - 1398\beta - 375)}{39916800}\alpha^4\eta^{11} \\ & + \frac{(4800\beta^3 - (12868 - 2400M)\beta^2 + (11184 - 4368M)\beta - 3000 + 1944M)}{5040}\alpha^3\eta^{10} \\ & + \frac{(16800\beta^3 - (39760 - 16800M)\beta^2 + (30688 - 30576M + 2352M^2)\beta - 7222 + 13608M - 2184M^2)}{20321280}\alpha^2\eta^9 \\ & + \frac{(26880\beta^3 - (55776 - 40320M)\beta^2 + (34608 - 65856M + 14112M^2)\beta - 5040 + 26880M - 13104M^2)}{13547520}\alpha\eta^8 \\ & + \frac{(16800\beta^3 - (26880 - 33600M)\beta^2 + (13440 - 43680M + 18480M^2)\beta)}{8467200}\eta^7, \end{aligned} \tag{27}$$

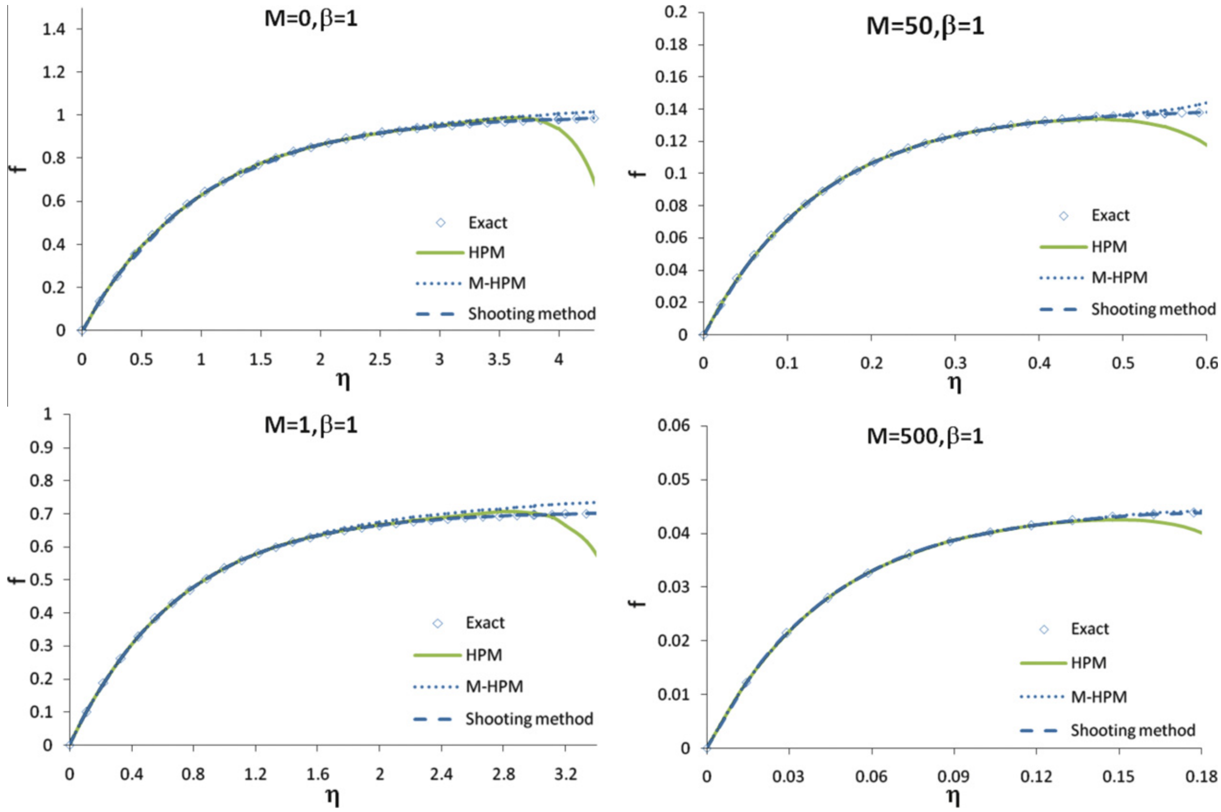

Figure 1 The results of f obtained for employing HPM and modified HPM as well as the numerical method suggested by shooting method.

