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Identifying the secondary school students' misconceptions about functions

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Abstract

It is very important to identify and correct the mistakes that students make about mathematical concepts. The purpose of this study is to detect the secondary school students' misconceptions about functions in the 2009-2010 academic year .Research data was collected 61 students, included 9th,10th and 11th grade students selected randomly, in Erzincan (2009-2010) academic year. Data collection tools consist of knowledge test and interviews. Frequency tables and percentages were used by analyzing students' explanations as answers to questions on the function information test. Pearson correlation was chosen to state the meaningful difference among the questions, if there is any, on the Function information test also included four different categories. Student, being believed to be in error, were interviewed with the help of data obtained from Function information test. As a result of the interview, it is most commonly observed that students make mistakes to understand whether a graphic is a function graphic or not, and about the demonstration of the table and function especially on the function information test.

Keywords: mathematical concepts, functions, misconceptions;

1. Introduction

It is considerably important to identify and eliminate mistakes made by students about mathematical concepts. That is because when we know about students' prior knowledge and their cognitive features that come along with them when we are educating them, this can assist us in knowing students more easily and having information regarding where they may make mistakes and what kind of mistakes they may make and how they may think. When we review the current literature, we come up different definitions on misconception. According to Murray, Schultz, Brown and Clement (1990), misconception can be observed at every age and educational level. Zembatdefines misconception as a perception or conception that is not compatible with the opinion commonly agreed on by experts on a particular subject (Zembat, 2008a, p.2). Smith, diSessa and Roshelle (1993, p.119) define misconception as the students' conception that produces a systematic pattern of errors. Michael (2002) defines misconception as the inconsistency between the concept that we want students to learn and the mental model that they build in their minds. In other words, misconception is the information that contradicts with currently accepted scientific theories (Clement, 1993). In a general aspect, misconception is not a mistake that can be considered simple or innocent and

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overlooked by teachers. It has an obstinate structure that does not give up insisting. The fact that a person having a misconception does not immediately accept, and instead, deny his/her mistake when he/she is confronted with his/her mistake or mistakes is one of the most evident features of misconceptions. According to Clement (1993), mistakes that are made are not random. Mistakes made systematically by students are considerably different from ordinary calculation errors and are in fact a conception and a cognitive structure that reveals (produces errors) and control itself (Oliver, 1989). In other words, mistakes made by students are only the sections that can be observed by us and the real essence is the cognitive sections that cause these mistakes and that we cannot fully understand from where and how they emerge (Nesher, 1987).

The concept of function comes first among the most important concepts that distinguish modern mathematics from classical mathematics (Kleiner, 1989). Students of all levels from elementary education to university somehow come across the concept of function and perform activities about this concept. For that reason, teaching the concept of function plays a key role in mathematics courses. According to Froelich,Bartkovich and Foerreester (1991), the concept of function, which is included in the content of algebra, analysis and geometry, is probably one of the most important elements of mathematics.

In our country, the introduction to the concept of functions is made by taking set theory as basis. The concept of function is given as a special relation after the concepts of ordered pairs, Cartesian products and relations have been given. After given orally, the definition of function is explained visually with a set matching diagram. Examples related with different representations of the definition of function, set matching diagrams, sets of ordered pair, equations and graphs are presented. Function machine, which represents input-output conditions in the new mathematics teaching program published in 2005 by the Ministry of National Education, is also utilized to show examples and the product. Studies in the field of mathematics teaching generally focus on the hardships experienced in teaching and learning the concept of function and the reasons for these hardships. Becker stated that the concept of function is one of the few concepts that form the school mathematics, and few concepts are misunderstood as much as the concept function or are not fully understood (Becker, 1991). Furthermore, it must also be expressed that students generally assign personal meanings to the concept of function and use idiosyncratic function definitions. In his research, Walton (1998) detected that many students make comparable definitions of function but they are not aware of what these definitions represent. Again in a similar study, it was set forth that the students cannot understand the concepts that are featured especially within operations in analysis courses (Gravemeijer & Doorman, 1999). Since, every learned concept in mathematics is closely related with previous or upcoming concepts (Altun, 2001), a difficulty experienced in learning a certain concept causes difficulties in learning many further concepts and paves the way for misconceptions. The concept of functions holds an important place in this regard. For that reason, we have tried to identify students' misconceptions about functions.

2. Method

2.1. Aim of the Study

This study aims to identify the function-related misconceptions of secondary school students who study in 2009-2010 school year:

- What are the misconceptions of secondary school students about the concept of functions?
- What are the relationships among misconceptions of secondary school students about the concept of functions according to the questions?

2.2. Sample

Data of the study have been collected from unbiasedly selected 9th, 10th and 11th grade students who study in (2009-2010) school year in Erzincan Province. 61 students have participated in the research.