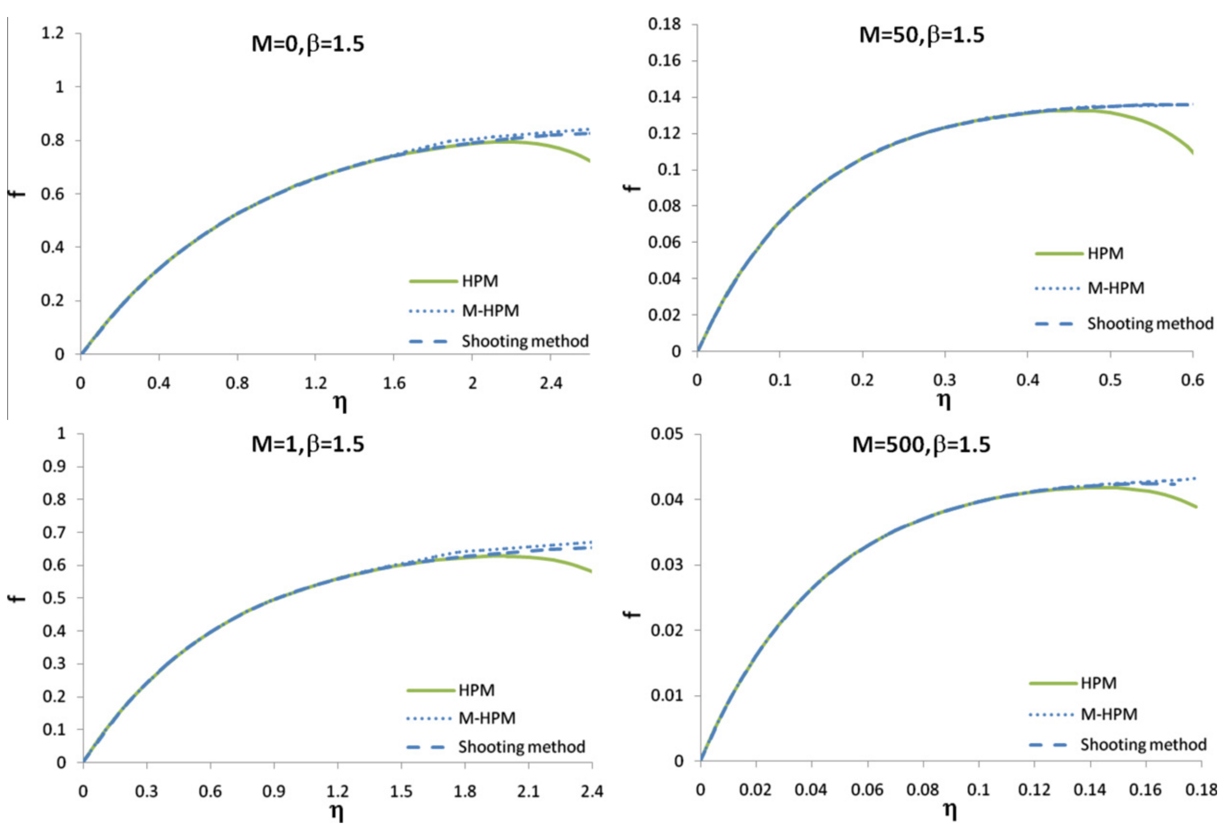

Figure 2 The variation of f for different values of M and $\beta = 1.5$ approximated by HPM, modified HPM and shooting method.

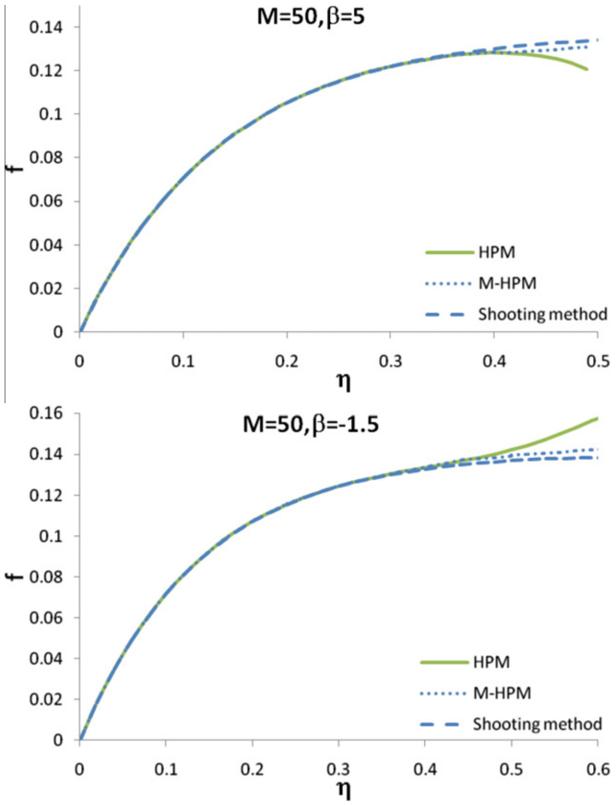

Figure 3 The variation of f for different values of β and $M = 50$ approximated by HPM, modified HPM and shooting method.

$$f = f_0 + f_1 + f_2 + f_3 + \cdots,\quad p \to 1. \tag{28}$$

To select $f''(0) = \alpha$ and η_∞, we begin with some initial guess value of α and draw f'. The solution process is repeated with another value of α until f' receive to zero. Also, η_∞ is obtained from highest value of η in f'. According to the new modified homotopy perturbation method, we first set

$$f_0 = \frac{1}{2}\alpha\eta^2 + \eta. \tag{29}$$

As suggested before we split zeroth iteration f_0 into two parts

$$H_0(\eta) = \eta \tag{30}$$

and

$$H_1(\eta) = \frac{1}{2}\alpha\eta^2. \tag{31}$$

According to the iteration algorithm, Eq. (9), we obtain

$$f_0 = \eta, \tag{32}$$

$$f_1 = \frac{1}{2}\alpha\eta^2 + \frac{1}{6}(M+\beta)\eta^3, \tag{33}$$

$$f_2 = \frac{1}{120}(M^2 + 3M\beta + 2\beta^2 - 2M - 2\beta)\eta^5 + \frac{1}{48}(2M - 2 + 4\beta)\alpha\eta^4, \tag{34}$$

$$f_2 = \left(\frac{M^3}{5040} - \frac{M^2}{504} - \frac{M}{630} - \frac{13M\beta}{2520} + \frac{11M^2\beta}{5040} + \frac{M\beta^2}{252} - \frac{\beta^2}{315} + \frac{\beta}{630}\right)\eta^7 + \left(\frac{M^2}{720} - \frac{M}{90} + \frac{M\beta}{72} + \frac{\beta^2}{72} - \frac{\beta}{60} + \frac{1}{240}\right)\alpha\eta^6 + \left(\frac{\beta}{60} - \frac{1}{120}\right)\alpha^2\eta^5, \tag{35}$$