2.3. Data Collection Tool

Data collection tools of the research consist of a function knowledge test and interviews. Utilized function knowledge test is composed of a total of 14 questions. Among these 14 questions, 6 questions were prepared by

Vinner & Drefyus (1989); 7 questions were prepared by Becker (1991) and one question was prepared by İşleyen (2005). This test is composed of four different dimensions (4 questions on recognizing the functions from graphical data; 4 questions on recognizing the functions from verbal expressions; 3 questions on table representation; and 3 questions on recognizing the functions from algebraic expressions). Students, who are believed to have misconceptions, have been detected in accordance with the data obtained from the function knowledge test, and these students have been interviewed. Interview data have been collected via face-to-face interviews which lasted about 20-30 minutes with each student in a private room. It is observed that the internal consistency coefficients (Cronbach's Alpha) of all items of the utilized function knowledge test are 0.67.

2.4. Analysis of Data

Answers given in the function knowledge test with explanations by students have been analyzed and examined. Answers given in the test by students have been graded as 2, 1 and 0 in terms of correctness. As for the answers given to each question by students, 2 points have been given if both the answer given to this question and its explanation were correct; 1 point has been given if the answer was correct but its explanation was wrong; and 0 point has been given if both the answer and its explanation were wrong. Analysis of the data has been performed with SPSS 12 package program. Pearson Correlation has been utilized to see whether there is a significant relationship among the questions in four different categories that are featured in the knowledge test. In line with the obtained data, the opinions of the course teachers have been taken using focus group interview method. Students, who are believed to have misconceptions, have been detected in accordance with the data obtained from the function knowledge test, and these students have been interviewed. Interview data have been collected via face-to-face interviews which lasted about 20-30 minutes with each student. Mutual questions and answers have been noted down during the interviews collectively by the assistant researcher and the researcher.

3. Findings

Students have been asked questions about the different representations of functions in the function knowledge test. Answers given to these questions have been evaluated in 4 different categories. Findings regarding students' recognizing whether or not a given graph is a function are presented in Table 1 below. The answers have been assessed as wrong when the explanation part was mathematically wrong even though the student chose the correct option.

Table 1. Frequency Distributions Regarding Students' Recognizing Functions from the Graphs

Question	Number of Students	Correct Answer	Wrong Answer	Unanswered	Correct Answer Percentage (P)
Question 1	61	40	13	8	65.57%
Question 2	61	28	11	22	45.90%
Question 3	61	21	8	32	34.42%
Question 4	61	14	21	26	22.95%
Total	244	103	53	88	42.21%

When Table 1 is examined, we see that 22.95% of the students could only correctly answer Question 4. This question is a little different from ordinary graph questions and was prepared in a way that it can be solved by the students who conceptually understand the definition of the concept of function. 4 students, who answered this question correctly, firstly combined the lines given in the graph and then stated that the graph drawn by them would be a function. It has been detected that these students had misconception of "continuity" which was set forth by the study conducted by Markovits, Eylon and Bruckhimmer (1986).

Especially in Question 4, some students chose a point among the points which were stated differently from the given points, and concluded that the given graph does not determine a function since this point has no appearance. On the other hand, some students answered that there won't be a function since the given graph is not continuous. It can be considered that this condition results from the fact that students always perceive the function as a continuous curve or line. In view of the answers given by the students to the first four questions in the function knowledge test, it has been observed that these students have difficulty in matching. The first one of these difficulties is the misconception that results from perceiving the requirement of functions to be one-to-one as the function requirement. The second difficulty is the misconception called symmetry confusion that results from the fact that students cannot tell the dependent variable from the independent variable. Students, who are included in the sample of our study, showed the tendency to combine the points given in the graph in Question 4 with a curve or line.

Table 2. Frequency Distributions Regarding Students' Recognizing Functions from Verbal Expressions

Question	Number of Students	Correct Answer	Wrong Answer	Unanswered	Correct Answer Percentage (P)
Question 5	61	3	9	49	4.91%
Question 6	61	20	14	27	32.78%
Question 7	61	2	8	51	3.27%
Question 14	61	24	9	28	39.34%
Total	244	49	40	155	20.08%

When Table 2 is examined, extreme difference is observed among the correct answers of the students in terms of ratio (for instance, while the correct answer percentage of Question 6is 32.78%, the correct answer percentage of Question 7is 3.27%). Since Question 7 featured a piecewise function, the students could not think of such a function. That is because the students could not perceive the piecewise function as a function. While some students defined the constant function in a very different way in Question 6, nearly all of the students who gave wrong answers confused constant function with unit function. While saying that there is such function in the test, they stated that $f(x) = x$ in the explanation part. In Question 7, students always looked for a function according to one formula and stated that such function could not exist when they could not find the function they were looking for.