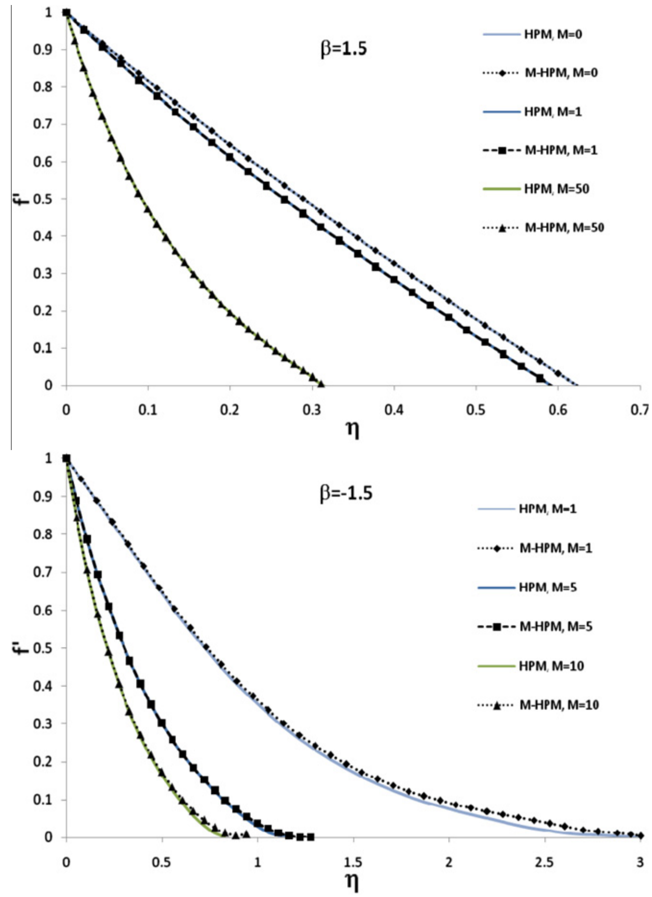

Fig. 4 Comparison of the solution (f') obtained by a new modified HPM and the standard HPM solution for different values of M and β.

Table 1 Comparison of the values of $f''(0)$ obtained by HPM, the modified HPM and the exact solution.

β	M	HPM	M-HPM	Exact solution
1	0	−1	−1	−1
	1	−1.41421	−1.41421	−1.41421
	5	−2.44948	−2.44948	−2.44948
	10	−3.31662	−3.31662	−3.31662
	50	−7.14142	−7.14142	−7.14142
	100	−10.0499	−10.0499	−10.04987
	500	−22.383	−22.383	−22.38302
	1000	−31.6386	−31.6386	−31.63858

$$f = f_0 + f_1 + f_2 + f_3 + \cdots,\quad p \to 1. \tag{36}$$

5. Results and discussion

Eq. (16) were solved analytically using the new modified HPM, standard HPM and numerically using the shooting method. It was shown in Figs. 1–3 the analytical, the exact solution and the numerical solution of f for different values of M and β. Furthermore, the comparison between the MHPM and the standard HPM solutions of f' was performed (Fig. 4). A very

Table 2 Variation in $f''(0)$ at the different values of β and M obtained by HPM, modified HPM and shooting method.

β	M	HPM	M-HPM	Shooting method	β	M	HPM	M-HPM	Shooting method
1.5	0	−1.1486	−1.1547	−1.1547	5	0	−1.9025	−1.9098	−1.9098
	1	−1.5252	−1.5252	−1.5252		1	−2.1529	−2.1528	−2.1528
	5	−2.5161	−2.5161	−2.5161		5	−2.9414	−2.9414	−2.9414
	10	−3.3663	−3.3663	−3.3663		10	−3.6956	−3.6956	−3.6956
	50	−7.1647	−7.1647	−7.1647		50	−7.3256	−7.3256	−7.3256
	100	−10.0664	−10.0776	−10.0776		100		−10.1816	−10.1816
	500	−22.3901	−22.3904	−22.3904		500		−22.4425	−22.4425
	1000	−31.6432	−31.6438	−31.6438		1000		−31.6806	−31.6806

good agreement was illustrated between the results obtained by the new modified HPM, HPM, exact solution and the numerical results for all values of η. The result of modified HPM has better than HPM in large amount of M.

Tables 1 and 2 clearly elucidate that present solution method namely modified HPM shows excellent agreement with the exact solution, standard HPM and numerical method and more convergent as compared with standard HPM. This analysis shows that MHPM suits for MHD viscous flow problems.

6. Concluding remarks

In this work, we carefully proposed an efficient modification of the HPM that accelerate the rapid convergence of series solution. A newly modified HPM was used to find analytical solutions of magnetohydrodynamics boundary-layer equation. Comparison of the present solution is made with the HPM, exact and shooting method solutions. An excellent agreement is achieved. The proposed method is employed without using linearization, discretization or transformation. It may be concluded that the MHPM is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. The method gives more realistic series solutions that converge very rapidly in physical problems.

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