5 of 9 students, who gave wrong answers in Question 14, had misconception and confused constant function with unit function just like in Question 6. In other words, they perceived the features of the unit function as the features of constant function. It has been observed that the students overlooked the expression "every number other than zero". For that reason, it has been observed that many students claimed such function could not be found since $0^2 = 0$. Students insisted in their conclusions during the performed interviews. Some students stated that it was not clear what was asked in this question.

Table 3. Frequency Distributions Regarding Students' Recognizing Functions from Table Data

Question	Number of Students	Correct Answer	Wrong Answer	Unanswered	Correct Answer Percentage (P)
Question 8	61	18	18	25	29.50%
Question 9	61	40	5	16	65.57%
Question 10	61	45	5	11	73.77%
Total	183	103	28	52	56.28%

When Table 3 is examined, it is observed that student success is lower especially in Question 8than the other questions. Question 8 and Question 9 are such questions that can be answered using the same logic. Although Question 9 can be written in $f(x) = x + 2$ format, Question 8 was prepared in a way that it could not be represented with such algebraic formula. Since students always look for a formula, they think that the given expression is not a function when they cannot find such formula. In Question 8, many of the students stated that there could not be a function since no formula was found. In Question 10, 3 students stated the fact that some elements could not be matched in the range violated the function condition while one student claimed the fact that two elements in the domain were matched with one element violated the function condition. It can be asserted that these three students had symmetric misconception. One student stated that there is no domain in Question 8, Question 9 and Question 10.

Table 4. Frequency Distributions Regarding Students' Recognizing Functions from Algebraic Data

Question	Number of Students	Correct Answer	Wrong Answer	Unanswered	Correct Answer Percentage (P)
Question 11	61	41	7	13	67.21%
Question 12	61	10	16	35	16.39%
Question 13	61	19	16	26	31.47%
Total	183	70	39	74	38.25%

When Table 4 is examined, it is observed that 8 of the students who gave wrong answers to Question 12 perceived one-to-one concept as the function condition. It is understood that the misconception found here is a second type symmetric misconception which is related with matching. Moreover, 5 students confused function continuity with function definition. It has been also observed that there are students who confused function continuity with function definition in Question 13.One of the most interesting answers given to Question 13 is the one saying "$f(x)=2x^3-5x+17$ expression is torn in x^2". It can be considered that the student, who gave this answer, perceived continuity as the decreasing powers of polynomial.

Table 5. Table of Pearson Correlation among the Groups

	Graph	Verbal	Table	Algebraic
Graph	1			
Verbal	0.285*	1 .		
Table	0.413**	0. 361**	1	
Algebraic	0.14	0.374**	0.323*	1

*p=0.05 **p=0.01 and N=61

When the Pearson Correlation of the performed function knowledge test has been examined, no significant relationship has been found between recognizing whether or not there is a function from graph and algebraic expressions although there is a significant relationship between other variables and the information collected regarding whether or not the given expressions are functions in view of graphs, verbal data, tables and algebraic data.

4. Results and Suggestions

When the answers given to the questions in the function knowledge test have been analyzed and reviewed, it has been observed that students have misconceptions about the concept of function. Similarly, the study of Polat and Şahiner (2007) also supports this result. In view of the performed interviews, students had some misconceptions like failure to understand whether or not the given graphs are function graphs, failure to correlate verbal expressions with the concept of function, experiencing confusion regarding whether or not the given algebraic expressions are functions. According to the result of the function knowledge test, it is clear that we cannot conclude whether or not the concept of function was conceptually learned by the students. These answers need to be examined in terms of conceptual and operational learning. Many conducted searches were based on these foundations. For instance, students mostly reached the solution by drawing vertical lines in finding whether or not a given graph represents a function graph in the conducted function knowledge test. If the vertical lines drawn towards this graph cross the graph on a single point, the given graph is a function graph. If the vertical lines drawn towards this graph cross the graph on more than one point, the given graph is not a function graph. If this is perceived as a rule, no further achievement can be made beyond operational learning. That is because the expression "each element in the domain is matched with only one element in the range" is the indication of this condition when the function definition is taken into account. Every student having conceptual learning must understand whether or not the given graph represents a function by drawing vertical lines instead of horizontal lines. Otherwise, it clear that memorizing this condition as a rule will not earn the student much knowledge. For that reason, whether or not students comprehend their correct answers must be dwelled upon instead of the correct answers given by them in a test. Finding the most suitable and the most explanatory examples for students is important for them to focus on the most abstract and the most general form of the concept. We can make use of technological facilities in order to eliminate problems caused in students' concept representations by the representations used in the National Education Program; form a better comprehension; and in order for students to go beyond forming piles of samples for algebraic expressions and graphs and form richer comprehensions. Consequently, it should not be forgotten that the real purpose is to make students learn a subject conceptually regardless of the utilized teaching method. Misconceptions observed in the concept of function should be monitored by mathematics teachers and instructors, and they should prevent their students from experiencing such kind of misconceptions in the subjects that will be taught.

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